THE NEWTONIAN FLUID

José Mª Franco and Pedro Partal
Dept. Ingeniería Química. Facultad de Ciencias Experimentales. Universidad de Huelva. Campus de "El Carmen". 21071 Huelva. Spain

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Contents

Summary

This chapter reviews the definition of a Newtonian fluid and main topics associated to the Newtonian fluid behavior, including the estimation of Newtonian viscosities. Once revised the Newton law of viscosity as originally stated in the *Principia Mathematica*, the concept of viscosity is introduced by means of the classical simple shear experiment and, afterwards, the three-dimensional form of Newton's law is analyzed. The mass and momentum conservation principles have been discussed in some detail. The study of these equations has been carried out from both macroscopic and microscopic points of view. The macroscopic approach results in relatively simple integral equations which are frequently applied to solve input/output macroscopic flow problems. On the contrary, the differential equations, like for instance the classical Navier-Stokes equation, were deduced from the microscopic approach. In this case, differential expressions have been directly obtained from the integral forms by applying the divergence theorem and the Leibnitz rule. Finally, existing methods in the literature to predict the viscosity of Newtonian fluids (gases and liquids) have been revised by emphasizing the influence of pressure and temperature on this property. In the case of gases, some molecular theories have been discussed. In particular, the treatment based on the elementary kinetic theory of gases has been detailed in order to better illustrate the concept of gas viscosity. In the case of liquids, some experimental correlations or

straightforward methods to evaluate the viscosity as a function of temperature and/or pressure have been considered.

1. Introduction

The great majority of most common fluids (liquids and gases) such as water, organic solvents, oils, air, steam, nitrogen or rare gases are Newtonian in wide temperature and pressure ranges. This fact justifies that classical textbooks on Fluid Mechanics are devoted to the study of Newtonian fluid dynamics. However, it must be taken into account that the Newtonian flow behavior, together with the Hookean elastic solid, represents the simplest extreme situation from a rheological point of view and frequently is considered a very particular case in the wide spectrum of rheological behaviors. Certainly, the three-dimensional form of Newton’s law is one of the multiple constitutive equations which may be used to describe the flow and deformation of matter (Larson, 1988) but it is the most applied to solve flow problems. Thus, as Barnes (2002) pointed out in the preface of his book, most of the flow problems found in industrial applications deals with a proper estimation or accurate measurement of viscosity. It is evident that the Newtonian flow behavior is mainly linked to the viscosity concept. In contrast to that found for non-Newtonian fluids, the Newtonian viscosity is a physical property which is only a function of pressure and temperature.

However, even considering the Newtonian fluid behavior, the effect of viscosity appears to be the main difficulty to solve the equations of motion, especially when they must be applied to complex geometrical configurations. In fact, in spite of the much earlier formulation of the Newton’s law of viscosity, up to the end of the nineteenth century, when Navier and Stokes included the viscous friction term in the equation of motion and Ludwig Prandtl formulated the boundary-layer theory, theoretical attempts to model the different flow problems were based on the assumption of a frictionless or inviscid fluid, which evidently led to unrealistic predictions. The difficulty extremely increases in the case of turbulent flows. In this chapter, discussions have been limited to the laminar flow regime in which the viscosity concept makes sense. Otherwise, in the turbulent flow regime, the concept of eddy viscosity must be introduced, which is out of scope here.

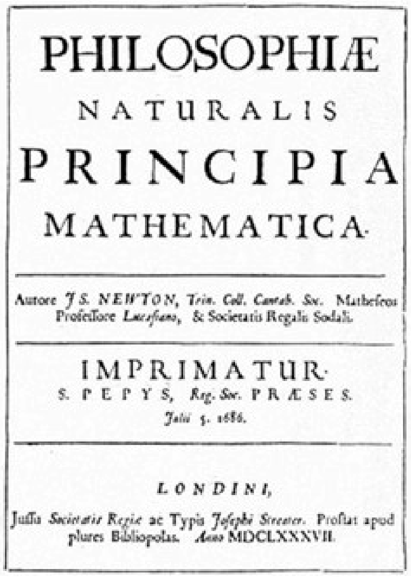
Finally, it must be emphasized that classical Fluid Mechanics is based on the concept of continuum or, in other words, the fluid is usually considered a continuous distribution of matter. However, the concept of viscosity is inevitably linked to the fundamentals of molecular momentum transport and, therefore, the molecular scale must be occasionally considered to better understand the concept of viscosity.

