

APL361 (2025-2026-I): Mid-term exam

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Maximum score is 120, time limit is 120 minutes. There are three problems in total. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

Important relations

- Axisymmetric incompressible flow in spherical coordinates:

$$1. \quad u_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta}, \quad u_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r},$$

$$2. \quad E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right)$$

$$3. \quad \nabla^2 \mathbf{u} = \left(\frac{\partial}{\partial r} \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u_r}{\partial \theta} \right) - \frac{2}{r^2 \sin \theta} \frac{\partial (u_\theta \sin \theta)}{\partial \theta} \right) \hat{e}_r + \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (u_\theta \sin \theta) \right) \right) \hat{e}_\theta$$

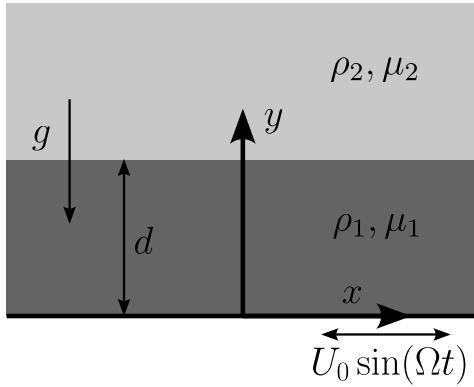
$$4. \quad \tau_{rr} = 2\mu \frac{\partial u_r}{\partial r}, \tau_{\theta\theta} = 2\mu \left(\frac{1}{r} \frac{\partial u_\theta}{\partial \theta} \right), \tau_{r\theta} = \mu \left(r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right).$$

- One root of $x^3 - 6x^2 - x + 30 = 0$ is $x = 3$.

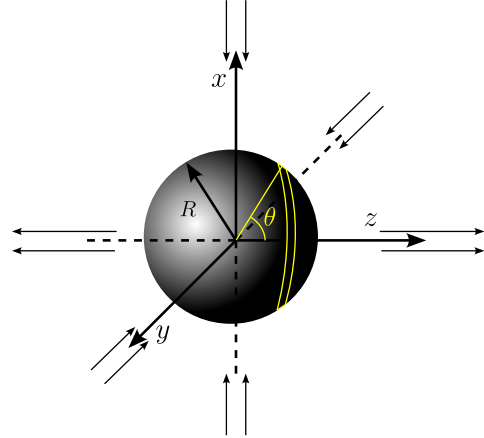
Problems

1. In class, we have solved for velocity which yields a prescribed vorticity field. A velocity dilatational field is defined as $\nabla \cdot \underline{u}$. Using the Helmholtz' decomposition theorem and Green's function in 3D, derive the velocity field which yields a prescribed dilatational field $D(\underline{x})$. That is, solve for $\underline{u}$ such that $\nabla \cdot \underline{u} = D(\underline{x})$. **(20)**
2. Consider an infinitely long vibrating plate under two fluid layer as shown below. A fluid of density ρ_1 and viscosity μ_1 forms a layer of height d and a fluid of density ρ_2 and viscosity μ_2 forms the layer on top extending to $y \rightarrow \infty$. The plate vibrates with speed $U_0 \sin(\Omega t)$ as shown. Gravity acts downwards. Pressure at the interface of two fluids is p_0 .
 - (a) Show that the flow is unidirectional. **(2)**
 - (b) Simplify the equations governing the flow velocity. **(3)**
 - (c) Derive all the boundary/interface conditions required to solve the flow velocity. **(5)**
 - (d) Derive the pressure variation in the whole region above the plate. **(5)**
 - (e) Solve for the complex velocity amplitudes of the oscillatory solution of the governing equations using the above derived boundary/interface conditions. **(20)**

- (f) Simplify the above solution for $\rho_2 = \frac{\rho_1}{K}$ and $\mu_2 = K\mu_1$. State the full real oscillatory solution. **(5)**



Problem 1



Problem 2

3. An extensional flow symmetric about z axis is defined by flow converging on z axis in xy plane. The velocity field is given by,

$$\underline{u} = -E \left(x\hat{i} + y\hat{j} - 2z\hat{k} \right),$$

where E is some parameter. The flow tends to extend the z axis, hence the name extensional flow.

- Calculate the velocity gradient for this flow and its symmetric and anti-symmetric components. **(3)**
- Consider a sphere of radius R in extensional flow of a fluid with viscosity μ and density ρ . State the velocity at $r \rightarrow \infty$ in spherical coordinate system. **(7)**
- What is the form of stream-function which satisfies the above velocity at $r \rightarrow \infty$? **(10)**
- Using the above form, solve for the stream-function for the extensional flow past the sphere. **(20)**
- Using the above solution, solve for the pressure force on the sphere. **(15)**
- What is the total force on the sphere due to the flow? **(5)**