

Practice problems : Boundary layers

Advanced Fluid Dynamics

Course coordinator - Prateek Gupta

Department of Applied Mechanics, IIT Delhi

1. Consider a steady flat-plate boundary layer as shown below. For boundary layers (in general), the displacement thickness and the momentum thickness are defined as (respectively),

$$\delta^*(x) = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy, \quad \Theta(x) = \int_0^\infty \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy.$$

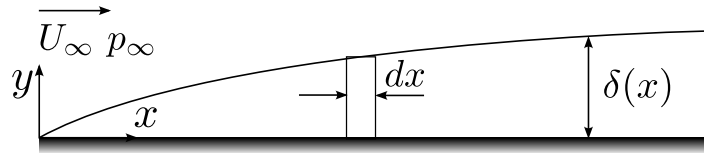


Figure 1: Flat plate boundary layer

- (a) Using the boundary layer equations, and the self-similarity assumption, $u(x, y) = U_\infty f'(\eta)$, derive the Blasius equation as done in class.
- (b) Numerically solve the Blasius equation. Using the solution, plot $\delta_{99}(x)$ vs x where $\delta_{99}(x)$ is the location of points where $u \approx 0.99U_\infty$.
- (c) Write δ^* and Θ using the self-similarity assumption and variable transformation. Plot vs x .
- (d) Derive the expressions for the skin-friction coefficient C_f and drag coefficient C_D for a plate of length L .
- (e) In the control volume of thickness dx and height $\delta(x)$, compute the average vorticity.
- (f) Integrate the x momentum equation (approximate form under boundary layer theory) in y from 0 to ∞ to obtain the following equation,

$$\frac{d}{dx} (U_\infty^2 \Theta) = \frac{\tau_w}{\rho_0},$$

where τ_w is the shear stress at the wall,

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}.$$

2. Air at flow rate Q emanates from a round orifice of radius a in still atmosphere. The orifice is an opening of a pipe in which fully-developed air-flow is established. An axisymmetric self-similar jet develops far away from the orifice.

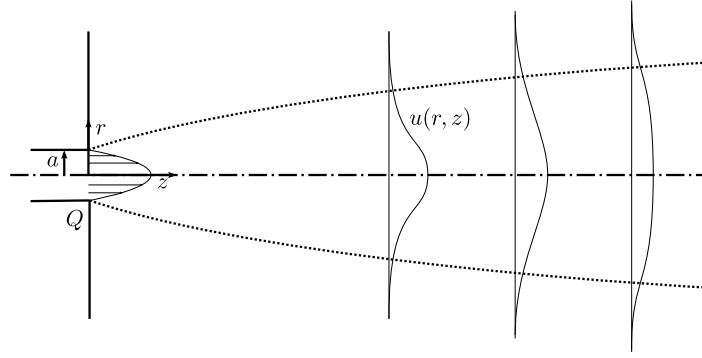


Figure 2: Axisymmetric free jet

- Using the boundary layer theory, derive the governing equations for the steady, laminar, axisymmetric jet.
 - Using a similarity variable $\eta = \frac{r}{z^\beta}$ and a streamfunction ansatz $\psi = \nu z F(\eta)$, find the value of β for which self-similar solution exists. What can you say about the increase of jet-width in this regime?
 - Using the similarity condition above, solve the boundary layer governing equations.
 - Plot the variation of centerline velocity $u(r = 0, z)$ vs z and the streamlines of the flow for various values of the flow rate Q .
 - Plot the variation of the volume flow rate vs z .
3. Solve the Falkner-Skan equation for boundary layers for the following cases and plot the (i) self-similar profiles, (ii) streamlines, (iii) shear-stress at the wall vs x ,

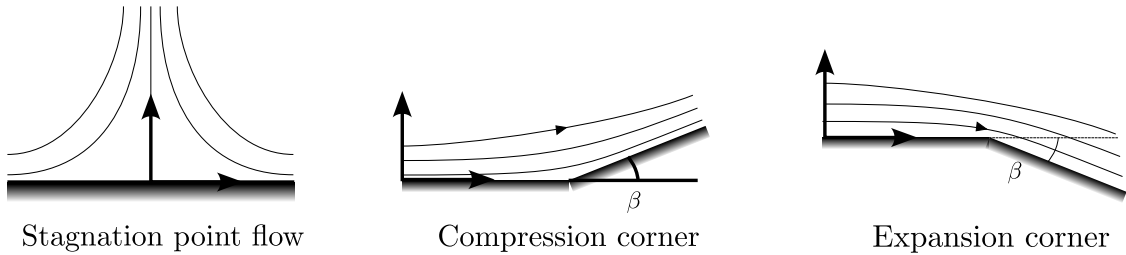


Figure 3: Planar Falkner-Skan solutions

- Stagnation point flow
 - Compression corner with angle $\beta = \pi/6$
 - Expansion corner with angle $\beta = \pi/6$
4. Consider a fluid exiting a narrow slot with its lower boundary being a planar wall along the x axis. This flow is known as a *wall jet* instead of a free jet.

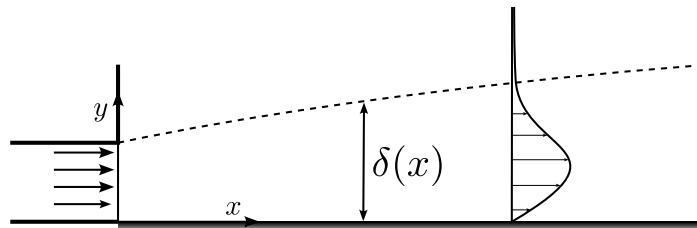


Figure 4: A planar wall jet

- To derive the similarity conditions, assume $\psi = ax^m f(\eta)$ where $\eta = y/bx^n$. Substitute in the boundary layer equation to obtain an equation in m and n .

- (b) Unlike a free jet, momentum flux in a wall jet is not constant in x . Draw a control volume to show that the wall-shear modifies the momentum flux in x .
- (c) To derive the second condition, integrate the momentum equation (with boundary layer approximation) in y from y to ∞ . Multiply the result with u and again integrate in y from 0 to ∞ . Using the continuity equation for manipulations, show that,

$$\int_0^{\infty} \left(u \int_y^{\infty} u^2 dy \right) dy = \text{const.}$$

- (d) Using the above condition, find values of m and n for a self-similar solution.
- (e) State the appropriate boundary conditions and solve the ODE. You may solve for another function g which satisfies $g^2 = f$. You will obtain an implicit solution.
- (f) Plot the profile of velocity at three different x locations.