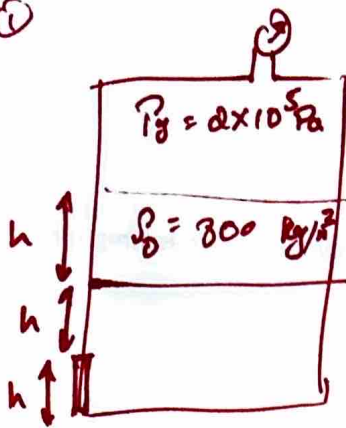


①



$P_0 = 10^5 \text{ Pa}$ (ATMOSPHERIC PRESSURE IS $\approx 101325 \text{ Pa}$)

$$\begin{aligned}
 & p_g + \rho_0 x g h + \rho_0 g h = p_0 \\
 & = 2.18 \times 10^5 \\
 & p_g + \rho_0 x g h + 2 \rho_0 g h \\
 & = 2 \times 10^5 + 800 \times 10 \times 1 + 2 \times 1000 \times 10 \times 1 \\
 & = 2 \times 10^5 + 8 \times 10^3 + 2 \times 10^4 \\
 & = 2.28 \times 10^5 \text{ Pa} = p_b
 \end{aligned}$$

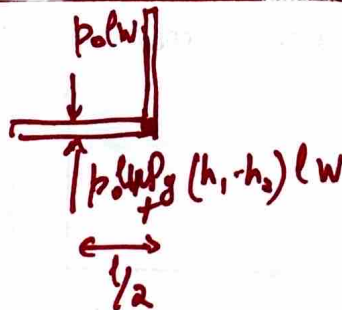
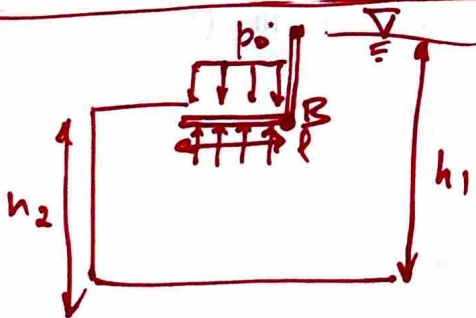
TOTAL FORCE

$$\left(p_u A + \int_0^h \rho_w g y dy - p_0 A \right) (-\hat{i})$$

$$\begin{aligned}
 & \left[(p_u - p_0) w h + \frac{\rho_w g h^2}{2} \right] (-\hat{i}) \\
 & = \left(0.59 \times 10^5 + 0.5 \times 10^4 \right) (-\hat{i}) \text{ N} + \frac{1}{2} \times 10^3 \times 10 \times 1^2 (-\hat{i}) \\
 & = 6.4 \times 10^5 \text{ N } (-\hat{i})
 \end{aligned}$$

LINE OF ACTION?

②

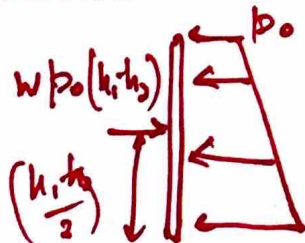


TOTAL VERTICAL FORCE

$$= \rho_g (h_1 - h_2) l w$$

MOMENT ABOUT B

$$\text{MOMENT FROM VERTICAL SECTION (ABOUT B)} = -\rho_g (h_1 - h_2) \frac{l^2}{2} w$$



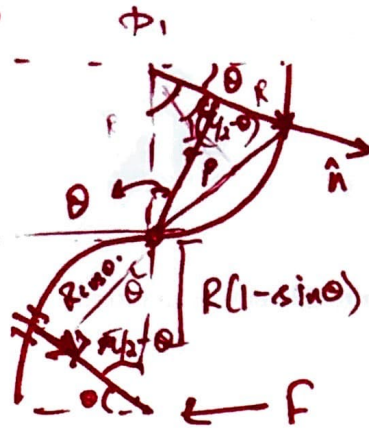
$$\begin{aligned}
 & -\cancel{p_0 w (h_1 - h_2)^2} + \cancel{p_0 w (h_1 - h_2)^2} + \int_0^{h_1 - h_2} \rho_g y^2 dy \\
 & = \rho_g (h_1 - h_2)^3 w
 \end{aligned}$$

