Tutorial 9 : Boundary layers

Prateek Gupta

Department of Applied Mechanics, IIT Delhi

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1. Boundary layers in practice transition from laminar to turbulent as shown in the figure below. The transition point for a smooth flat plate depends on the free stream velocity, kinematic viscosity, and the Reynolds number. The turbulent boundary layers are usually thicker than the laminar boundary layers due to enhanced diffusivity of momentum caused by the turbulent eddies.

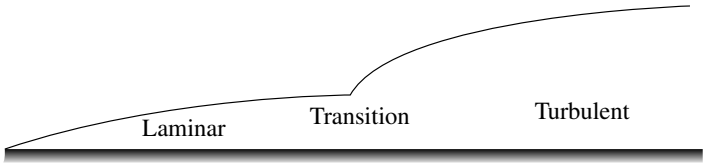


Figure 1: *Laminar to turbulent transition on a flat plate boundary layer*

   (a) From the Blasius solution, it is calculated that in the laminar region, $\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}$. Consider that the transition occurs at $Re_x = 5 \times 10^5$. Calculate the transition point thickness δ_{trans} in terms of the unknowns.

   Answer: $\delta_{trans} = 5\sqrt{\mathrm{Re}_{cr}}\,\dfrac{\nu}{U} = 3.54 \times 10^3\,\dfrac{\nu}{U}$.

   (b) Assume that the Von-Karman integral equation applies in the turbulent region as well. The velocity profile in the boundary layer is given as $u/U = \left(\frac{y}{\delta}\right)^{1/7}$ for $y/\delta < 1$. Experimentally, the skin friction coefficient is known to have the correlation with Re_δ: $C_f \approx 0.020 Re_\delta^{-1/6}$. Using the above information, calculate $\delta_{turb}(x)$ in terms of unknowns.

   Answer: $\delta_{turb}(x) = 0.162\left(\dfrac{\nu}{U}\right)^{1/7} x^{6/7}$.

   (c) Substitute your answer from (a) in (b) to find the correlation of δ_{turb} with Re_x.

   Answer: $\dfrac{\delta_{turb}}{x} = \dfrac{0.162}{\mathrm{Re}_x^{1/7}}$.

2. Assume that the velocity in the laminar boundary layer on a flat plate has the profile,

$$\frac{u}{U_\infty} = \sin\left(\frac{\pi y}{2\delta}\right).$$

   Using von Karmann momentum integral, show that

$$\frac{\delta}{x} = \frac{4.795}{\sqrt{\mathrm{Re}_x}},\; C_f = \frac{0.655}{\sqrt{\mathrm{Re}_x}}$$

   Answer: $\theta/\delta = 2/\pi - 1/2$.

3. Air flows between two parallel flat plates as shown in the figure. The upper plate is porous in a section and additional air is injected through this surface at a speed V. As a result, the free stream velocity U maybe considered to vary in x as, $U(x) = U_\infty\left(1 + \frac{x}{\ell}\right)$ where ℓ is some length scale. Assume a linear polynomial approximation of velocity in the boundary layer ($u/U = \eta$) and estimate the variation of boundary layer thickness $d\delta(x)/dx$ with the Reynolds number $Re_x = \frac{U_\infty x}{\nu}$. Compare this with what you would obtain for a constant U with x.

Answer: $\dfrac{d\delta}{dx} = \dfrac{6\nu}{U_\infty \delta}\left(1 + x/\ell\right)^{-11}$, $\delta^2 = 1.2\dfrac{\nu\ell}{U_\infty}\left[1 - (1 + x/\ell)^{-10}\right]$; constant U: $\dfrac{d\delta}{dx} = \dfrac{1.73}{\sqrt{\mathrm{Re}_x}}$.

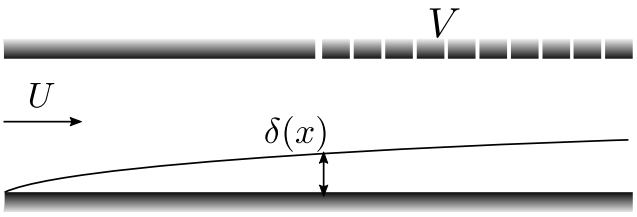


Figure 2: *Effect of a porous wall on a boundary layer*

4. A sharp plate with $L = 50$ cm and $b = 3$ m is parallel to a stream of velocity 2.5 m/s. Find the drag on one side of the plate, and the boundary layer thickness δ at the trailing edge for (a) air and (b) water at 20° C and 1 atm. See Table A.2 from Frank M. White's textbook for values of physical properties. Take transition Reynolds number as $\mathrm{Re}_x = 5 \times 10^5$.

Answer: (a) Air: $F \approx 0.026$ N, $\delta \approx 8.7$ mm. (b) Water: $F \approx 14.1$ N, $\delta \approx 10.8$ mm.

5. A thin equilaterial triangle is immersed parallel to a $U_\infty = 1$ m/s stream of air at standard conditions. Given that the flow transitions at $Re_x = 5\times10^5$, estimate the skin-friction drag on this plate. Assume velocity profile is quadratic in the boundary layer in the laminar regime and $u/U_\infty = \eta^{1/7}$ in the turbulent regime. Use $\nu = 1.5\times10^{-5}$ m^2/s and $\rho = 1.23$ kg/m^3.

Answer: $D = 0.453\,\rho U^2\sqrt{\nu/U}\;s^{3/2} \approx 6.1 \times 10^{-3}$ N.

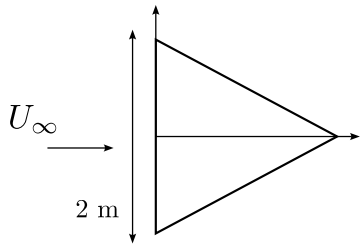


Figure 3: *Top view of flow on a triangular flat plate*

6. Repeat the Von Kármán-Pohlhausen method for a flat-plate zero pressure gradient boundary layer using the polynomial approximation,

$$\frac{u}{U_\infty} = \sum_{k=1}^{5} a_k \eta^k,$$

where η is simply the similarity variable $\eta = y/\delta(x)$. Estimate $\delta(x)$ and $\tau_w(x)$ which is the wall shear stress. Calculate the total drag force on a flat plate of length L. Also compute the skin-friction coefficient C_f defined as the ratio,

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}.$$

Answer: $\dfrac{u}{U} = \frac{5}{2}\eta - 5\eta^3 + 5\eta^4 - \frac{3}{2}\eta^5$, $\delta = \dfrac{7.04\,x}{\sqrt{\mathrm{Re}_x}}$, $\tau_w = \dfrac{0.355\,\rho U^2}{\sqrt{\mathrm{Re}_x}}$, $C_f = \dfrac{0.711}{\sqrt{\mathrm{Re}_x}}$, $C_D = \dfrac{1.42}{\sqrt{\mathrm{Re}_L}}$.

7. (F.M. White's book) Derive modified forms of the laminar boundary layer equations for the case of axisymmetric flow along the outside of a circular cylinder of constant radius R as shown below. Consider two special cases (a) $\delta \ll R$ and (b) $\delta \sim R$. Clearly write the boundary conditions in both the cases.

Answer: (a) $\delta \ll R$: Same as Cartesian equations with $y = r - R$; $u = v = 0$ at $y = 0$, $u \to U$ far. (b) $\delta \sim R$: $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = U\frac{dU}{dx} + \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$; $u = v = 0$ at $r = R$, $u \to U$ as $r \to \infty$.

