

# APL361 (2025-2026-I): Quiz-1

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Maximum score is 40. There are 3 questions in total. Time limit is 60 minutes. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

1. Evaluate/Prove using index notation (no points for expanding individual summation terms and calculating/brute force), **(14)**

(a)  $\delta_{ij}\delta_{ij}$  **(1)**

(b)  $\epsilon_{ijk}\epsilon_{ijk}$  **(2)**

(c)  $\underline{u} \cdot \nabla \underline{u} = \nabla \left( \frac{|\underline{u}|^2}{2} \right) - \underline{u} \times \underline{\omega}$  where  $\underline{\omega} = \nabla \times \underline{u}$  **(3)**

- (d) Determinant of a second order tensor  $\underline{T}$  and components of it's inverse can be written as,

$$\det(\underline{T}) = \frac{1}{6} \epsilon_{ijk} \epsilon_{lmn} T_{li} T_{mj} T_{nk}, \text{ and } [T^{-1}]_{ij} = \frac{1}{2\det(\underline{T})} \epsilon_{jpk} \epsilon_{ikl} T_{pk} T_{ql}.$$

Show that  $\frac{\partial(\det(\underline{T}))}{\partial T_{ij}} = \det(\underline{T}) [T^{-1}]_{ji}$ . **(4)**

- (e) Let  $\underline{r} = r_i \hat{e}_i = x_i \hat{e}_i$  be the position vector of a point, with magnitude  $r$ . Simplify,  $\nabla^2 (r^p)$ . **(4)**

2. Electrohydrodynamics is the study of electric field driven fluid flow of a dielectric fluid (fluid with bulk charge density). For a steady electric field  $\underline{E}$  on a dielectric fluid of permittivity  $\epsilon$ , the Coulombic force per unit volume is given by  $\underline{F}_e = \underline{E} \nabla \cdot (\epsilon \underline{E}) - \frac{1}{2} |\underline{E}|^2 \nabla \epsilon$ . Steady electric field implies  $\nabla \times \underline{E} = 0$ . Show that the force  $\underline{F}_e$  can be written as,

$$F_{ei} = \frac{\partial T_{ij}}{\partial x_j}.$$

Determine  $T_{ij}$  tensor. (Hint:- Start by simplifying  $\epsilon \underline{E} \times (\nabla \times \underline{E})$ ). **(12)**

3. The rate of deformation work done is given by  $\phi = T_{ij} \frac{\partial u_i}{\partial x_j}$ , where  $\underline{T}$  is a symmetric stress tensor and  $\underline{u}$  is velocity.

- (a) Show that  $\phi = T_{ij} S_{ij}$  ( $\underline{S}$  is the symmetric part of velocity gradient tensor) **(2)**.

- (b) For a particular fluid, the stress tensor is given by  $T_{ij} = -p\delta_{ij} + 2\mu S_{ij} - \frac{2}{3}\mu (\nabla \cdot \underline{u}) \delta_{ij}$ , where  $p$  is a field and  $\mu$  can be taken as constant. Show that  $\phi + p \nabla \cdot \underline{u} \geq 0$ . (Hint:- Think of type of quantities which are always  $\geq 0$ ) **(12)**