

Practice problems : Hydrodynamic stability

APL361 Prateek Gupta

Department of Applied Mechanics, IIT Delhi

1. (Rayleigh-Plateau instability) Consider the instability of a column of an inviscid fluid (a capillary jet) of radius $a(z, t)$. In this problem, we will account for the Capillary effects (surface tension) forces as well. Use cylindrical coordinates (z, r, θ) . Given the flow is assumed inviscid, the governing equations are,

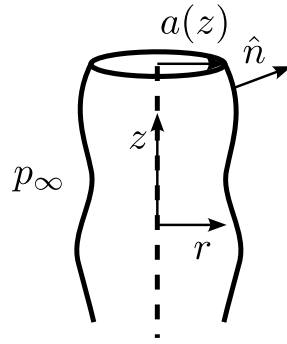


Figure 1: A capillary jet.

$$\nabla \cdot \mathbf{u} = 0,$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla p}{\rho}.$$

The pressure p inside the capillary jet is related to the atmospheric pressure p_∞ via the Laplace-Young equation,

$$p = p_\infty + \sigma \nabla \cdot \hat{n}$$

at the surface, where σ is the constant surface tension coefficient (for variable surface tension, the equation is slightly different).

- (a) Let the capillary jet be falling down with a uniform constant speed of U_0 . In the frame moving with the jet, derive the linearized stability equations. Assume the radius of jet is given by $a(z, t) = a_0 + \xi'(z, t)$. Clearly specify the use of surface kinematic boundary conditions.
- (b) Using the normal mode assumption of the form,

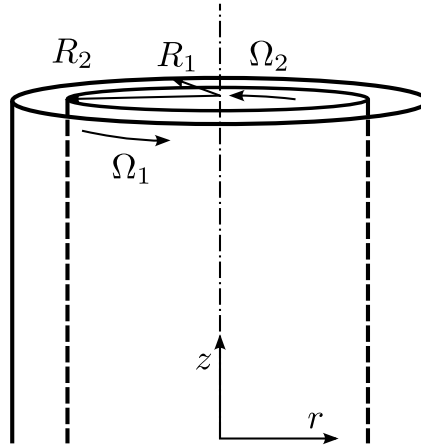
$$p' = \hat{p}(r) e^{i(\omega t + kz + n\theta)},$$

show that the governing equation for $\hat{p}$ is,

$$\frac{d^2 \hat{p}}{dr^2} + \frac{1}{r} \frac{d\hat{p}}{dr} - \left(k^2 + \frac{n^2}{r^2} \right) \hat{p} = 0.$$

- (c) State the boundary conditions for $\hat{p}(r)$. Using Bessel's functions, solve the above equation.
- (d) Solve the eigenvalue relation for ω and plot the variation of $i\omega$ vs the wavenumber of perturbations k .

2. Solve for the Kelvin-Helmholtz instability as done in class, considering $\rho_1 \neq \rho_2$.
3. Solve the problem covered in class for differentially heated plates with temperature T_0 at $y = -h/2$ and $T_0 - \Delta T$ at $y = h/2$. Reduce the governing equations as coupled differential eigenvalue problem. Solve using the Chebyshev polynomials based code shared.
4. Use the Chebyshev polynomials based code to plot various eigenmodes of a laminar channel flow (Poiseuille flow). Sketch the flow features induced by the first three perturbations.
5. (Taylor-Couette flow instability) Flow between two cylinders rotating relative to each other undergoes instability for some values of relative rotation rates. Consider two very long cylinders of radii R_1 and $R_2 < R_1$ rotating with Ω_1 and Ω_2 about their axis as shown. Fluid of viscosity μ and density ρ is filled between the two.



- (a) Assuming homogeneous z direction, solve for the axisymmetric base flow of fluid between the cylinders.
- (b) Linearize the governing equations about the axisymmetric base flow. Assume perturbations are also axisymmetric.
- (c) Assume the following normal mode assumption for all the variables,

$$p' = \hat{p}(r)e^{i(\omega t + kz)}.$$

Substituting the normal modes, obtain the differential eigenvalue problem. State the boundary conditions for all modes.