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CASCADE FLOW PROBLEMS

by

H. SCHLICHTING

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This Report was presented at the Tenth Meeting of the Wind Tunnel and Model Testing Panel, held from February 18th to 21st, 1957, in Paris.

SUMMARY

In the Institute of Fluid Mechanics of the Engineering University of Braunschweig a considerable amount of research work on cascades has been done in recent years. Some of this work is reported very briefly here.

(i) An extensive program of theoretical calculations of loss coefficients of two-dimensional cascades in incompressible flow has been carried out, applying boundary layer theory to cascade flow. There is good agreement between theory and experiment.

(ii) Using the results of (i), and also results of secondary flow losses, the characteristic curves of an axial flow compressor (pressure coefficient and efficiency coefficient against mass flow coefficient) have been calculated purely theoretically. The tendency of these curves agrees with what is to be expected.

(iii) Some results of pressure distribution measurements on cascades at high subsonic Mach numbers are presented. These have been carried out in the new Variable Density High Speed Cascade Wind Tunnel of the Deutsche Forschungsanstalt für Luftfahrt, Braunschweig. This wind tunnel allows independent variation of Reynolds number and Mach number.

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SOMMAIRE

L'Institut de la Mécanique des Fluides de l'Ecole des Arts et Métiers de Brunswick a effectué beaucoup de recherches au sujet des grilles d'aubes au cours de ces dernières années. Ce rapport donne un exposé de certains des travaux entrepris.

(i) Détermination par le calcul des coefficients de perte des grilles planes en régime incompressible, avec application à l'écoulement par les grilles de la théorie concernant la couche limite. Il existe un bon accord entre théorie et expérience.

(ii) Détermination théorique des courbes caractéristiques d'un compresseur axial (coefficients de pression et de rendement donnés en fonction de l'écoulement massique), en appliquant les résultats obtenus à (i) ainsi que ceux relatifs aux pertes en écoulement secondaire. L'allure de ces courbes correspond à celle à laquelle on doit s'attendre.

(iii) Exposé de certains résultats suivant des mesures relatives à la répartition de pression sur des grilles d'aubes pour une distribution de nombres de Mach subsoniques élevés, effectuées dans la nouvelle soufflerie à grille d'aubes à grande vitesse et à densité variable de la Deutsche Forschungsanstalt für Luftfahrt, Brunswick. Cette soufflerie permet la variation indépendante des nombres de Reynolds et de Mach.

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NOTATION

d	distance between blades
c	blade chord
β_B	blade angle
β_1	angle of inflow
β_2	angle of outflow
w_1	velocity of inflow
w_2	velocity of outflow
w_a	axial component of velocity
Δw_u	difference between circumferential velocity components behind and in front of cascade
$\Delta w_u/w_a$	deflection of flow through cascade
ρ	density
ν	kinematic viscosity
Δh	loss in total head
ζ	$= \Delta h \Big/ \left(\frac{\rho}{2}\right) w_a^2$, dimensionless loss coefficient
R	$= w_2 c/\nu$, Reynolds number
a_1, a_2	velocity of sound in front of cascade and behind cascade, respectively
M_1, M_2	Mach numbers ($= w_1/a_1$ and w_2/a_2 respectively)
p_0	stagnation pressure in front of cascade
p_1	static pressure in front of cascade
p	static pressure on blade surface
c_p	$= \frac{p - p_1}{p_0 - p_1}$, dimensionless pressure coefficient

Axial compressor

u	circumferential velocity of rotor at radial station r

u_A	circumferential velocity of rotor at tip, $r = r_A$
V	volume flow per unit time
H, H_{th}	actual head and theoretical head, respectively
Φ	$= V/(\pi r_A^2 u_A)$, dimensionless flow coefficient
Ψ	$= 2gH/u_A^2$, pressure coefficient
η	$= H/H_{th}$, efficiency coefficient

CASCADE FLOW PROBLEMS

H. Schlichting*

1. INTRODUCTION

The problem of the flow through cascades is an important one in the whole field of turbo-machinery, but it is a pure aerodynamic problem. The subject, in which considerable progress has been made in recent years, seems important enough to be discussed at a Meeting of the Wind Tunnel Panel, perhaps partly in conjunction with the Combustion and Propulsion Panel.

Some research work in this field has been done in Germany in recent years, and some new equipment has been built, so that Germany would be able to contribute some papers, if it is decided to discuss this subject at a future meeting.

This paper gives, very briefly, some ideas on the work done on the subject at Braunschweig University in recent years.

2. SCOPE OF INVESTIGATION

The main incentive of these investigations is that real progress in the flow problems of turbo-machines will be achieved only by a deeper knowledge of complex flow phenomena. This requires extensive theoretical calculations which, however, need careful correlation with experiments. A general survey of these investigations has been given in previous papers[1,2].

The very complex cascade flow problem has been split up as follows:-

(i) Two-dimensional flow through cascades.

- (a) Incompressible and inviscid flow.
- (b) Incompressible, viscous flow.
- (c) Compressible flow.

(ii) Three-dimensional flow through cascades.

- (a) Secondary flow effects at blade root and blade tip.
- (b) Effects due to radial divergence of the blades in cascades of rotational symmetry.

3. INCOMPRESSIBLE CASCADE FLOW

For two-dimensional cascades, the main object of the investigations has been to find a way to calculate theoretically the loss coefficients of the cascade, since they depend on the geometrical and aerodynamic parameters of the cascade. This has been

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* *Institut für Stromungsmechanik, Technische Hochschule, Braunschweig, Germany and Deutsche Forschungsanstalt für Luftfahrt, Institut für Aerodynamik, Braunschweig, Germany*

achieved by applying boundary layer theory to the cascade flow. But before dealing with this problem, it was necessary to improve the methods of calculating the incompressible and inviscid flow through a cascade. Convenient solutions of the 'Indirect Problem' and the 'Direct Problem', have been published in two VDI-Forschungshefte[3,4]. These solutions have been used in an extensive programme of theoretical calculations of loss coefficients, in connection with a large amount of experimental work, mainly to check the theoretical results. Figures 1 and 2 show some examples of the loss coefficient plotted against the dimensionless deflection. In theory fully turbulent flow in the boundary layer has been assumed, and in the experiments this was achieved by a turbulence wire near to the leading edge of the blade. The results of Figure 1 are for cascades of blades with the symmetrical profile NACA 0010, whereas in Figure 2 the blades have the cambered profile NACA 8410. In both cases the solidity ratio d/c and the blade angle β_B have been varied. The agreement between theory and experiment is very satisfactory.

