Department of Mechanical Engineering
Indian Institute of Technology New Delhi
Ist Semester – 2018 - 2019

MCL704: Applied Mathematics of Thermo-Fluids
Problem Set: 1

Prove the following vector and tensor identities and discuss their useful applications.
Single bar denotes vectors
Double bar denotes tensors

1. $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$
2. $(\vec{u} \times \vec{v}) \times \vec{w} = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{v} \cdot \vec{w})\vec{u}$
3. $(\vec{u} \times \vec{v}) \cdot (\vec{w} \times \vec{x}) = (\vec{u} \cdot \vec{w})(\vec{v} \cdot \vec{x}) - (\vec{u} \cdot \vec{x})(\vec{v} \cdot \vec{w})$
4. $(\vec{u} \times \vec{v}) \times (\vec{w} \times \vec{x}) = (\vec{u} \cdot (\vec{v} \times \vec{x}))\vec{w} - (\vec{u}.(\vec{v} \times \vec{w}))\vec{x} = (\vec{u} \cdot (\vec{w} \times \vec{x}))\vec{v} - (\vec{v} \cdot (\vec{w} \times \vec{x}))\vec{u}$
5. $\frac{d}{du}(\overline{\overline{A}} \cdot \overline{\overline{B}}) = \overline{\overline{A}} \cdot \frac{d\overline{\overline{B}}}{du} + \frac{d\overline{\overline{A}}}{du} \cdot B$
6. $\frac{d}{du}(\overline{\overline{A}} \times \overline{\overline{B}}) = \overline{\overline{A}} \times \frac{d\overline{\overline{B}}}{du} + \frac{d\overline{\overline{A}}}{du} \times \overline{\overline{B}}$
7. $\frac{d}{du}[\overline{\overline{A}} \cdot (\overline{\overline{B}} \times \overline{\overline{C}})] = \frac{d\overline{\overline{A}}}{du} \cdot (\overline{\overline{B}} \times \overline{\overline{C}}) + \overline{\overline{A}} \cdot \left(\frac{d\overline{\overline{B}}}{du} \times \overline{\overline{C}}\right) + \overline{\overline{A}} \cdot \left(\overline{\overline{B}} \times \frac{d\overline{\overline{C}}}{du}\right)$
8. $\vec{a} \cdot \frac{\vec{a}}{du} = a\frac{da}{du}$
9. $\vec{a} \cdot \frac{d\vec{a}}{du} = 0$ iff $||\vec{a}|| = const$
10. $\int_{P_1}^{P_2} \overline{\overline{A}} \cdot dx = -\int_{P_2}^{P_1} \overline{\overline{A}} \cdot dx$
11. $\int_{P_1}^{P_2} \overline{\overline{A}} \cdot dx = \int_{P_1}^{P_3} \overline{\overline{A}} \cdot dx + \int_{P_3}^{P_2} \overline{\overline{A}} \cdot dx$, where P_3 is in between P_1 and P_2
12. $\nabla(\overline{\overline{A}} + \overline{\overline{B}}) = \nabla\overline{\overline{A}} + \nabla\overline{\overline{B}}$
13. $\nabla \cdot (\overline{\overline{A}} + \overline{\overline{B}}) = \nabla \cdot \overline{\overline{A}} + \nabla \cdot \overline{\overline{B}}$
14. $\nabla \times (\overline{\overline{A}} + \overline{\overline{B}}) = \nabla \times \overline{\overline{A}} + \nabla \times \overline{\overline{B}}$
15. $\nabla \cdot (\alpha\overline{\overline{A}}) = (\nabla\alpha) \cdot \overline{\overline{A}} + \alpha(\nabla \cdot \overline{\overline{A}})$
16. $\nabla \times (\alpha\overline{\overline{A}}) = (\nabla\alpha) \times \overline{\overline{A}} + \alpha(\nabla \times \overline{\overline{A}})$
17. $\nabla \times (\nabla \times \overline{\overline{A}}) = \nabla(\nabla \cdot \overline{\overline{A}}) - \nabla^2\overline{\overline{A}}$
18. $\nabla \cdot (\vec{v} \times \vec{w}) = (\nabla \times \vec{v}) \cdot \vec{w} + \vec{v} \cdot (\nabla \times \vec{w})$
19. $\nabla \times (\vec{v} \times \vec{w}) = (\vec{w} \cdot \nabla)\vec{v} - (\nabla \cdot \vec{v})\vec{w} - (\vec{v} \cdot \nabla)\vec{w} + (\nabla \cdot \vec{w})\vec{v}$
20. $\nabla(\vec{v} \cdot \vec{w}) = (\vec{w} \cdot \nabla)\vec{v} + (\vec{v} \cdot \nabla)\vec{w} + \vec{w} \times (\nabla \times \vec{v}) + \vec{v} \times (\nabla \times \vec{w})$

21. $\vec{v}+\vec{0}=\vec{0}+\vec{v}=\vec{v}$ Existence of additive identity

22. $\vec{v}+(-\vec{v})=(-\vec{v})+\vec{v}=0$ Existence of additive inverse

23. $\vec{v}+\vec{w}=\vec{w}+\vec{v}$ Commutative Law

24. $\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$ Associative Law

25. $\alpha(\vec{v}\ +\ \vec{w})\ =\ \ \alpha\vec{v}\ +\ \alpha\,\vec{w}$ Distributive law

26. $(\alpha+\beta)\vec{v}=\alpha\vec{v}+\beta\vec{v}$ Distributive law

27. $\alpha(\alpha\vec{v})=(\alpha\beta)\vec{v}$ Associative Law

28. $1\vec{v}=\vec{v}$ Multiplication with scalar identity

29. $\vec{u}.\vec{v}=\vec{v}.\vec{u}$ Commutative

30. $u(\alpha\vec{v}+\beta\vec{w})=\alpha\vec{u}.\vec{v}+\beta\vec{u}.\vec{w}$ Linearity

31. $\vec{v}.\vec{v}\geq 0\ and\ \vec{v}.\vec{v}=0\ \Leftrightarrow \vec{v}=0$ Positivity

$Prove\ the\ following\ dyadic\ products$

32. $\vec{v}\otimes\vec{w}\neq\vec{w}\otimes\vec{v}$

33. $\vec{u}\otimes(\alpha\vec{v}+\beta\vec{w})\neq\alpha\vec{u}\otimes\vec{v}+\beta\vec{u}\otimes\vec{w}$

34. $(\alpha\vec{u}+\beta\vec{v})\otimes\vec{w}=\alpha\vec{u}\otimes\vec{w}+\beta\vec{v}\otimes\vec{w}$

35. $(\vec{u}\otimes\vec{v}).\vec{w}=(\vec{v}.\vec{w})\vec{u}$

36. $\vec{u}.(\vec{v}\otimes\vec{w})=(\vec{u}.\vec{v}).\vec{w}$

37. Prove for identity tensor $\bar{\bar{I}}$, following holds: $\bar{\bar{I}}.\vec{v}=\vec{v}.\bar{\bar{I}}=\vec{v}$

38. Prove for orthogonal tensor $\bar{\bar{Q}},$ $\bar{\bar{Q}}\bar{\bar{Q}}^T=\bar{\bar{I}}\ or\ \bar{\bar{Q}}^{-1}=\bar{\bar{Q}}^T$

39. $\vec{v}\times\vec{w}=-\vec{w}\times\vec{v}$ Anti-symmetry

40. $\vec{u}\times(\alpha\vec{v}+\beta\vec{w})=\alpha\vec{u}\times\vec{v}+\beta\vec{u}\times\vec{w}$ Linearity