GATE ROTATES FOR

$$\frac{\rho_g (h_1 - h_2)^3 w}{3} \geq \frac{\rho_g (h_1 - h_2)^3 w}{2}$$

(SUBSTITUTE VALUES)

IN EXAM



$$\int w(p_1 + \rho_g R \sin \theta) R d\theta \hat{n} \rightarrow \text{FORCE TOP CURVE}$$

$$w \int_0^{\pi/2} (p_1 + \rho_g R \sin \theta) R d\theta R \cos \theta \rightarrow \text{MOMENT}$$

$$p_1 R^2 w + \rho_g w R^3 \int_0^{\pi/2} \sin \theta \cos \theta d\theta$$

$$= p_1 R^2 w + \frac{\rho_g w R^3}{2}$$

BOTTOM CURVE

$$\int_0^{\pi/2} ((p_1 + \rho_g R) + \rho_g R(1 - \sin \theta)) R \cos \theta w R d\theta$$

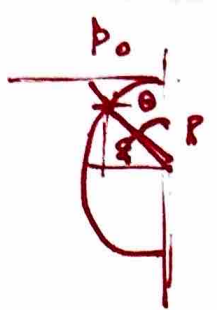
$$= (p_1 + \rho_g R) R^2 w + \rho_g R^3 w \int_0^{\pi/2} (1 - \sin \theta) \cos \theta d\theta$$

$$= R^2 w p_1 + \rho_g R^3 w + \rho_g R^3 w (1 - \frac{1}{2})$$

$$= p_1 R^2 w + \frac{3}{2} \rho_g R^3 w$$

NET MOMENT REQUIRED BY F

$$= \rho_g R^3 w = F R \Rightarrow \frac{F}{w} = \rho_g R^2$$

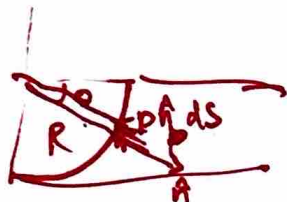


$$F = \int \left[p_0 + \rho_g R (1 - \cos \theta) \right] (R d\theta w)$$

$$F_{\text{LEFT}} = \cancel{p_0 R w} + \rho_g R^2 w \int_0^\pi \sin \theta (1 - \cos \theta) d\theta$$

$$F_{\text{LEFT}} = 2 p_0 R w \hat{i} + \rho_g R^2 w \left(2 \right) \hat{i} - \rho_g R^2 w \left[-\frac{\pi}{2} \right] \hat{j}$$

$$= 2 p_0 R w \hat{i} + 2 \rho_g R^2 w \hat{i} + \frac{\pi}{2} \rho_g R^2 w \hat{j}$$



$$F_{\text{RIGHT}} = w \int_0^{\pi/2} (p_0 + \rho_g R \sin \theta) R d\theta (-\cos \theta \hat{i} + \sin \theta \hat{j})$$

$$= -p_0 R w \hat{i} + w \int_0^{\pi/2} \rho_g R^2 \sin \theta \cos \theta d\theta (-\hat{i}) + p_0 R w \hat{j} + w \int_0^{\pi/2} \rho_g R^2 \sin^2 \theta d\theta \hat{j}$$

$$= -p_0 R w \hat{i} + \frac{\rho_g R^2 w}{2} \int_0^{\pi/2} \sin 2\theta d\theta (-\hat{i}) + p_0 R w \hat{j} + \frac{\rho_g R^2 w}{2} \int_0^{\pi/2} (1 - \cos 2\theta) d\theta \hat{j}$$