These results on two-dimensional cascades, and some more on secondary flow losses, have finally enabled calculation, purely theoretically, of the characteristic curves of an axial flow compressor[5]. An example of this kind of calculations is given in Figure 3 and Figure 4. In Figure 3 the blading and the velocity vectors at different cross sections are given. Figure 4 shows the pressure coefficient $\Psi = 2gH/u_A^2$ and the efficiency coefficient η for the single-stage compressor plotted against the dimensionless flow coefficient $\Phi = V/(\pi r_A^2 u_A)$. The general shape of these curves agrees with what is expected, but no experiments to compare with these theoretical calculations have yet been carried out.

4. COMPRESSIBLE CASCADE FLOW*

In the past two years we have been engaged mainly on problems of three-dimensional cascade flow and of compressible cascade flow. Very extensive results have been accumulated on three-dimensional effects, but this subject is still far from being accessible to theory, and will not be dealt with here. Compressibility effects, however, are considered in a little more detail.

During the past year a special High Speed Cascade Wind Tunnel of the continuous flow type, for basic research work on compressibility effects, has been completed. A brief description of this tunnel was given at the Rome meeting of the Agard Wind Tunnel Panel[6]. A special feature of it is that the Mach number and the Reynolds number can be varied independently. This is regarded as very important for basic research on cascades, because the aerodynamic coefficients in most cases depend considerably on both the Mach number and the Reynolds number of the blade. The independent variation of Mach number and Reynolds number is achieved by installing the cascade wind tunnel in a tank which can be evacuated from 1 atm down to 0.1 atm. The Mach number range is from $M = 0.2$ to about 1.1. The blade length $l = 300$ mm and the blade chord $c = 60$ mm. In the meantime, since the Rome meeting, the tunnel has come into full operation.

During the past year an extensive programme of pressure distribution measurements has been carried out on cascade blades at high subsonic speeds. Figures 5 and 6 show the cascade

*These tests were done by Dr.-Ing. N. Scholz and Dipl.-Ing. K.H. Grewe.

geometry of the whole programme. The cascade geometry and the blade profile are the same as in the earlier low speed investigations. Pressure distributions on all these cascades were measured at different angles of inflow and for Mach numbers from $M = 0.2$ to the choking Mach number. The Reynolds number is constant over the whole range of Mach numbers, $R_2 = w_2 c/\nu = 3x10^5$. The results given in the following figures refer to the three cascades marked in solid black in Figure 5 and 6.

In Figure 7 the cascade is unstaggered and of NACA 0010 profile, the angle of inflow being $\beta_1 = 90^o$ (zero lift) and $\beta_1 = 100^o$. With increasing Mach number the pressure distribution remains quite normal up to about $M_1 = 0.65$ following the Prandtl-Glauert rule. For $\beta_1 = 100^o$ and $M_1 = 0.65$ a shock wave first appears on the suction side of the blade. At $M_1 = 0.69$ choking has occurred, and no further increase of Mach number is possible. With the onset of choking the pressure distribution changes completely, as can be seen from Figure 7 for $M_1 = 0.69$. This is accompanied by a sudden increase of the pressure difference across the cascade, p_1-p_2, which is also given in Figure 7.

In Figure 8 similar results are presented for a compressor cascade of blade profile NACA 8410 and blade angle $\beta_B = 135^o$. Here also the pressure distribution does not change its character up to $M_1 = 0.7$. The choking Mach number is rather different for the two angles of inflow, being $M_1 = 0.75$ for $\beta_1 = 142^o$, but $M_1 = 0.90$ for $\beta_1 = 148^o$. Due to the considerable amount of stagger, the increase of the pressure drop through the cascade is not as steep as with the unstaggered cascade.

In Figure 9 some results for a turbine cascade are given, the blade angle being $\beta_B = 45^o$ and the blade profile NACA 8410 again. Here the change of the pressure distribution, when approaching the choking Mach number, is not so abrupt as for the compressor cascade. This must be attributed to the favourable pressure gradient of the turbine cascade. If in this case the Mach number is referred to the outflow velocity, the choking Mach number is about $M_2 = 0.8$.

A convenient method has been developed for calculating the pressure distribution according to the Prandtl-Glauert rule. In Figure 10 the experimental results of Figure 7 for the unstaggered cascade of NACA 0010 profile are compared with theoretical calculations. For high subsonic Mach numbers the compressibility effect on the pressure distribution is considerable. For those Mach numbers where no shock waves occur, the agreement of the theoretical and experimental pressure distributions is very satisfactory.

Later work on these cascades will include measurements of loss coefficients.

It is hoped that measurements of this kind, which have apparently not been done systematically before, will contribute considerably in giving a clearer insight into the behaviour of flow in a compressor and a turbine at high speeds.

