Department of Mechanical Engineering

Indian Institute of Technology New Delhi

I Semester -- 2019 – 2020

MCL 704 *APPLIED MATHEMATICS FOR THERMOFLUIDS*

Problem Set 3: Basics of Second Order Linear ODEs

Classify the following linear differential equations as constant coefficient or variable coefficient equations.

1. $y'' + \omega^2 y = 0$

2. $y' = \frac{y}{x}$

3. $y'' + 4y'' + 4y' = e^x$

4. $x^2y'' + xy' - 3y = 0$

5. $(1 - x^2)y'' - 2xy' + 6y = 0$

Examine whether the following functions are linearly independent in the region $x > 0$.

6. $x, 3x + 1, 2x + 2$

7. $x, x^2 - 1,\ x^2 - 2x + 1$

8. $sinx, sin2x, sin3x$

9. $cosx, cos2x, cos3x$

10. $1, cosx, sinx$

11. $x, \frac{1}{x}$

12. $x^2, \frac{1}{x^2}$

13. $e^x, coshx, sinhx$

14. $e^{-x}, coshx, sinhx$

15 lnx, lnx^2, lnx^3

16. Show that $y_1 = cosx$ and $y_2 = 2sinx - cosx$ are linearly independent solutions of $y'' + y = 0$. Express the solution $\emptyset(x) = sinx$ as a linear combination of $y_1 and\ y_2$.

17. Show that $y_1 = cos\omega x$ and $y_2 = sin\omega x$ are linearly independent solutions of $y'' + \omega^2 y = 0$.

18. Show that $y_1 = e^{-x}cos\omega x$ and $y_2 = e^{-x}sin\omega x$ are linearly independent solutions of $y'' + 2y' + 2y = 0$.

19. Show that $y_1 = e^{-x}$, $y_2 = e^{2x}$ are linearly independent solutions of $y'' - y' - 2y = 0$.

20. Show that $y_1 = 1$, $y_2 = x$, $y_3 = e^{-x}$ are linearly independent solutions of $y''' + y'' = 0$.