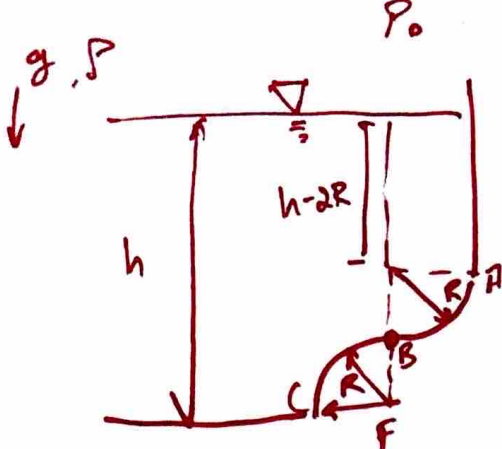
$$F_{\text{LEFT}} + F_{\text{RIGHT}}$$

$$= p_0 R w \hat{i} + \frac{3 \rho_g R^2 w}{2} \hat{i} + p_0 R w \hat{j} + \frac{3 \rho_g R^2 w \pi}{4} \hat{j}$$

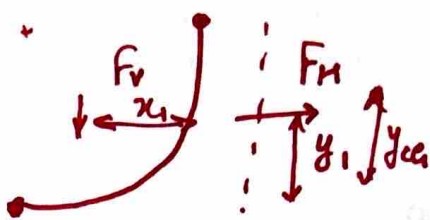
$$= -R_c$$

$$= -p_0 R w \hat{i} - \frac{\rho_g R^2 w}{2} \hat{i} + p_0 R w \hat{j} + \frac{\rho_g R^2 w \pi}{4} \hat{j}$$

COMPARE THIS WITH THE SIMPLIFIED INTUITION (NO VERTICAL FORCE FROM LEFT) & ARGUE WHY ITS WRONG.



FORCES WILL BE BALANCED BY
HINGE REACTIONS. WE ONLY NEED
MOMENTS
ASSUME WIDTH W



$$F_h = p y_{CG} A_{\text{PROJECTED}}$$

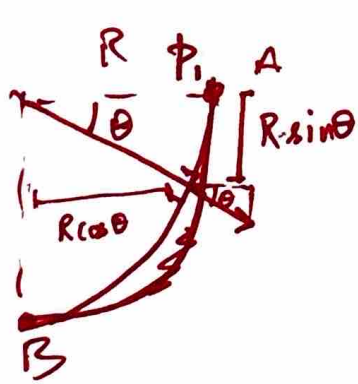
$$A_{\text{PROJECTED}} = R w$$

$$F_h = \frac{p R R w y}{2} = \frac{p R^2 w y}{2}$$

$$y_1 = \frac{R}{2} + \frac{I_{xx \text{ PROJECTED}}}{\left(\frac{R}{2}\right) A_{\text{PROJECTED}}} \quad \text{FROM B}$$

$$= \frac{R}{2} + \frac{\frac{1}{12} W R^3}{\frac{R^2 W}{2}} = \frac{R}{2} + \frac{R}{6} = \frac{2R}{3} \quad \text{FROM B}$$

$$F_v = (h-2R) p_0 R w + p_0 \frac{\pi R^2 w}{4}$$



$$w \int_0^{\pi/2} (p_1 + p_0 R \sin \theta) R d\theta R \cos \theta \sin \theta$$

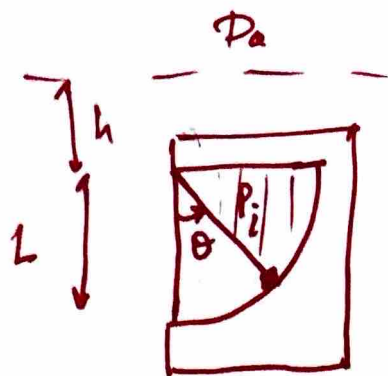
$$= w \int_0^{\pi/2} p_1 R^2 \sin \theta \cos \theta d\theta$$