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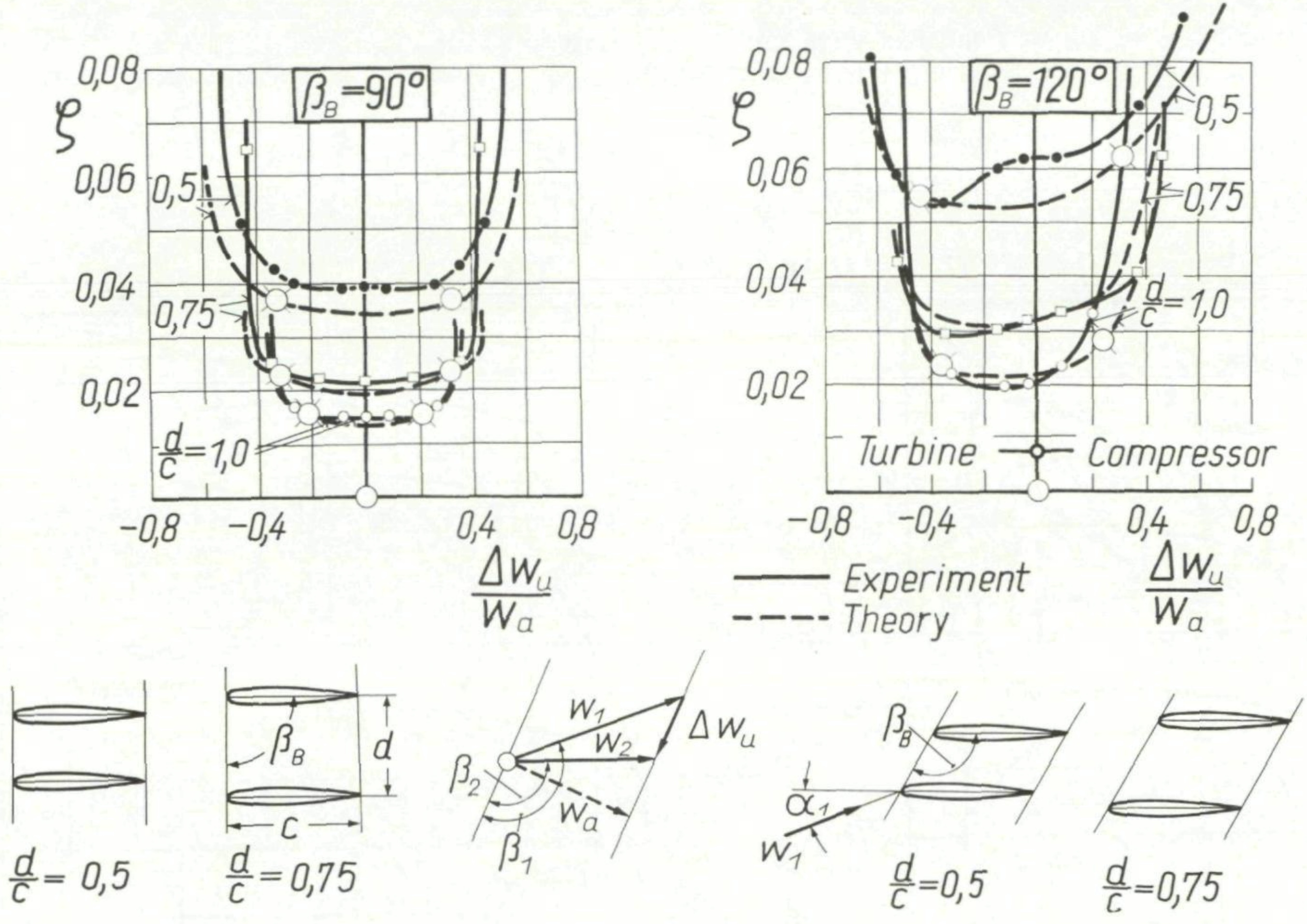


Fig. 1 Loss coefficient $\zeta = \Delta h/(\frac{1}{2}\rho w_a^2)$ for cascades of various solidity ratios d/c and various blade angles β_B; blade section NACA 0010; Reynolds number $R_2 = w_2 c/\nu = 5 \times 10^5$, fully turbulent boundary layer

The circles with projected lines on the theoretical curves indicate the beginning of separation

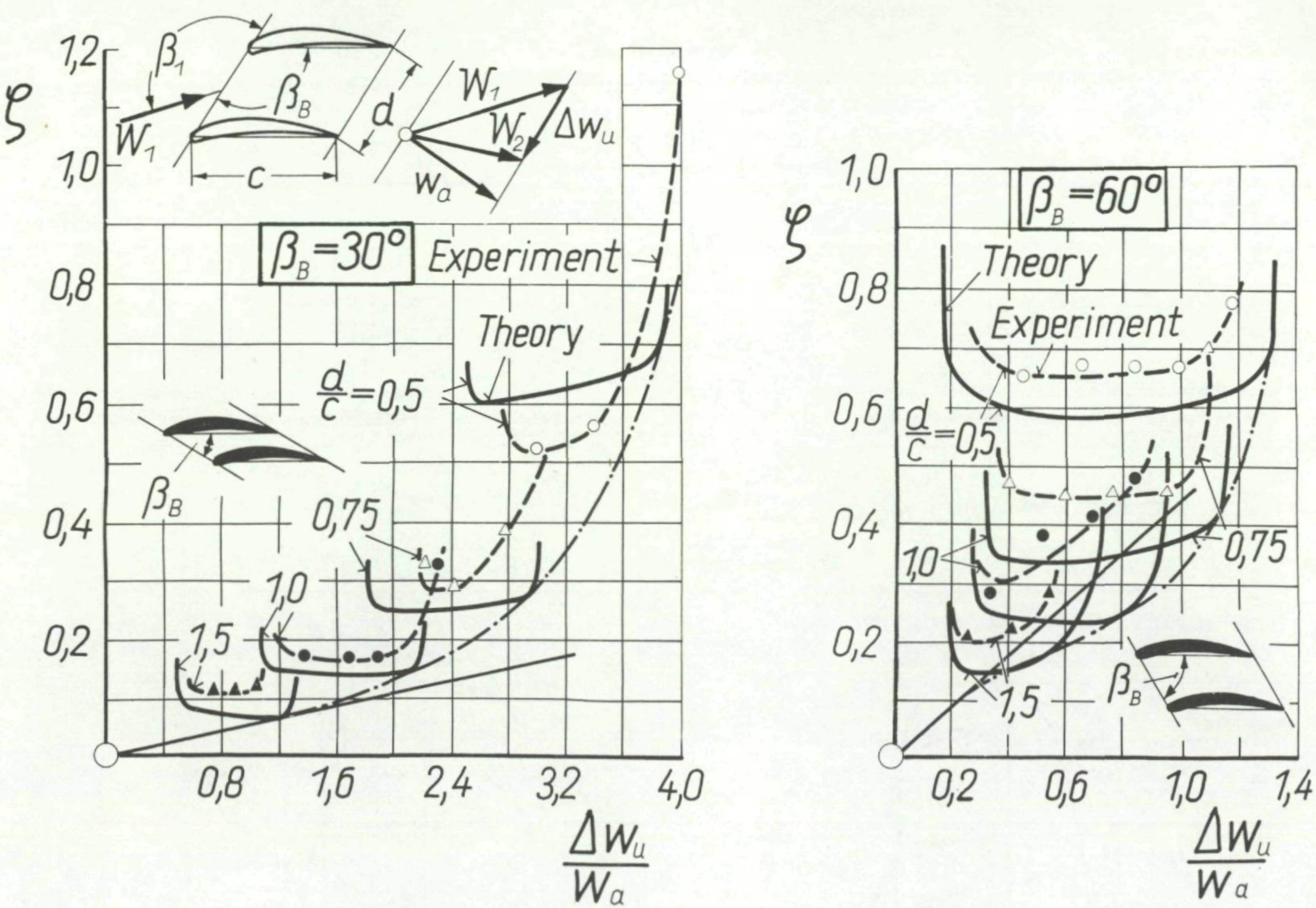


Fig. 2 Loss coefficients $\zeta = \Delta h/(\frac{1}{2} \rho w_a^2)$ of turbine cascades of various solidity ratios d/c and various blade angles β_B; blade section NACA 8410; Reynolds number $R_2 = w_2 c/\nu = 5\text{x}10^5$