2. Newton’s Law of Viscosity

2.1. The *Principia Mathematica*

In 1687, Isaac Newton in the *Philosophiae Naturalis Principia Mathematic*a (Figure 1), probably the greatest and more influencing work in the history of science, paid attention to fluid flow and shear, among many other physical phenomena. In the second book of the *Principia*, in Section IX *“The circular motion of fluids”*, the following hypothesis is enunciated in the original Latin language:

Resistentian, quae oritur ex defectu lubricitatis partuim fluidi, caeteris paribus, proportionalem esse velocitati, qua partes fluidi separantur ab invicem.



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.

Figure 1. Title page of the first edition of Newton's *Philosophiae Naturalis Principia Mathematica*, located at the Wren Library in the Trinity College, Cambridge.

This hypothesis in the first English edition, translated by Andrew Motte in 1729, appears as:

The resistance arising from the want of lubricity in the parts of a fluid, is, others things being equal, proportional to the velocity with which the parts of the fluid are separated from one another.

Later on, in other English translations the expression *want of lubricity* was replaced by *lack of slipperiness.* In summary, in Section IX of the *Principia*, Newton postulated the idea of what a viscous fluid is, illustrated with some basic experiments or theorems (propositions LI and LII). Thus, we can assume that *the lack of slipperiness* (o*r the want of lubricity*) *in the parts of a fluid* is what we associate with the term "viscosity", *the resistance* is the stress and *the velocity with which the parts of the fluid are separated from one another* clearly refers to the velocity gradient. Therefore, as Barnes et al. (1989) pointed out, the viscosity concept is related to the measure of internal friction and the resistance to flow since 1687.

2.2. The Viscosity Concept

The viscosity concept as it was formulated by Newton arises when a layer of fluid is made to move in relation to another layer. Thus, the greater the internal friction, the greater the stress (force per unit area) required to promote this movement. The viscosity concept is generally illustrated by means of the simple shear experiment (Figure 2). In the simple shear experiment the fluid is confined between two large parallel plates of area A, in order to ignore edge effects, separated by a distance Y. At a certain time, a force, F, is exerted on the lower plate so that it moves relative to the other with a constant velocity, V. Immediately, assuming that the flow is laminar, the layers of fluid closer to the lower plate start to flow and a transient (time-dependent) velocity profile is established. Afterwards, if the force, F, is kept constant to maintain the motion of the lower plate, the steady-state velocity profile, $u_x(y)$, is attained that, in this particular case, is linear:

$$\frac{u_x}{V} = \frac{y}{Y} \qquad (1)$$

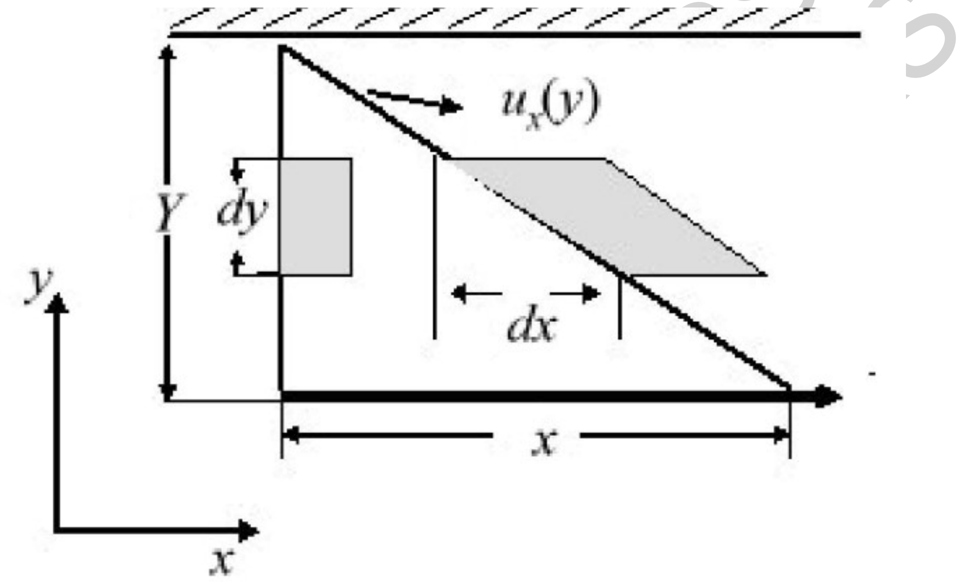


Figure 2. The simple shear experiment.