$$+ w p_0 R^3 \int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta$$

→ MOMENT OF
VERTICAL
FORCE
ABOUT B

$$= \frac{w p_1 R^2}{2} + \frac{p_0 R^3 w}{3}$$

$$\text{MOMENT OF } F_v = \frac{w R^2}{2} (h-2R) p_0 + p_0 \frac{R^2 w}{3}$$



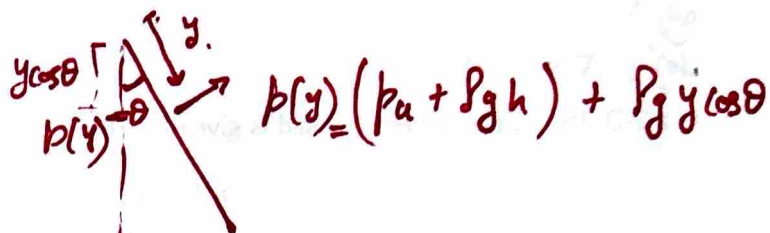
$$P_i V_i = p_a \frac{\pi L^2}{4} W$$

$$V_i = \left(\frac{\pi}{4} - \theta\right) L^2 W \Rightarrow P_i = \left(\frac{\frac{\pi}{4}}{\frac{\pi}{4} - \theta}\right) p_a$$

$$P_i = \left(\frac{\pi}{\pi - 2\theta}\right) p_a$$

IGNORE p_a at gh

FOR ANY HEIGHT y



$$\int_0^L y W b(y) dy \rightarrow \text{MOMENT ABOUT HINGE FROM OUTSIDE}$$

$$P_i L W \frac{L}{2} \rightarrow \text{MOMENT} \quad \text{"} \quad \text{FROM INSIDE}$$

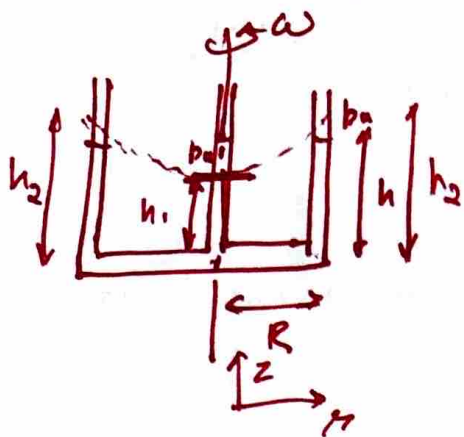
$$\begin{aligned} \Rightarrow \int_0^L \left((p_a + p_g h) + p_g y \cos \theta \right) y W dy \\ = p_a \frac{W L^2}{2} + p_g h \frac{W L^2}{2} + p_g \frac{W \cos \theta L^3}{3} \\ = \left(\frac{\pi}{\pi - 2\theta} \right) p_a \frac{W L^2}{2} \end{aligned}$$

$$\Rightarrow 1 + \left(\frac{p_g h}{p_a} \right) + \frac{2 p_g L \cos \theta}{p_a} = \frac{\pi}{\pi - 2\theta}$$

$$\frac{p_g h}{p_a} = \left(\frac{\pi}{\pi - 2\theta} \right) - 1 - \frac{2 p_g L \cos \theta}{p_a}$$

$$h = \left(\frac{\pi}{\pi - 2\theta} \right) \frac{p_a}{p_g} - \frac{p_a}{p_g} - 2 L \cos \theta$$

⑥



$$\nabla p = \rho(\underline{g} - \underline{a})$$

$$\underline{a} = -\omega^2 R \hat{e}_r$$

$$\nabla p = \rho(-g \hat{e}_z + \omega^2 R \hat{e}_r)$$

$$\frac{\partial p}{\partial z} = -\rho g, \quad \frac{\partial p}{\partial r} = \rho \omega^2 R$$