	Section I (Tip)	Section II	Section III	Section IV (Hub)
r/r_A	1,0	0,85	0,7	0,55
d/c	1,30	1,11	0,91	0,71
β_B	131°	121°	113°	106°
$\Delta c_u/c_m$	0,575	0,547	0,589	0,695

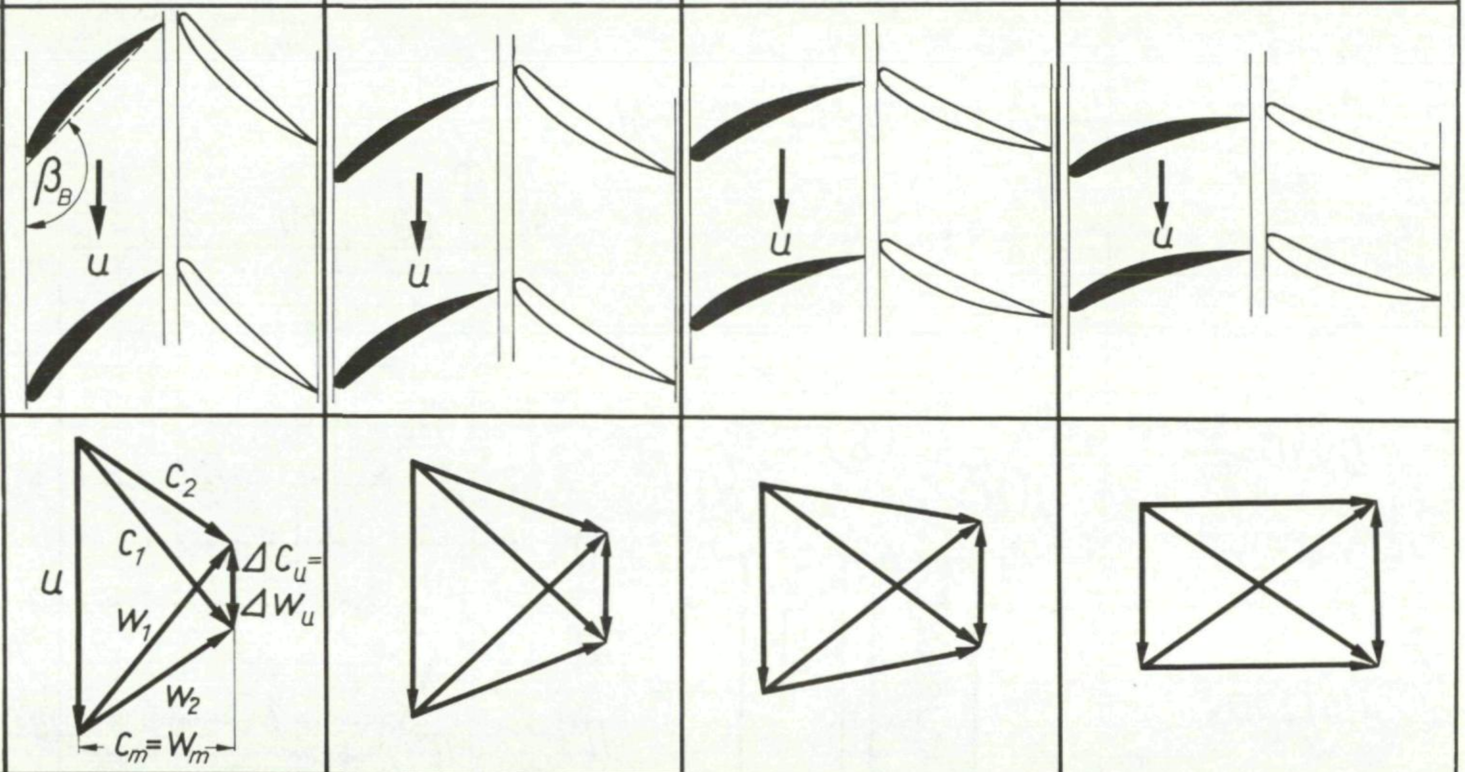
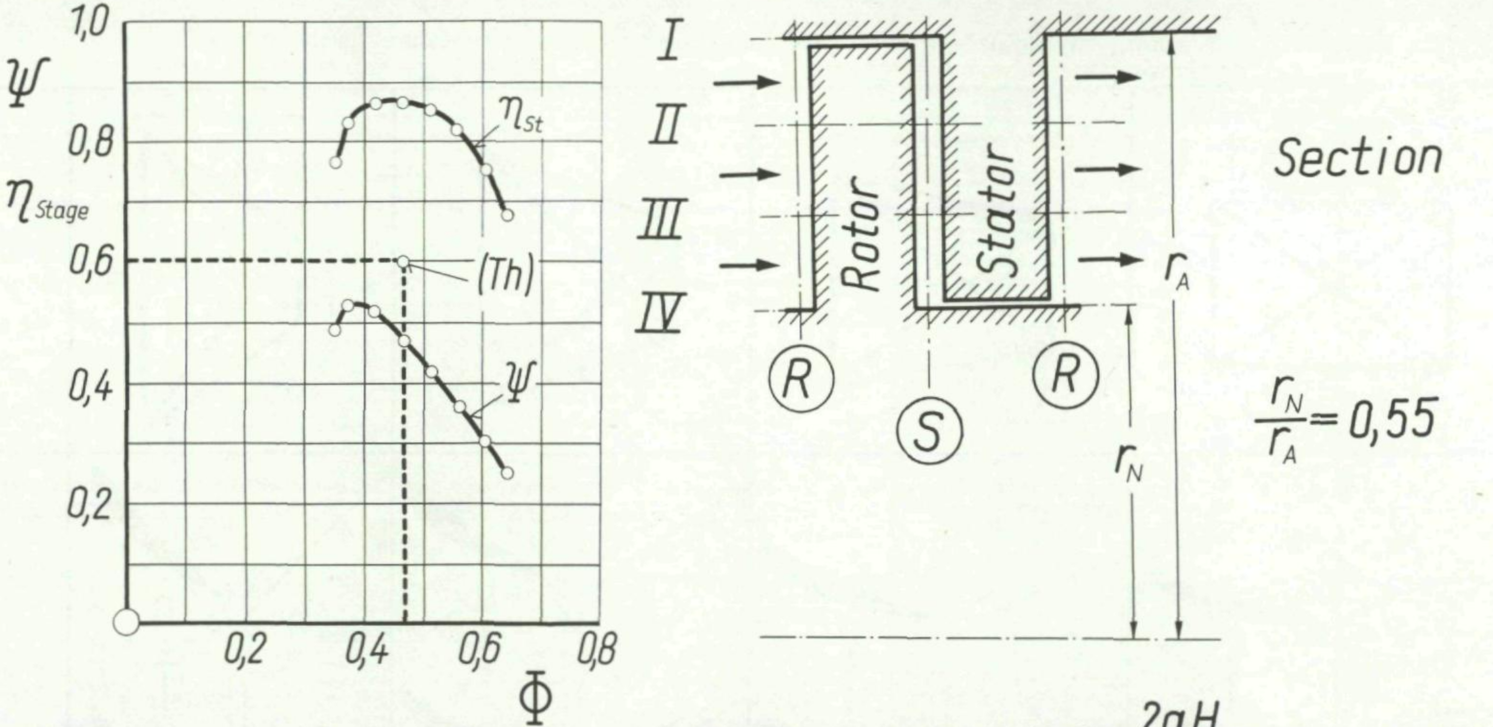


