Department of Mechanical Engineering
Indian Institute of Technology New Delhi
Ist Semester 2018-2019

MCL704: Applied Mathematics of Thermo-Fluids
Problem Set: 2

Exercises

1. Solve the equation

$$\frac{dy}{dx} = \frac{e^{-x}}{y}.$$

2. Solve the following equation subject to the condition $y(0) = 1$:

$$\frac{dy}{dx} = 3x^2 e^{-y}$$

3. Find the general solution of the following equations:

(a) $\frac{dy}{dx} = 3,$ (b) $\frac{dy}{dx} = \frac{6\sin x}{y}$

4. (a) Find the general solution of the equation

$$\frac{dx}{dt} = t(x-2).$$

(b) Find the particular solution which satisfies the condition $x(0) = 5$.

5. Some equations which do not appear to be separable can be made so by means of a suitable substitution. By means of the substitution $z = y/x$ solve the equation

$$\frac{dy}{dx} = \frac{y^2}{x^2} + \frac{y}{x} + 1$$

6. The equation

$$iR + L\frac{di}{dt} = E$$

where R, L and E are constants arises in electrical circuit theory. This equation can be solved by separation of variables. Find the solution which satisfies the condition $i(0) = 0$.

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Exercises

1. Solve the equation $\frac{d}{dx}(yx^2) = x^3$.
2. Solve the equation $\frac{d}{dx}(y\mathrm{e}^x) = \mathrm{e}^{2x}$ given the condition $y(0) = 2$.
3. Solve the equation $\mathrm{e}^{2x}\frac{dy}{dx} + 2\mathrm{e}^{2x}y = x^2$.
4. Show that the equation $x^2\frac{dy}{dx} + 2xy = x^3$ is exact and obtain its solution.
5. Show that the equation $x^2\frac{dy}{dx} + 3xy = x^3$ is not exact.
   Multiply the equation by x and show that the resulting equation **is** exact and obtain its solution.

Exercises

1. Solve the equation $x^2\frac{dy}{dx} + x\ y = 1$.
2. Find the solution of the equation $x\frac{dy}{dx} - y = x$ subject to the condition $y(1) = 2$.
3. Find the general solution of the equation $\frac{dy}{dt} + (\tan t)\ y = \cos t$.
4. Solve the equation $\frac{dy}{dt} + (\cot t)\ y = \sin t$.
5. The temperature θ (measured in degrees) of a body immersed in an atmosphere of varying temperature is given by $\frac{d\theta}{dt} + 0.1\theta = 5 - 2.5t$. Find the temperature at time t if $\theta = 60°\mathrm{C}$ when $t = 0$.
6. In an LR circuit with applied voltage $E = 10(1 - \mathrm{e}^{-0.1t})$ the current i is given by

   $$L\frac{di}{dt} + Ri = 10(1 - \mathrm{e}^{-0.1t}).$$

   If the initial current is i_0 find i subsequently.