$$p = -\rho g z + \frac{\rho \omega^2 R^2}{2} + C$$

At

$$-\rho g h_1 + \frac{\rho \omega^2 (0)^2}{2}$$

$$= -\rho g h_2 + \frac{\rho \omega^2 R^2}{2}$$

$$2h_2 + h_1 = 3h$$

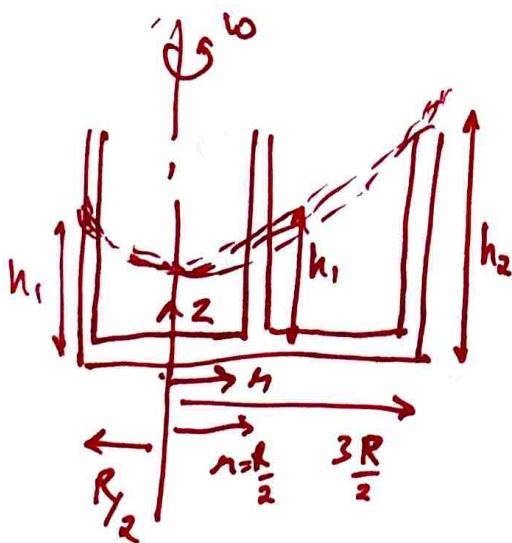
$$h_2 - h_1 = \frac{\omega^2 R^2}{2g}$$

$$\Rightarrow \rho g (h_2 - h_1) = \frac{\rho \omega^2 R^2}{2}$$

$$h_2 - h_1 = \frac{\omega^2 R^2}{2g}$$

$$h_2 = h + \frac{\omega^2 R^2}{2g}$$

$$h_1 = h - \frac{\omega^2 R^2}{2g}$$



$$p = -\rho g z + \frac{\rho \omega^2 R^2}{2} + C$$

$$-\rho g h_1 + \frac{\rho \omega^2 R^2}{2} \frac{1}{4} = -\rho g h_2 + \frac{\rho \omega^2 9R^2}{2} \frac{1}{4}$$

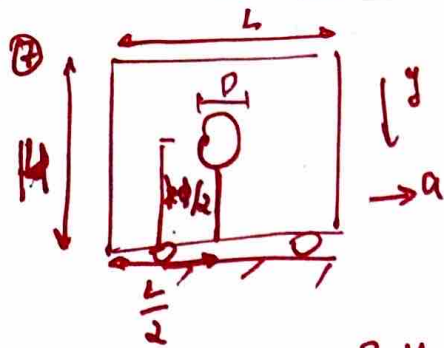
$$h_2 - h_1 = \frac{\omega^2 R^2}{2g} \left(\frac{9}{4} - \frac{1}{4} \right) = \frac{\omega^2 R^2}{g}$$

$$2h_1 + h_2 = 3h$$

$$h_2 - h_1 = \frac{\omega^2 R^2}{g}$$

$$\Rightarrow h_1 = h - \frac{\omega^2 R^2}{3g}$$

$$h_2 = h + \frac{2\omega^2 R^2}{3g}$$

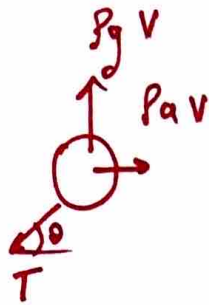


$$\nabla p = \rho(-g\hat{j} - a\hat{i})$$

$$= -\rho(g\hat{j} + a\hat{i})$$

FORCE ON BALLOON DUE TO WATER

$$-\int \nabla p dV = (\rho g\hat{j} + \rho a\hat{i})V$$

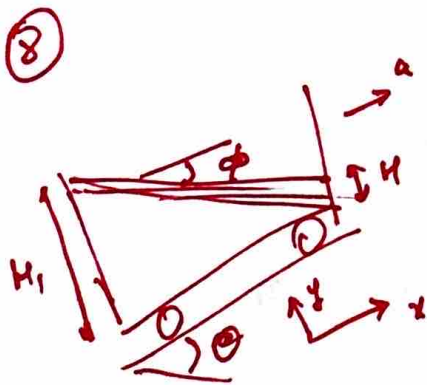


$$-T \cos \theta + \rho a V = \rho_{ne} a V$$

$$T \sin \theta + \rho_{ne} g V = \rho g V$$

$$\tan \theta = \frac{(\rho - \rho_{ne})gV}{(\rho - \rho_{ne})aV} = \frac{g}{a}$$