An intuitive analysis of this experiment suggests the basic idea that the force per unit area applied on the lower surface is proportional to velocity, V, and inversely proportional to the distance Y:

$$\frac{F}{A} = \mu \frac{V}{Y} \qquad (2)$$

where the proportionality constant is the viscosity. Since the velocity profile between the two plates is linear, every infinitesimal segment is represented by the same relationship. Therefore, Eq. (2) may be written in the differential form

$$\tau_{yx} = \mu\left(-du_x / dy\right) \qquad (3)$$

given that the force is exerted to the area perpendicular to the y axis in the positive direction of the x axis and, consequently, the only component of the stress tensor is τ_{yx}. Here, we have applied the criteria followed in the book of Bird et al (2002) of considering positive the stress in the directions of the coordinate axis and, consequently, the velocity gradient is negative, i.e. u_x decreases as y increases. It must be noticed that, according to Bird et al (2002), the shear stress acting on every differential volume of the fluid can be interpreted as the flux (flow per unit area) of momentum in the positive y direction, since the layers of fluid in motion impart a certain momentum to the adjacent layers of lower velocity. In this sense, the negative velocity gradient can be considered the driven force for momentum transport.

Defining the strain as

$$\gamma = dx/dy = x/Y \qquad (4)$$

the strain rate for simple shear can be easily related with the velocity gradient

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{d\left(dx/dy\right)}{dt} = \frac{d\left(dx/dt\right)}{dy} = \left(-\frac{du_x}{dy}\right) \qquad (5)$$

and Eq. (3) can be written in a more popular form for rheologists in terms of the strain rate

$$\tau_{yx} = \mu\,\dot{\gamma} \qquad (6)$$

Since Eq. (3) expresses mathematically the basic ideas pointed out in the *Principia*, it is called the Newton's law of viscosity which is frequently used to define the viscosity as the ratio between the shear stress and the velocity gradient. This means that a high viscous fluid requires a higher stress to achieve the same velocity gradient than a low viscous fluid. Then, μ is a material property which, in words of Newton, takes into account the want of lubricity (or the lack of slipperiness) in the parts of a fluid. Fluids which obey this law are called Newtonian. The viscosity of Newtonian fluids is affected by temperature and pressure. The effect of pressure and temperature will be discussed in Section 4.

$$\mu = \mu\,(P,T) \qquad (7)$$

Units of viscosity in the SI and cgs systems are Pa s (Pascal second) and Poise (1P=1 g cm^{-1} s^{-1}), respectively. Another common unit employed is the centipoise (1 cP = 10^{-2} P= 1 mPa s) because the viscosity of water at 20ºC is approximately 1cP. The viscosity values at 20ºC for more common Newtonian fluids (gases and liquids) are compared in the book of Barnes (2002). Thus, for instance, it is interesting to point out that some of the most usual gases like air, steam or ammonia have a viscosity of around 100 times inferior than water or, on the contrary, glycerin and honey have a viscosity of around 1500 and 10^{4} times higher than water, respectively.

Going back to the concept of momentum transport, Eq. (3) is frequently found for incompressible fluids in the form

$$\tau_{yx} = \nu\left(-\frac{d\,\rho u_x}{d\,y}\right) \qquad (8)$$

where $\nu = \mu/\rho$ is called the kinematic viscosity, which, by analogy with the heat and mass transport, is considered the momentum diffusivity (Bird et al, 2002; Welty et al., 2001). The usual unit employed for the kinematic viscosity is the centistoke (1 cSt = 1 $mm^2s^{-1} = 10^{-6}\ m^2s^{-1}$; 1 Stoke=1 cm^2s^{-1}).

2.3. The 3-D Newton's Law

We have shown that Newton's hypothesis about *"The circular motion of fluids"*, stated in the *Principia* in 1687, is in agreement with the one-dimensional relationship between stress and shear rate deduced from the simple shear experiment. However, it was not until 1845 when Stokes, based on the works of Navier (1821) and Poison (1831), formulated the three-dimensional mathematical form of Newton's law. The 3-D generalization of Newton's law of viscosity is, in fact, the set of nine relations between stresses and velocity gradients, which are usually called the Stokes relations:

$$\overline{\overline{\tau}} = \mu\left[\overline{\overline{\nabla u}} + \overline{\overline{\left(\nabla u\right)^{\mathrm{T}}}}\right] + \left(\frac{2}{3}\mu - \kappa\right)\cdot\left(\vec{\nabla}\cdot\vec{u}\right)\cdot\overline{\overline{I}} \qquad (9)$$

