

Tutorial 5 : Differential analysis

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1. A plate of radius R is pushed towards a stationary plate of radius R below with a constant velocity v_p . Oil of density ρ is being squeezed out between the plates.

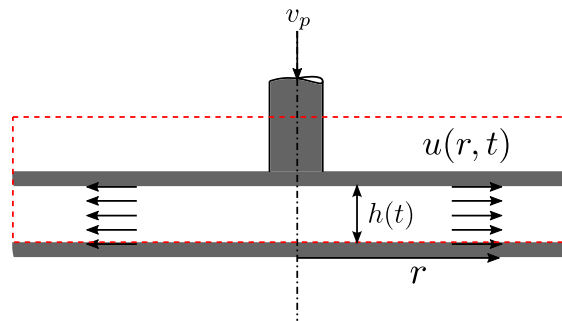


Figure 1: Oil between plates

- (a) Using the control volume shown, calculate the velocity $u_R(t)$ of exit of fluid between the plates.
 - (b) Compute the velocity distribution $u(r, t)$ between the plates, assuming flow is incompressible and uniform in the normal direction.
2. Compute the velocity profile of a double film falling down the inclined plane of incline angle θ with the horizontal as shown above. The fluid at the bottom 1 has density and viscosity ρ_1, μ_1 and similarly the one on top 2 has ρ_2 and μ_2 .

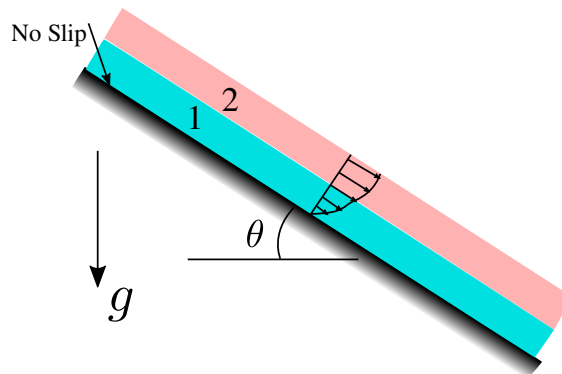


Figure 2: Double film

3. The flow from a sluice gate of height h flows over an inclined plane of angle θ as shown. The height of the reservoir behind it H remains almost constant since the area of the reservoir is large compared to the sluice gate area. Compute the thickness of the flow stream down the inclined plane.

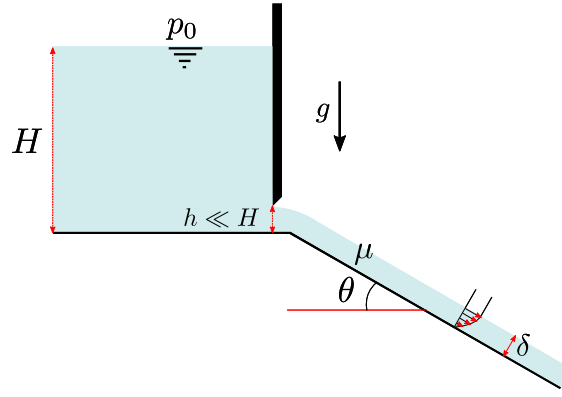


Figure 3: Sluice flow over an inclined plane

4. A fluid flows radially through a long and thin inner porous cylinder of radius R_1 , to an outer, concentric porous cylinder of radius R_2 . The non-gravitational pressure at R_1 (at the outer surface of inner cylinder) is p_1 and the total volume flow rate is Q . For steady and laminar flow between the two cylinders, simplify the governing equations (make suitable additional assumptions) to solve for pressure p in terms of p_1 and Q .

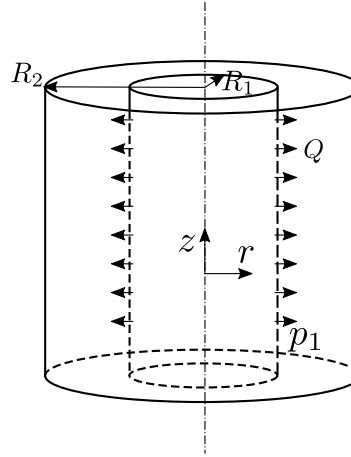


Figure 4: Radial flow through a porous cylinder

5. A cylinder of radius R_1 is being extruded out from a concentric stationary cylinder of radius R_2 at speed v_0 . The space between the two cylinders is filled with a viscous fluid of density ρ and viscosity μ . Simplify the governing equations using suitable assumptions and solve for the force required to keep the inner cylinder moving at speed v_0 . Assume long enough cylinders.