LEANS FORWARD



$$\nabla p = \rho(\underline{g} - \underline{a})$$

$$= \rho(-g \cos \theta \hat{j} - (a + g \sin \theta) \hat{i})$$

$$\frac{\partial p}{\partial y} = -\rho g \cos \theta, \quad \frac{\partial p}{\partial x} = -\rho(a + g \sin \theta)$$

$$p = -\rho g \cos \theta y - \rho(a + g \sin \theta)x + c$$

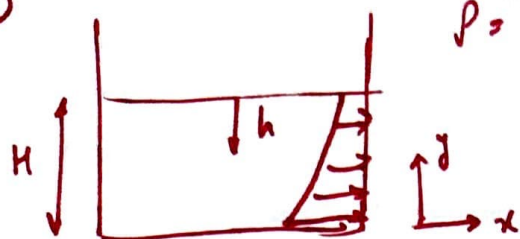
$$y = \frac{-p}{\rho g \cos \theta} - \frac{a + g \sin \theta}{g \cos \theta} x + \frac{c}{\rho g \cos \theta}$$

$$y = -\tan \phi x + K$$

$$\Rightarrow \tan \phi = \frac{a + g \sin \theta}{g \cos \theta} \Rightarrow \frac{3}{100} = \frac{a + 10/2}{10 \sqrt{3}/2}$$

$$a = 10(0.866)(0.03) - 5 < 0$$

⑨



$$\rho = \rho_0(1 + ky)$$

$$\nabla p = \rho g$$

$$= -\rho_0(1 + ky)g \hat{j}$$

$$h = H - y$$

$$= -\rho_0(1 + k(H - y))g \hat{j}$$

$$p = -\rho_0(y + kHy - \frac{ky^2}{2})g + C$$

$$p_0 = -\rho_0(H + kH^2 - \frac{kH^2}{2})g + C$$

$$C = p_0 + \rho_0 g (H + kH^2(\frac{1}{2} - 1))$$

$$p = p_0 + \rho_0 g (H - y)$$

$$+ \rho_0 g k H (H - y) - \rho_0 g k (\frac{H^2}{2} - \frac{y^2}{2})$$

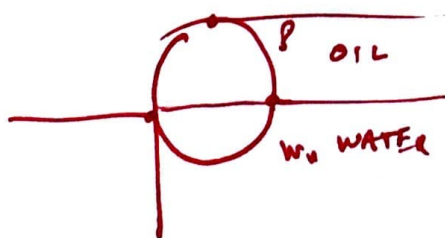
Force

$$w \int_0^H (p_0 + \rho_0 g (H - y) + \rho_0 g k H^2 - \rho_0 g k y^2 - \rho_0 g k H y) dy$$

$$p_0 w H + \rho_0 g \frac{H^2 w}{2} + w \rho_0 g k \frac{H^3}{2} - \rho_0 g k \frac{H^3 w}{2} + \rho_0 g k \frac{H^3 w}{6}$$

$$= p_0 w H + \rho_0 g \frac{H^2 w}{2} + \rho_0 g k \frac{H^3 w}{6}$$

⑩



$$F_v = w \int_0^{\pi/2} \rho g R^2 d\theta (1 - \sin\theta) \sin\theta$$

$$= w \rho g R^2 \left[\int_0^{\pi/2} (-\cos\theta) d\theta - \int_0^{\pi/2} \sin^2\theta d\theta \right]$$

