

APL361 (2025-2026-I): Quiz-3

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Maximum score is 40. There is only 1 problem. Time limit is 60 minutes. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

Important relations

- Axisymmetric incompressible flow in spherical coordinates :

$$u_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta}, u_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r},$$

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right)$$

$$\nabla^2 \mathbf{u} = \left(\frac{\partial}{\partial r} \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u_r}{\partial \theta} \right) - \frac{2}{r^2 \sin \theta} \frac{\partial (u_\theta \sin \theta)}{\partial \theta} \right) \hat{e}_r + \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (u_\theta \sin \theta) \right) \right) \hat{e}_\theta$$

- For ODE : $(1 - \eta^2)f'''' - 4\eta f''' + 4f'' = 0$, consider two solutions $f = \eta^3$ and $f = \eta$.

Problem

1. Consider incompressible flow in a cone with polar angle θ_0 due to a source of constant flow rate Q_0 at the apex (corner) of the cone. Assume purely radial flow. We will solve for the flow away from the apex.

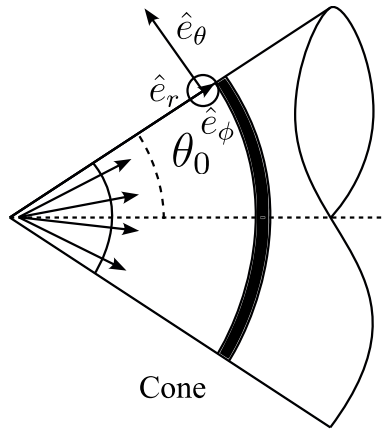


Figure 1: Flow due to a source at the apex of a cone

- (a) If constant flow rate Q_0 is maintained, then what is the form of the streamfunction? **(5)**
- (b) Assume low Reynolds number axisymmetric flow. Convert the coordinates from (r, θ) to (r, η) where $\eta = \cos \theta$. State the governing equation in terms of r and η . **(5)**
- (c) Solve for the streamfunction. Using the appropriate boundary conditions, solve for the radial velocity u_r . **(20)**
- (d) If the pressure at $r \rightarrow \infty$ is p_∞ , then find the pressure variation inside the cone. **(10)**