where $\overline{\overline{\nabla u}}$ is the velocity gradient tensor, $\overline{\overline{\left(\nabla u\right)^{\mathrm{T}}}}$ is the transpose of the velocity gradient tensor, $\vec{\nabla}\cdot\vec{u}$ is the divergence of velocity, $\overline{\overline{I}}$ is the identity tensor and κ is the so called "dilatational" viscosity. κ takes into account the contribution to stress as a consequence of density changes during the flow. It is demonstrated that κ is zero for ideal monoatomic gases. On the other hand, as detailed in Section 3.1., for incompressible fluids

$$\overline{\nabla}\cdot\vec{u} = 0\,(10)$$

and, consequently, the term containing this divergence, which affects the normal stress components, can be removed,

$$\overline{\overline{\tau}} = \mu\left[\overline{\overline{\nabla u}} + \overline{\overline{\left(\nabla u\right)^{\mathrm{T}}}}\right] \qquad (11)$$

which is valid for most of liquids. The term in brackets is what rheologists know as the strain rate tensor (Macosko, 1994)

$$\overline{\overline{\tau}} = \mu\,\overline{\overline{\dot{\gamma}}} \qquad (12)$$

which takes exactly the same form of Eq. (6) but expressed in three dimensions. Eq. (9), or the simplification for incompressible fluids (Eq. (12)), is called the Newtonian constitutive equation and represents the complete flow behavior for a Newtonian fluid, i.e. the relationship between stress and strain rate under any type of flow.

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Biographical Sketches

José M Franco was born in Seville (Spain), February 19, 1969. Graduated in Industrial Chemistry at the University of Seville (Spain) in 1992. PhD received in the Faculty of Chemistry at the University of Seville (Spain) in 1995 on “Rheology of food emulsions”.

Professor of Chemical Engineering at the University of Huelva (Spain) since 1995. Invited professor at the Technical University of Hamburg (Germany) and at the Technical University of Valencia (Spain). Head of the Chemical Engineering Department in the University of Huelva (2007-present). Director of the R&D Services at the University of Huelva (2000-2002). He coordinated the PhD studies on “Complex Fluid Engineering” at the University of Huelva (2003-07). External examiner of the University of Wales for the “Environmental Management” and “Environmental Science” studies (2003-2006). Director of the postgraduate program on “Formulation and Product Engineering” of the University of Huelva (2006-2007). The research activity was focused on Food Colloids, Emulsion Technology, Rheology, Processing of non-Newtonian Fluids, Lubricants, Liquid Crystals and Product Development. Researcher in more than 20 research projects sponsored by the administration and the industry, author of more than 50 papers in peer reviewed journals, author of several reviews on Food and Emulsion Rheology, co-author of several chapters in specialized books, co-editor of 2 books on Rheology, author of more than 30 communications in International Conferences. Honored with the Andalusian government award for young researchers (2003).

Prof. Franco is Vice-president (2006-present) of the Spanish Rheological Society and member of the *Real Sociedad Española de Física y Química*. Member of the Organizing Committee of “Eurorheo 2002-01, Joint Meeting of British, Italian, Portuguese and Spanish Rheologists” (Málaga, Spain, 2002) and “Ibereo-08, Iberian Meeting on Rheology” (Madrid, Spain, 2008).

Pedro Partal is Professor of Chemical Engineering at the Chemical Engineering Department, University of Huelva. He was born in Sevilla in 1968 and received his bachelor’s degree in Industrial Chemistry from University of Sevilla, Spain, in 1991. He completed his Ph.D. in 1995 with Dr. Antonio Guerrero at University of Sevilla. He was assistant professor at University of Huelva from 1994 to 2000 and Secretary of the Chemical Engineering Department of the University of Huelva (1997-2000). He became Director for Research Affairs at University of Huelva from 2000 to 2003 and Director of the R&D Centre on Food Technology of the University of Huelva, CIDERTA, in 2004. He is involved in PhD studies on “Complex Fluid Engineering” at the University of Huelva. He is external examiner of the National Evaluation and Foresight Agency (Ministry of Science and Innovation, Spain) in Food Technology (2000-2009).

His research activity is related to the Product Design and Development, with specific interest in the microstructure, rheology and processing of complex materials (food emulsions and gels, biopolymers, bioplastics, bitumen, lubricants, surfactants, etc.). In general, he is involved in topics related to fluid flow and heat transfer of non-Newtonian fluids. Researcher in at least 39 research projects sponsored by the public funds or by the industry, he is coauthor of more than 60 papers in peer reviewed journals, coauthor of a review on Food and Emulsion Rheology, coauthor of several chapters in specialized books, coeditor of a book on rheology entitled “Progress in Rheology: Theory and Applications”, and author of more than 40 communications in International Conferences.