

Tutorial 8 : Stokes flow and potential flow

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1. Consider pressure driven viscous flow of a fluid of viscosity μ and density ρ in a hyperbolic slot as shown below. The length of the slot is ℓ , the thickness $h(x)$ varies as $h(x) = \frac{h_i}{(1+\epsilon x/\ell)}$ where $\epsilon \ll 1$. The width of the slot inside the page is unity. The flow exits from the outlet of height h_o and the outlet pressure is p_o . Consider the origin at the bottom end of the inlet as shown.

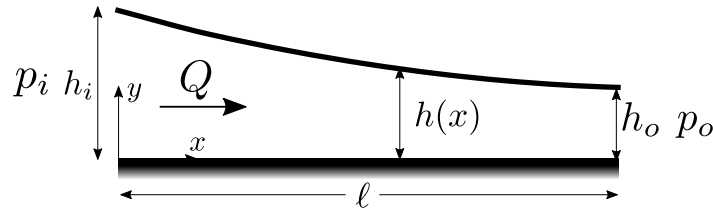


Figure 1: Pressure driven flow in a hyperbolic slot

- (a) Non-dimensionalize the Navier-Stokes equations and the continuity equation using ℓ as scale of x and h_i as scale of y and suitable scales for velocity and pressure. Derive the conditions on Re_ℓ and h_i/ℓ under which the x momentum governing equation reduces to the approximate form,

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{dp}{dx}.$$

- (b) Write down the boundary conditions for u .
 - (c) Solve the approximate form of the x direction momentum equation derived above. Using this, determine the inlet pressure p_i required to maintain a flow rate Q in the slot. Use binomial approximation ($(1+x)^n \approx 1+nx$ for $x \ll 1$) to simplify your calculations.
 - (d) Find the total horizontal force on the lower plate. Use binomial approximation to simplify your calculations.
2. In the above problem, find the change in the solution if the top hyperbolic wall is moving in x direction with U_0 and y direction with V_0 .
 3. In class, while non-dimensionalization for Stokes' flow (lubricating flow) analysis, we took a time scale defined by velocity scale and the length scale. In some problems, a separate time scale can be imposed externally, for instance in the shown oscillating plate Stokes' flow.
 - (a) Non-dimensionalize the momentum and continuity equations. For time scale, take $1/\Omega$ as the characteristic time scale. Within the assumptions made for the lubricating flow, derive the governing equations.
 - (b) Derive the conditions under which the unsteady term can be neglected. Compare against the conditions under which the unsteady term is important.

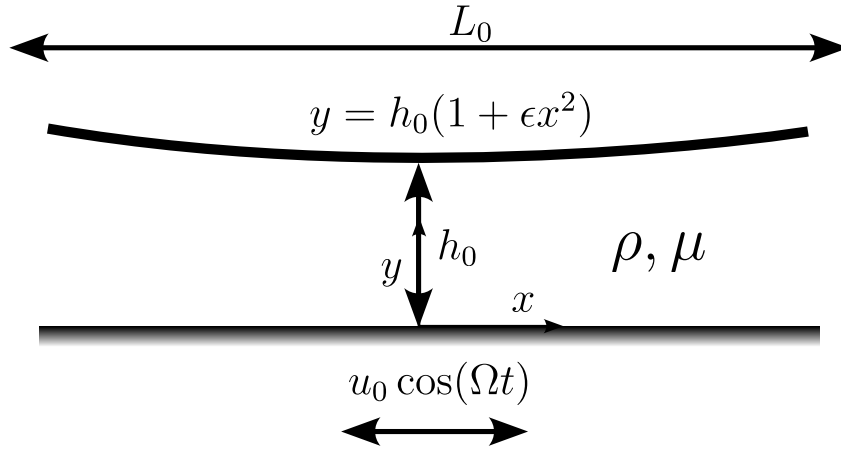


Figure 2: Oscillating Stokes' flow

- (c) Under the conditions the unsteady term is important, derive the solution for x, y velocities and the pressure difference.
4. For an ideal vortex, circumferential velocity u_θ is defined as $\Gamma/2\pi r$. However, such a velocity field is difficult to use in models due to very large velocity near $r \rightarrow 0$. To mitigate this problem, a Rankine vortex is used for which the tangential velocity is defined as,

$$u_\theta = \frac{\Gamma}{2\pi} \begin{cases} r/a^2 & r \leq a \\ 1/r & r > a \end{cases}.$$