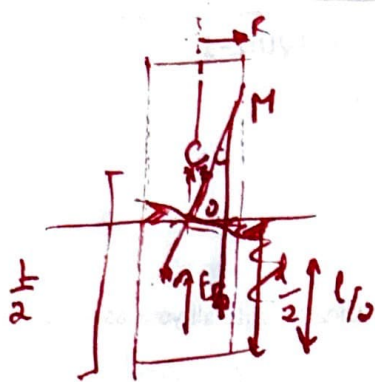
$$= w \rho g R^2 \left[1 - \frac{\pi}{4} \right]$$



$$\uparrow F_v = \rho_w g \frac{\pi R^2 w}{2}$$

$$mg = \rho_w g \frac{\pi R^2 w}{2} - \rho g \frac{\pi R^2 w}{4} \left(1 - \frac{\pi}{4} \right) \rightarrow \frac{mg}{w} \text{ FIND}$$

u



$$MC = \frac{I_0}{V_{sub}} - BC$$

$$BC = \left(\frac{L}{2} - \frac{l}{2} \right)$$

$$I_0 = \frac{\pi R^4}{4}$$

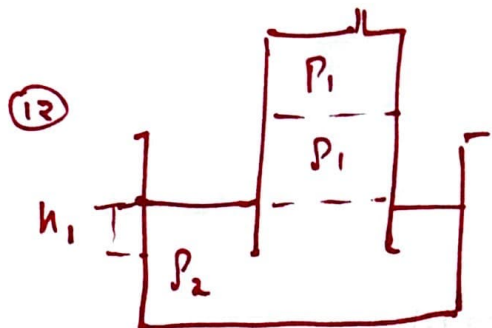
$$V_{sub} = \pi R^2 l$$

$$\rho_w g \pi R^2 l = \rho_l g \pi R^2 L$$

$$\frac{\pi R^4}{4 \pi R^2 l} > \frac{L}{2} - \frac{l}{2} \Rightarrow \frac{R^2}{L^2} > 2 \frac{L}{L} - \frac{2l^2}{L^2}$$

$$\frac{\rho_s}{\rho_w} = \frac{l}{L}$$

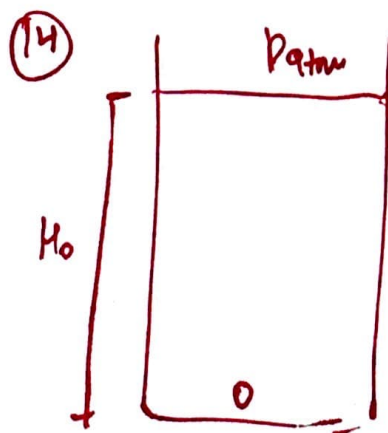
$$\Rightarrow \frac{R}{L} > \sqrt{2 \frac{\rho_s}{\rho_w} \left(\frac{\rho_s}{\rho_w} \right)}$$



$$P_{atm} + \rho_2 h_1 g = P_1 + \rho_1 g h_2 + \rho_2 g h_1$$

$$P_1 = P_{atm} - \rho_1 g h_2$$

13 COVERED IDU CLASS



INITIALLY

$$P_{bubble} = P_{atm} + \rho g H_0 + \frac{2\sigma}{R}$$

~~P_0 + \rho~~

$$P_0 R_0^3 = P(z) R(z)^3$$

ISOTHERMAL CHANGES

$$P(z) = P_{atm} + \rho g (H_0 - z) + \frac{2\sigma}{R(z)}$$

$$P_0 = P_{atm} + \rho g H_0 + \frac{2\sigma}{R_0} \quad \text{SURFACE TENSION}$$

$$(P_{atm} + \rho g H_0) R_0^3 + 2\sigma R_0^2 = (P_{atm} + \rho g (H_0 - z)) R^3 + 2\sigma R^2$$

b) SMALL CHANGES (LINEARIZATION)

$$R^3 (p_{\text{atm}} + \rho_g H_0) + (2\sigma) R^2 = \text{constant} + \rho_g R^3 z$$

$$3R (p_{\text{atm}} + \rho_g H_0) dR + 4\sigma R^2 dR = 3\rho_g R^2 z dR + \rho_g R^3 dz$$

$$\left. \frac{dR}{dz} \right|_{R_0, z=0} = \frac{\rho_g R_0^3}{3R_0 (p_{\text{atm}} + \rho_g H_0) + 4\sigma R_0^2}$$