Fig.3 Blade sections of the single-stage axial flow compressor of Figure 4



Pressure coefficient, $\Psi = \frac{2gH}{u_A^2}$

$\eta_{Stage} = \frac{H}{H_{th}}$

Mass flow coefficient, $\Phi = \frac{V}{\pi \cdot r_A^2 u_A}$

(Th): $\Psi_{th} = 0{,}6$; $\Phi_{th} = 0{,}466$

Degree of reaction, $\varkappa(r) = \frac{\Delta p_{rotor}(r)}{\Delta p_{stage}(r)} = const. = 0{,}5$

Fig. 4 Characteristic curves of a single-stage axial flow compressor, as calculated theoretically from cascade data, by N. Scholz[5]. Pressure coefficient $\Psi = 2gH/u_A^2$ and efficiency coefficient $\eta = H/H_{th}$ against mass flow coefficient $\Phi = V/(\pi\, r_A^2 u_A)$

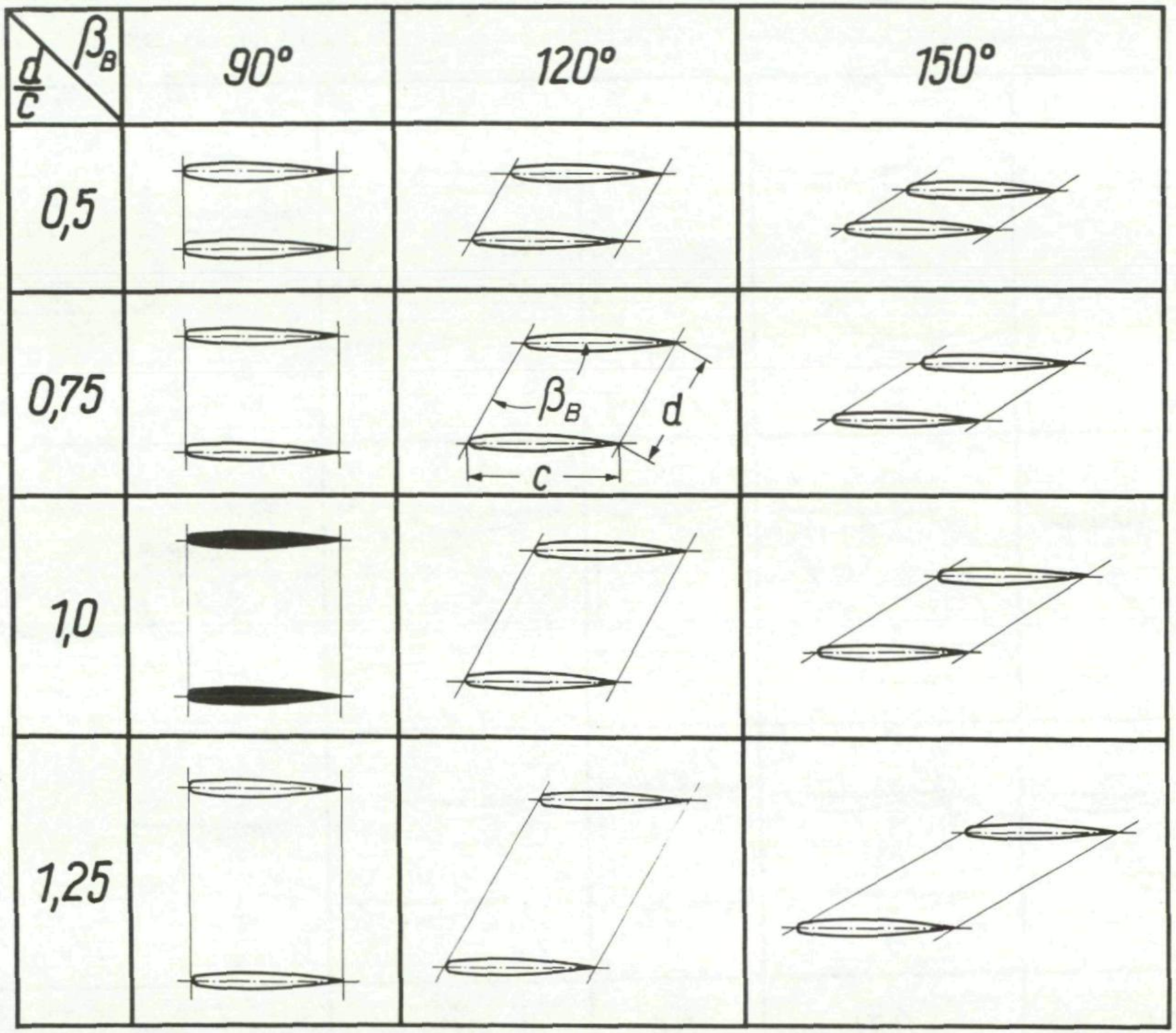


Fig. 5 Cascade geometry for pressure distribution measurements in the High Speed Cascade Wind Tunnel; blade section NACA 0010

