

Tutorial 4 : Statics and surface tension

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- Find the pressure at the bottom of the tank. Find the force on the gate if its width is $w = 0.5$ m ($\rho_w = 1000 \text{ kg/m}^3$, $\rho_a = 1 \text{ kg/m}^3$).

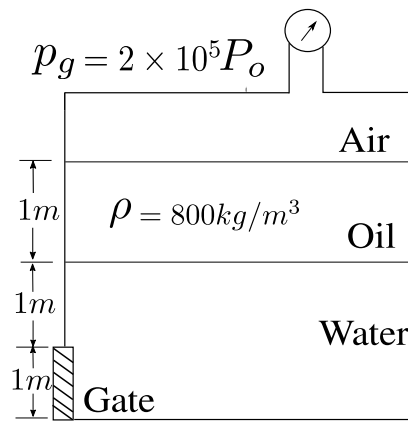


Figure 1: Problem 1.

- Gate ABC in the figure has a fixed hinge line at B and is 2 m wide into the paper. The gate will open at A to release water if the water depth is high enough. Compute the depth h for which the gate will begin to open.

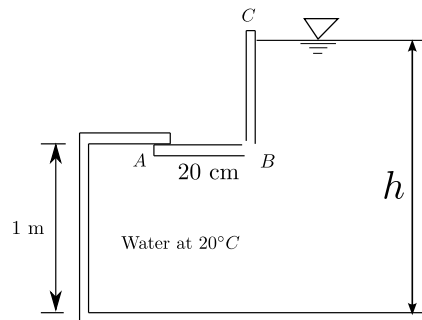


Figure 2: Problem 2.

- Given h , R , g and water density ρ , determine the horizontal force F per unit of length normal to the paper required to keep the S-shaped gate shown in figure 3 closed. Neglect friction in the bearing.

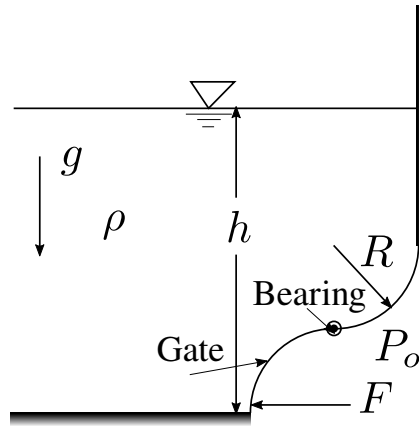


Figure 3: Problem 3.

4. A 1 m diameter log $\rho/\rho_w = 0.8$, 2 m long dams water as shown in figure 4. Compute the net vertical and horizontal reactions at point C.

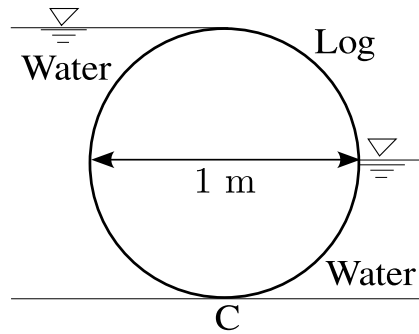


Figure 4: Problem 4.

5. The container shown in figure 5 (length b perpendicular to the sheet) has a massless door of height L which is freely hinged at the top. Initially the pressure of the air inside the container is atmospheric P_a , the same as outside, and the door hangs vertically with $\theta = 0$. Suppose now the container is pushed slowly down under water (density ρ). Assuming the pressure p_1 of the air in the container is related to the volume V_1 by $p_1 V_1 = \text{constant}$, derive an expression for the depth h of the container at which the door will be turned inward at an angle θ .