$$R(z) = R(z=0) + \left. \frac{dR}{dz} \right|_{z=0} z + \dots$$

OR

$$(R_0 + R')^3 (p_{\text{atm}} + \rho_g H_0) + 2\sigma (R_0 + R')^2 = \text{const} + \rho_g (R_0 + R')^3 z$$

$$R_0^3 \left(1 + \frac{3R'}{R_0}\right) (p_{\text{atm}} + \rho_g H_0) + 2\sigma R_0^2 \left(1 + \frac{2R'}{R_0}\right)$$

$$= \text{const} + \rho_g R_0^3 \left(1 + \frac{3R'}{R_0}\right) z$$

NOTE $\text{const} = (p_{\text{atm}} + \rho_g H_0) R_0^3 + 2\sigma R_0^2$

So constant cancels RHS terms without (')

$$\Rightarrow 3R_0^2 R' (p_{\text{atm}} + \rho_g H_0) + 4\sigma R_0 R' = \rho_g R_0^3 z$$

c) $F_B = \rho_w \frac{4}{3} \pi R^3 g$, $F_g = \rho_{\text{air}} \frac{4}{3} \pi R^3 g$

$$p_{\text{air}}(z) = \frac{p_0 \rho_0}{\rho(z)} \quad \text{OR} \quad p_{\text{air}}(z) R(z)^3 = p_0 R_0^3$$

$$m \frac{dz}{dt^2} = \rho_w \frac{4}{3} \pi R^3 g - m g.$$

$$\rho_{\text{air}} = \frac{\rho_0 R_0^3}{R^3} = \frac{\rho_0 R_0^3}{(R_0 + R')^3}$$

$$\approx \rho_0 \left(1 - \frac{3R'}{R_0}\right)$$

$$\frac{d^2 z}{dt^2} = \frac{\rho_w R(z)^3}{\rho_0 R_0^3} g - g \quad \text{NOTE } K \propto \left. \frac{dR}{dz} \right|_{R_0} > 0$$



SINCE RUBBLE EXPANDS

FOR SMALL $\frac{\rho_0}{\rho_w}$, $\frac{d^2 z}{dt^2} \approx Kz + C$

$$\Rightarrow z = \frac{C}{K} (e^{\sqrt{K}t} - 1) \Rightarrow z = \frac{AC}{K} + D$$

$$z(t=0) = 0$$

$$\Rightarrow A + D = 0$$

$$z = Ae^{\sqrt{K}t} + Be^{-\sqrt{K}t} + D$$

$$\frac{d^2 z}{dt^2} = AKe^{\sqrt{K}t} - KBe^{-\sqrt{K}t} + 0 = K(Ae^{\sqrt{K}t} + D)$$

$$z(t=0) = 0 \Rightarrow A + B + D = 0$$

$$\dot{z}(t=0) = 0 \Rightarrow A\sqrt{K} - B\sqrt{K} = 0 \Rightarrow A = B$$

$$\Rightarrow D = -\frac{A}{K}$$

$$z = A(e^{\sqrt{K}t} + e^{-\sqrt{K}t}) - \frac{2A}{K}$$

$$= 2A(\cosh \sqrt{K}t - 1)$$

$$\frac{d^2 z}{dt^2} = 2AK \cosh \sqrt{K}t = 2AK(\cosh \sqrt{K}t - 1) + C$$

$$C = 2KA$$

$$A = \frac{C}{2K}$$

$$\Rightarrow z = \frac{C}{K} (\cosh \sqrt{K}t - 1) \rightarrow \text{grows exponentially in time}$$

Compute time constant for values given.

$$d) \quad \frac{d^2 z}{dt^2} = \frac{\rho_w R(z)^3}{\rho_0 R_0^3} g - g \approx \underbrace{6\pi\eta R(z)}_{\text{LINEARIZE}} \frac{dz}{dt} + (R_0 + e') \frac{dz}{dt} \approx R_0 \frac{dz}{dt}$$

$$\frac{d^2 z}{dt^2} = Kz + C \quad \text{STEADY VELOCITY}$$

$$DV = \int (Kz + C)$$