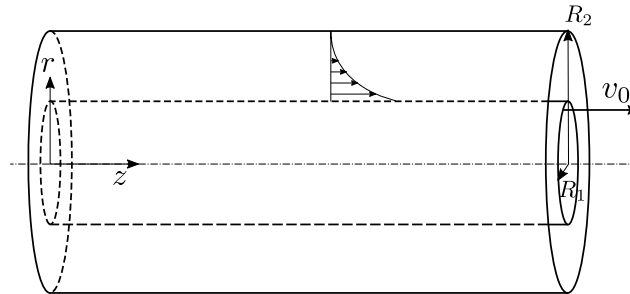


Figure 5: Extrusion flow

6. In a falling-cylinder viscometer, a solid cylindrical slug of radius κR falls down with a terminal velocity v_0 in a long vertical cylinder of radius R , closed at both ends. The slug's axis coincides that of cylinder at all times.
- (a) Find the velocity profile of the fluid in the gap between the two cylinder walls $u_z(r)$ in terms of v_0 .

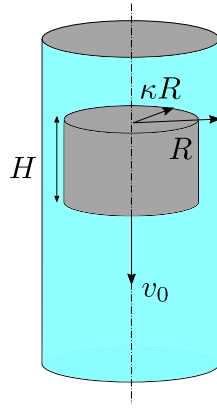


Figure 6: Falling cylinder viscometer

- (b) Assume that the slug is of density ρ_s and the fluid is of density ρ such that $\rho_s \gg \rho$. Balance the force on the slug and find the expression of viscosity in terms of parameters that can be measured in the system.
7. Water in an overflowing pipe falls down the external surface of the pipe of radius R making an annular film of thickness δ . If the volume flow rate of water in the pipe is Q , calculate δ in terms of known variables.

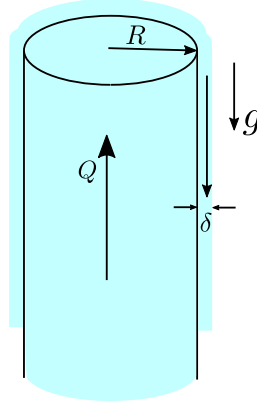


Figure 7: Overflowing pipe

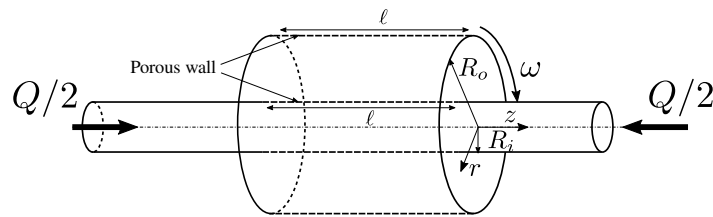


Figure 8: Rotating filter

8. A centrifuge may be used to filter sediments from a water supply. Consider the setup shown in figure ?? where the water is supplied at flow rate Q from both ends to a pipe which has a permeable wall in a particular section. The outer cylinder, also made of permeable wall is rotating at an angular velocity ω . Clear water transpires through both the pipe and cylinder getting filtered.. The transpiration velocity is measured to be V from the outside cylinder. Assume axisymmetric and flow and evaluate $u_\theta(r)$ and $u_r(r)$ between the cylinders. Follow below steps,
- (a) Apply continuity equation and find u_r . What is the transpiration velocity through the inner cylinder in terms of V ? Can you find V in terms of Q assuming steady state.
- (b) Apply θ momentum equation. Evaluate $u_\theta(r)$. (Hint: If you obtain an ODE which is seemingly unsolvable, substitute $u_\theta = Cr^n$ and solve)

- (c) Apply r momentum equation and evaluate the pressure distribution $p(r)$ between the two cylinders.
How does the pressure change correlate with the direction of the flow?