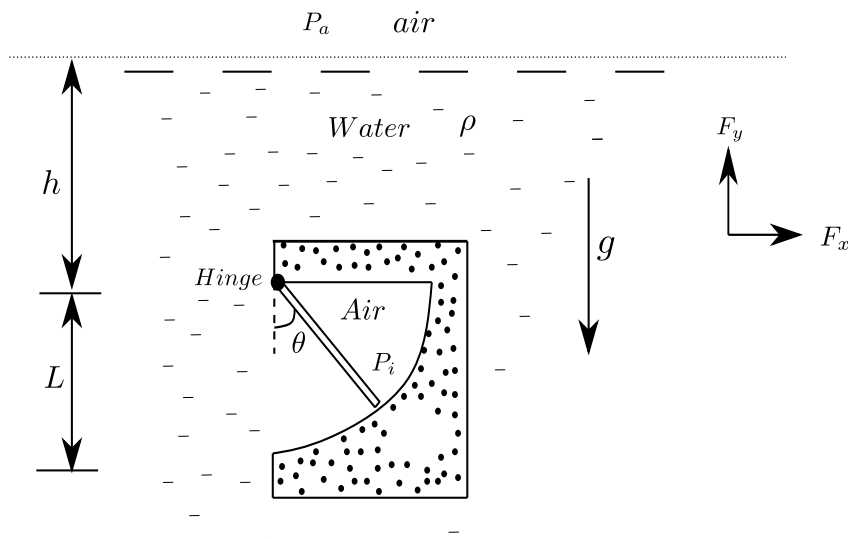


Figure 5: Problem 5.

6. The tube arrangement shown rotates about axis AA at $600/\pi$ rpm. Determine the levels in columns C_1 , C_2 , C_3 in the new position of equilibrium. If the axis is shifted to BB , what are the levels? The tubes are long enough to prevent spillage and the level h is when the tube is at rest.

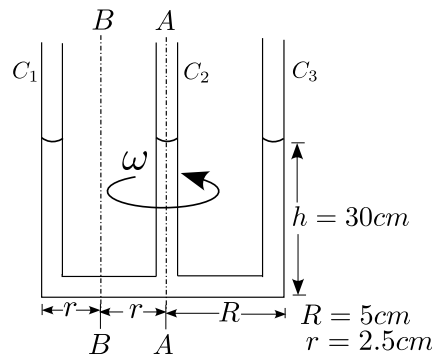


Figure 6: Problem 6.

7. The tank in figure 7 is filled with water and has a vent hole at A. The tank is 1 m wide into the paper. Inside the tank, a balloon 10 cm in diameter, filled with Helium at 130 kPa, is tethered centrally by a string. If the tank accelerates to the right at 5 m/s^2 in rigid body motion, at what angle will the balloon lean? Will it lean to the left or to the right?

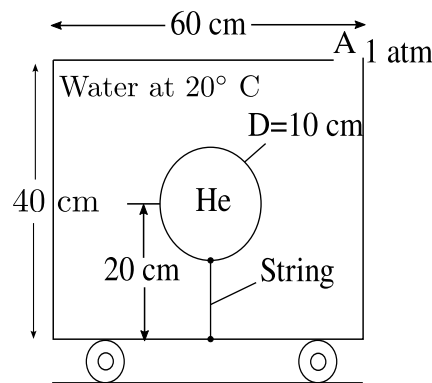


Figure 7: Problem 7.

8. A tank of liquid is moving with constant acceleration up a 30° inclined plane. Assuming rigid-body motion, compute,

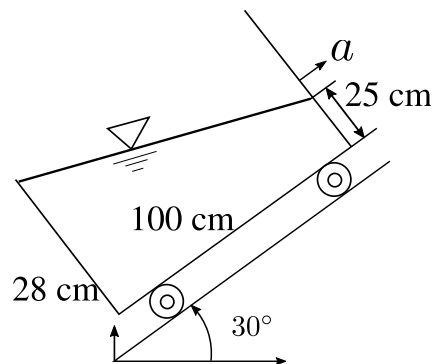


Figure 8: Problem 8.

- The value of acceleration a .
- Whether the acceleration is up or down, and

- (c) The gauge pressure at point A if the fluid is mercury.
9. A salt water tank is in the shape of a cuboid with its depth being 10 m. Due to salt, the density varies with the depth as $\rho = \rho_0(1 + kh)$, where h is the depth, $\rho_0 = 1000 \text{ kg/m}^3$ and $k = 0.03 \text{ m}^{-1}$.
- (a) Find the pressure distribution in the tank as a function of h .
- (b) Find the force on any one of the sides of the tank.
10. A log of diameter 1 m supports water and oil ($\rho/\rho_w = 0.82$) as shown. Find the weight per unit length of the log. Assume contact with the wall as frictionless.