Assuming no radial velocity exists, is the above flow model irrotational? If not, calculate the vorticity field. Calculate the streamfunction and velocity potential for this model.

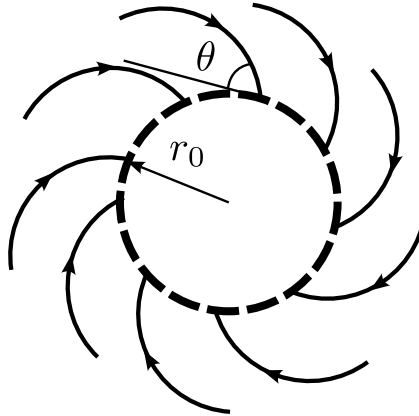


Figure 3: Sink and an ideal vortex

5. Let us model a tornado with superposition of a sink and an ideal vortex. Let the circulation around the tornado be given as $\Gamma = 8500 \text{ m}^2/\text{s}$ and that the pressure at $r = r_0 = 40 \text{ m}$ is 2200 Pa less than the far-field pressure (atmospheric).
- Estimate the appropriate sink strength.
 - Pressure at $r = 15 \text{ m}$.
 - Angle θ at which the flow enters the circle or radius r_0 .
6. For a superposition of a free stream and a vortex pair at $(0, \pm a)$ of circulation $\mp \Gamma$, obtain the streamfunction, velocity potential, and the location of stagnation points along the x axis using the complex potential.
7. For a 2D flow, the velocity potential function ϕ is given by $\phi = x^2 - y^2$.

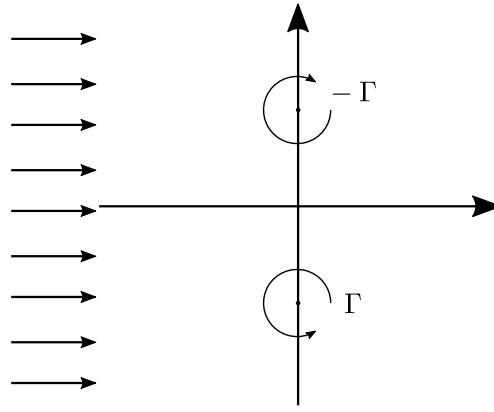


Figure 4: Vortex pair in a uniform stream

- (a) Determine the velocity components in x and y directions.
 - (b) Show that velocity components satisfy flow continuity and irrotationality.
 - (c) Determine the stream function
 - (d) Sketch the constant ϕ and ψ lines.
8. Uniform flow past a vortex closed to a wall can be solved using this problem. Apply *method of images* to it to convert into a situation shown in above figure. Assume the plate is at $y = 0$. Calculate the total upward force on the plate (per unit length).
 9. For a flow past a rotating cylinder of radius a , the complex velocity potential for a uniform flow, a doublet, and an ideal vortex can be added.
 - (a) Using the superposed complex potential, calculate u_r and u_θ . Relate the circulation, Γ with the angular speed Ω of the cylinder.
 - (b) Determine the location of stagnation points on the cylinder surface (where velocity vanishes).
 - (c) Using Bernoulli's equation, determine the pressure at the surface of the cylinder given that pressure far upstream is p_∞ .
 - (d) Integrate the pressure to calculate the net lift and drag force on the cylinder.
 10. Using the method of images, find the velocity potential and stream function due to a source of strength m at a distance a from the wall in $+x$ direction.