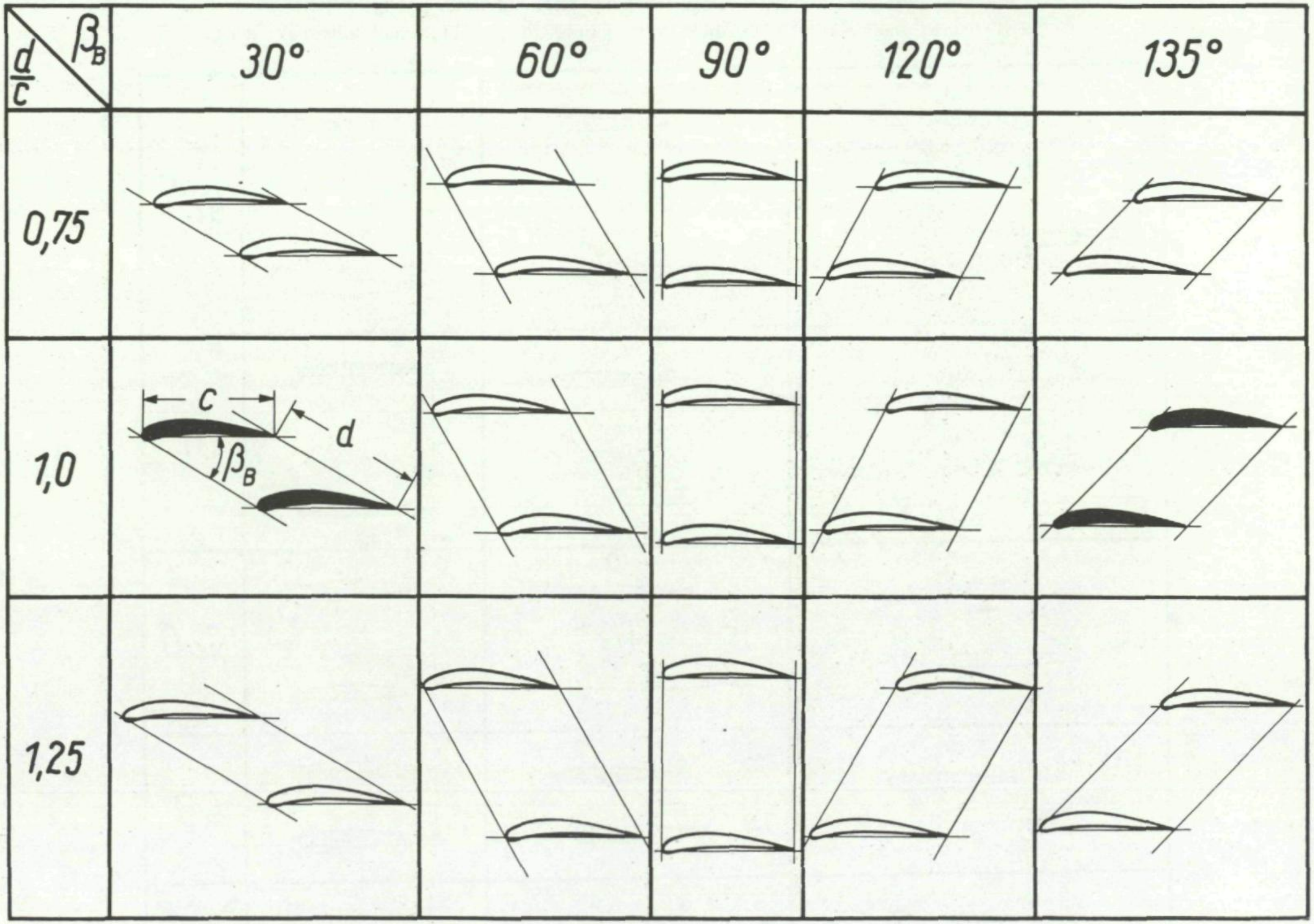


Fig. 6 Cascade geometry for pressure distribution measurements in the High Speed Cascade Wind Tunnel; blade section NACA 8410

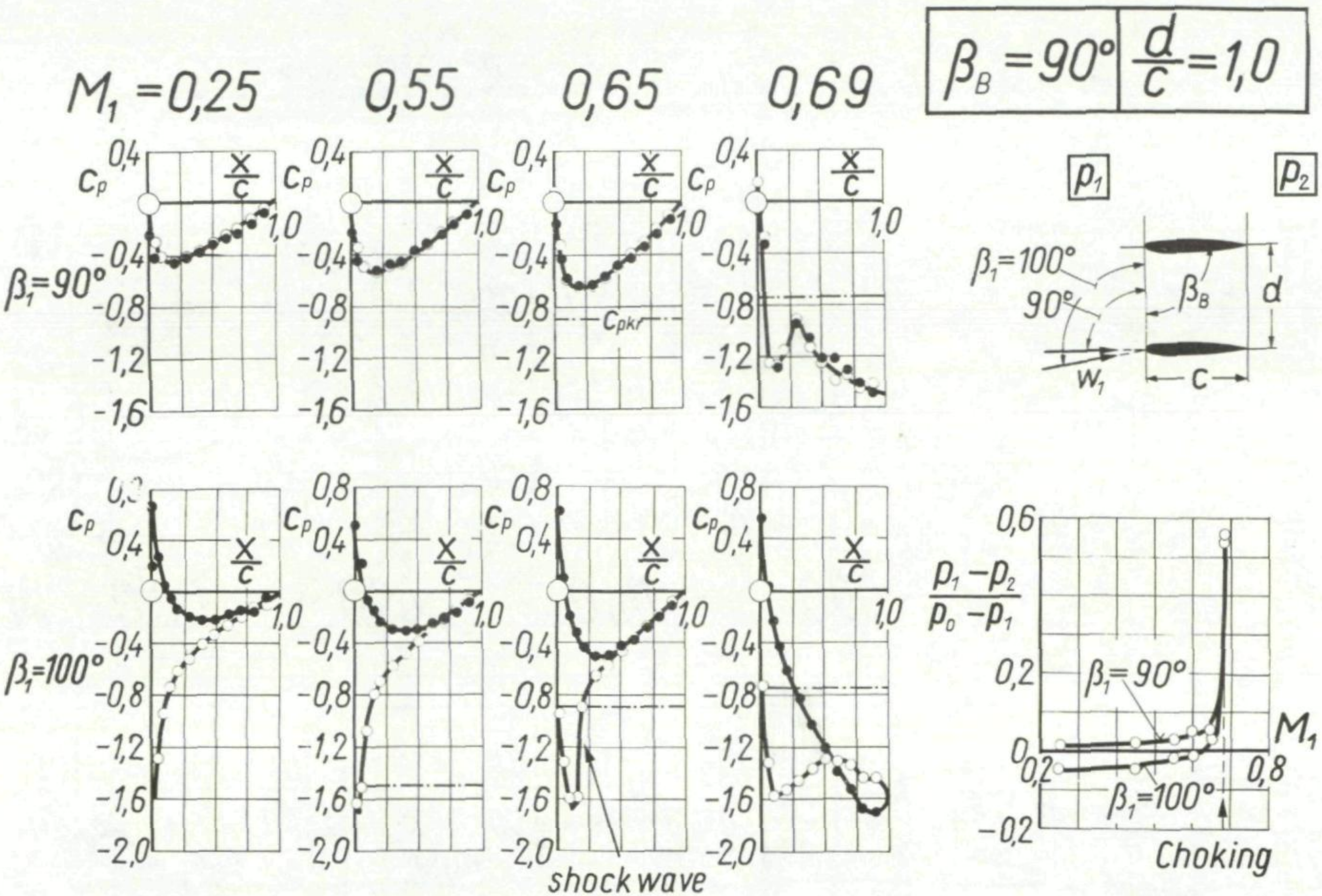


Fig. 7 Pressure distribution measurements of cascades in compressible flow; blade section NACA 0010; Reynolds number $R_2 = w_2 c/\nu = 3\text{x}10^5$; see Figure 5

$$\text{Pressure coefficient } c_p = \frac{p-p_1}{p_0-p_1}$$

$M_1 = w_1/a_1$ = Mach number of inflow.