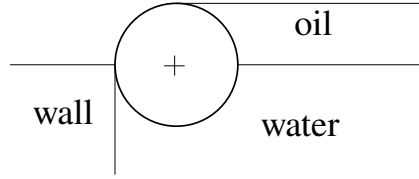


Figure 9: Problem 10.

11. Consider a homogeneous right circular cylinder of length L , radius R , and density ρ_s , floating in water of density ρ_w . Show that it is stable with its axis vertical if

$$\frac{R}{L} > \sqrt{2 \frac{\rho_s}{\rho_w} \left(1 - \frac{\rho_s}{\rho_w} \right)}.$$

12. Obtain the gauge pressure P_1 in terms of two densities ρ_1 and ρ_2 and level differences h_1 and h_2 shown in the figure.

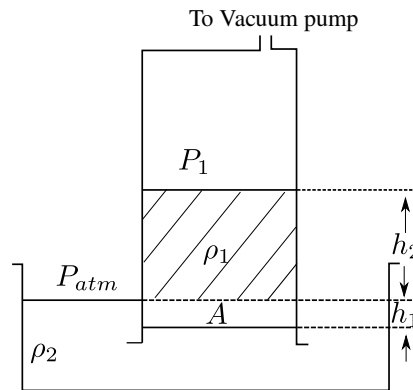


Figure 10: Problem 12.

13. (F.W. White, Problem 1.68) Water-air interface near a plane wall makes a shape $\eta(x)$ as shown. Ignore air density. Density of water is ρ and surface tension coefficient of the interface is σ . Wetting angle at $x = 0$ is known to be θ . For $x \rightarrow \infty$, a horizontal surface is observed.

- (a) Using $-\frac{1}{2} \nabla \cdot \hat{n}$ expression for the mean curvature, assuming planar shape in depthwise direction, derive an expression for the mean curvature.

- (b) Assuming small slope, approximate the mean curvature expression above.
- (c) What is the maximum height h at the wall.

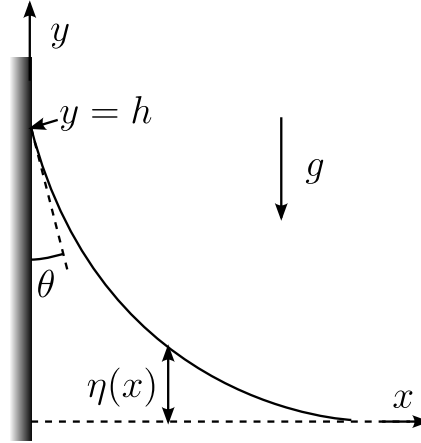


Figure 11: Problem 13

14. A bubble of air starts rising from the bottom in water of depth H_0 . Let the surface tension coefficient be σ . Initially, the bubble pressure is in equilibrium. Pressure on the surface of water column is p_{atm} . Density of water is ρ and density of air is ρ_a . As the bubble rises, it expands due to decreasing pressure outside.
- (a) For an initial radius R_0 and pressure p_0 , derive an equation for $R(z)$ where z is the instantaneous height of the bubble. Assume the bubble's expansion is isothermal and all the matter is at 300 K.
- (b) Assume small changes in air pressure $p_0 + p'$, density $\rho_0 + \rho'$, and bubble radius $R_0 + R'$. Assuming small changes, derive a linearized relation between R' and z . For linearization, substitute above expansions, apply binomial theorem, and ignore any products of primed quantities.
- (c) Solve for the bubble's height in time ignoring drag force. Plot the bubble's height vs time. Take (all values in SI units) $\rho_{\text{water}} = 1000$, $\mu = 10^{-3}$, $\sigma = 0.072$, $p_{\text{atm}} = 10^5$, $R_0 = 5 \times 10^{-4}$, $H_0 = 10$, $g = 9.81$, $\rho_{\text{air}} = 1.2$ at 300 K and atmospheric pressure.
- (d) Accounting for drag force (Stokes drag), derive an expression for bubble's height and velocity in time. Compare with previous part's solution.