

Tutorial 7 : Dimensional analysis

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1. A dam of height H is overflowing. Water approaches the top of the dam at uniform velocity V . A volumetric flow rate Q is measured below the dam. First, list any additional parameters that might be relevant in this type of problem. Second, using dimensional analysis, propose a relationship between Q and the remaining parameters. From this analysis, can you conclude how the volumetric flow rate Q should scale, specifically, with the dam height H ? [Hint: don't overthink it; only 1 or 2 other physical parameters might matter.]
2. In this problem, we consider G.I. Taylor's analysis of the Trinity Test. The story goes that, only using dimensional analysis, physicist Geoffrey I. Taylor was able to determine the yield of the first nuclear explosion (Trinity test) ever tested by the U.S. Army. However, Taylor did much more than just use dimensional analysis (see his original paper¹). In this problem, we are interested in determining how the radius of the explosion cloud R is related to the energy of the explosion E , the time since the explosion occurred t , pressure of the surrounding gas p , and the temperature of the surrounding gas T . Assume that the surrounding air is an ideal gas such that the density $\rho = p/RT$. Determine the dimensionless groups which relate E and R .



Figure 1: Trinity test fireball [Wikipedia]

3. The flow rate Q , torque, T , and power P , developed by a centrifugal pump are known to be dependent on the pressure rise ΔP , μ , ρ , D and ω . Here, D is the diameter of the pump and ω is the rotational speed. Use D , ρ , ω as repeating variables and determine the relevant non-dimensional groups for each of the three relations. For geometrically similar pumps operating under the same conditions, how do the torque and power depend on the rotational speed?

¹Taylor, Geoffrey Ingram. "The formation of a blast wave by a very intense explosion I. Theoretical discussion." Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences 201.1065 (1950): 159-174.

4. In turbulent flows the smallest length scale η is known to be dependent on the kinematic viscosity ν and the energy transfer rate ϵ (m^2/s^3). Determine the non-dimensional groups relevant to the problem. Typically in laboratory flows (with air) $\epsilon = 1 \text{ m}^2/\text{s}^3$. Assuming that the non-dimensional group(s) is (are) approximately 1, estimate η for these flows. The kinematic viscosity of air can be taken as $\nu = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$.
5. An airship is to operate at 20m/s in air at standard conditions. A model of 1/20 scale is built and tested at the same air temperature to determine drag. What criterion should be considered to obtain dynamic similarity? If the model is tested at 75 m/s, what pressure should be used in the wind tunnel (assume viscosity to be only a function of temperature and air as an ideal gas)? If the model drag force is 250 N, what will be the drag on the prototype?
6. In some speed ranges, vortices are shed from the rear of bluff cylinders placed across a flow. The vortices alternately leave the top and bottom of the cylinder causing an alternating force normal to the freestream velocity. The vortex shedding frequency, f , is thought to depend on ρ, d, V , and μ . Use dimensional analysis to develop a functional relationship for f . Vortex shedding occurs in standard air on two cylinders with a diameter ratio of 2. Determine the velocity ratio for dynamic similarity, and the ratio of vortex shedding frequencies.
7. Governing equations of fluid flows assuming inviscid flow in a rotating frame of reference (rotating with $\underline{\Omega}$) are given by,

$$\frac{D\underline{v}}{Dt} + 2\underline{\Omega} \times \underline{v} = -\frac{1}{\rho} \nabla p.$$

These equations, along with the continuity equations are used in atmospheric sciences. Use pressure difference Δp , stratosphere height h to non-dimensionalize the equations. Obtain the important dimensionless groups.

8. For unsteady, 2D, compressible inviscid flows, differential form of governing equation can be written as,

$$\frac{\partial^2 \psi}{\partial t^2} + \frac{\partial}{\partial t}(u^2 + v^2) + (u^2 - c^2) \frac{\partial^2 \psi}{\partial x^2} + (v^2 - c^2) \frac{\partial^2 \psi}{\partial y^2} + 2uv \frac{\partial^2 \psi}{\partial x \partial y} = 0,$$

where ψ is the stream function such that $\underline{v} = \nabla \times \psi$. u and v are the velocity components in x and y directions and c is the local speed of sound in the flow. Using ℓ and c_0 as the length and velocity scales, where c_0 is the speed of sound in air at STP, obtain the dimensionless groups that characterize this equation.

9. The aerodynamic forces on a tall building are to be determined by measurements on a model in a wind-tunnel. The wind field approaching the building is one having a constant velocity gradient dv/dz , as shown below. The independent variables of the flow are thus, dv/dz , the building height H and the air density ρ . The wind-tunnel test will be conducted at a high enough Reynolds number so that the air viscosity is unimportant. Measurements will be made of the moment T of the aerodynamic drag force about the base of the building and the pressure $p(z)$ as a function of the height z above the base.



Figure 2: Aerodynamic forces on a building

- (a) Using the dimensional analysis, express the form of the functional relationship between the dimensionless moment T and any remaining dimensionless independent variables.

- (b) A model test is run for $(dv/dz)_m = 1 \text{ s}^{-1}$, $H_m = 1 \text{ m}$, and $\rho_m = 1.25 \text{ kg/m}^3$. The measured value of $T_m = 0.03 \text{ N} \cdot \text{m}$. What would be the numerical value of the moment T_p on a prototype of $H_p = 100 \text{ m}$, $\rho_p = 1.25 \text{ kg/m}^3$ and $(dv/dz)_p = 0.30 \text{ s}^{-1}$?
- (c) Using the dimensional analysis, express the form of the functional relationship between the dimensionless pressure $p(z)$ and any remaining variables.
- (d) Under the same conditions as the test for part (b), the model pressure rise $(p(z) - p_\infty)_m$ is measured to be 0.1 Pa at $z_m = H_m/2$. At what value of z_p can you calculate the pressure rise for the prototype? Calculate this value of pressure rise.
10. The boundary-layer thickness, δ , on a smooth flat plate in an incompressible flow without pressure gradients depends on the freestream speed, U , the fluid density, ρ , the fluid viscosity, μ , and the distance from the leading edge of the plate, x . Express these variables in dimensionless form.