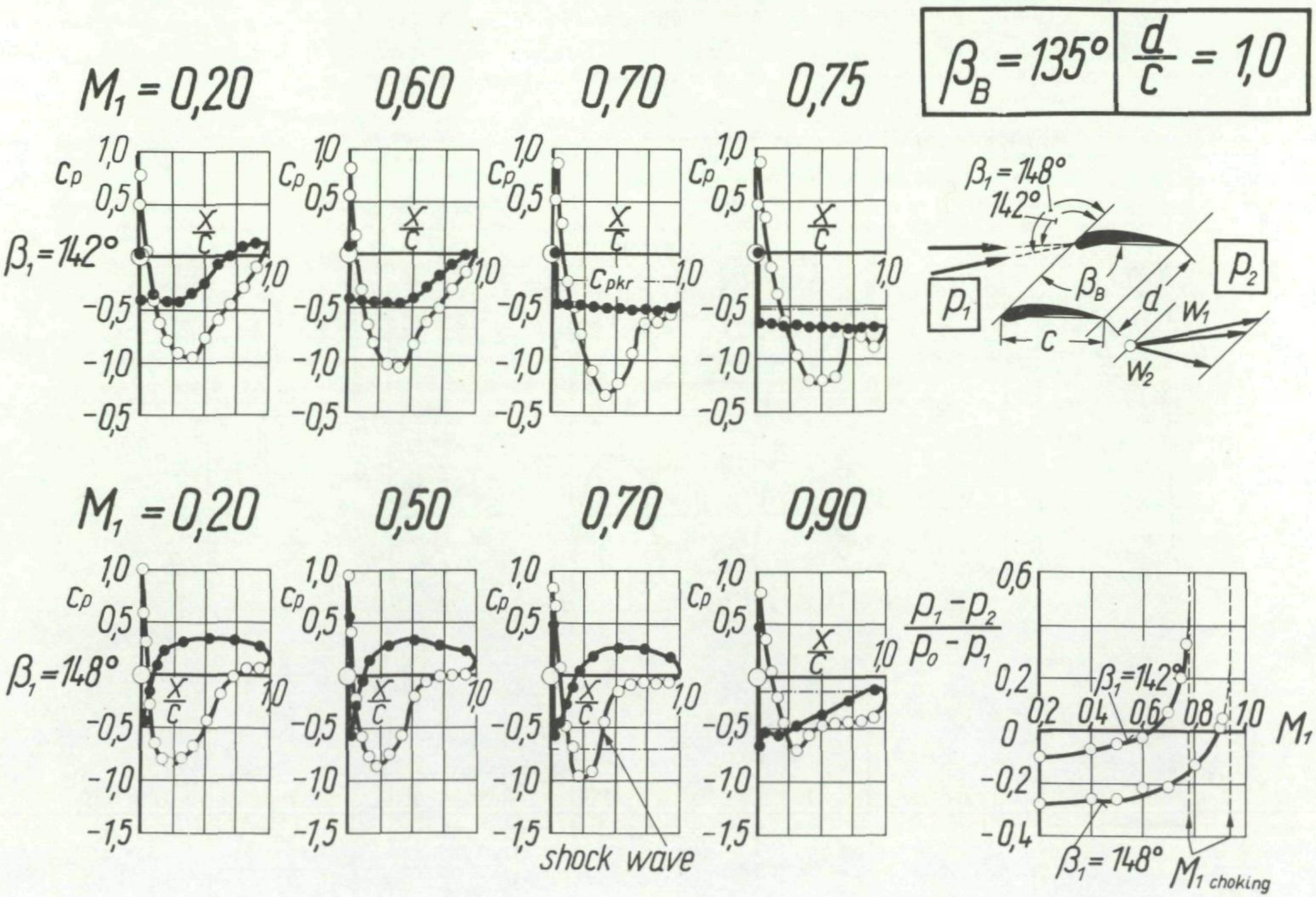


Fig.8 Pressure distribution measurements of compressor cascades in compressible flow; blade section NACA 8410; Reynolds number $R_2 = w_2 c/\nu = 3x10^5$; see Figure 6

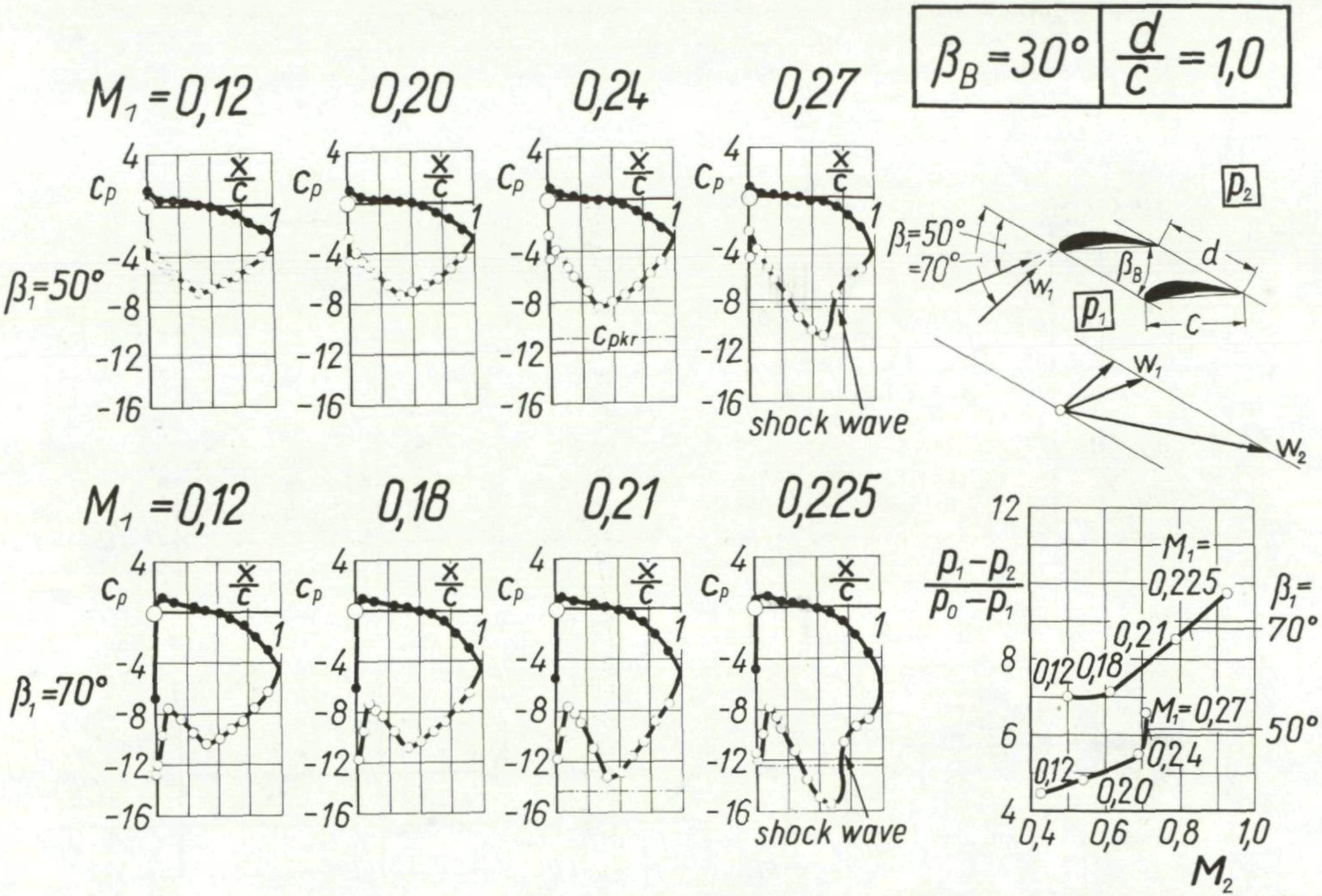


Fig. 9 Pressure distribution measurements of turbine cascades in compressible flow; blade section NACA 8410; Reynolds number $R_2 = w_2c/\nu = 5x10^5$; see Figure 6

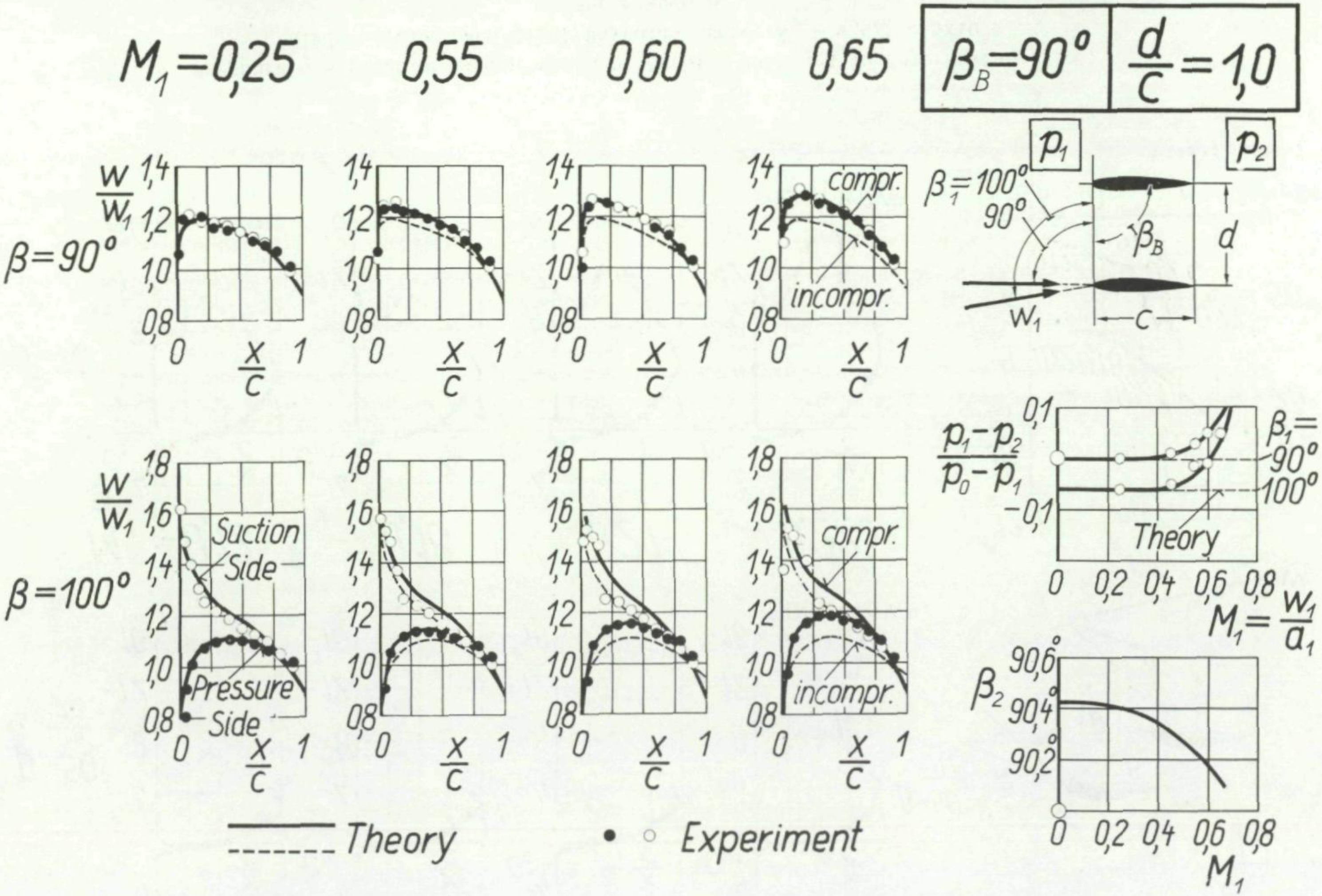


Fig. 10 Comparison of theoretical and experimental velocity distribution of cascades in compressible flow. Theory from Prandtl-Glauert rule; experiments, see Figure 7. Blade section NACA 0010; Reynolds number $R_2 = w_2 c/\nu = 3 \times 10^5$

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