

September 16, 2026

Maximum score is 100. There are 5 problems in total. Time limit is 120 minutes. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

Useful equations

- Radial component of momentum equation in cylindrical coordinates: $\rho \left(\frac{\partial u_r}{\partial t} + \underline{v} \cdot \nabla u_r - \frac{u_\theta^2}{r} \right) = -\frac{\partial p}{\partial r} + \mu \left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right) + f_r$ for $\underline{v} = u_r \hat{e}_r + u_\theta \hat{e}_\theta + u_z \hat{e}_z$.
- Axial component of momentum equation in cylindrical coordinates: $\rho \left(\frac{\partial u_z}{\partial t} + \underline{v} \cdot \nabla u_z \right) = -\frac{\partial p}{\partial z} + \mu \nabla^2 u_z + f_z$
- $\underline{v} \cdot \nabla = u_r \frac{\partial}{\partial r} + \frac{1}{r} \frac{\partial}{\partial \theta} + u_z \frac{\partial}{\partial z}$
- $\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$.
- Viscous stress components in cylindrical coordinates
 1. $\sigma_{rr} = 2\mu \frac{\partial u_r}{\partial r}$
 2. $\sigma_{\theta\theta} = 2\mu \left(\frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_\theta}{r} \right)$
 3. $\sigma_{zz} = 2\mu \frac{\partial u_z}{\partial z}$
 4. $\sigma_{r\theta} = \mu \left(\frac{1}{r} \frac{\partial u_r}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) \right) = \sigma_{\theta r}$
 5. $\sigma_{\theta z} = \mu \left(\frac{1}{r} \frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z} \right) = \sigma_{z\theta}$
 6. $\sigma_{rz} = \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) = \sigma_{zr}$

1. The tube arrangement shown rotates about axis AA at $600/\pi$ rpm. Determine the levels in columns C_1 , C_2 , C_3 in the new position of equilibrium. If the axis is shifted to BB , what are the levels? The tubes are long enough to prevent spillage and the level h is when the tube is at rest. Take $g = 10 \text{ m}^2/\text{s}$ and $\rho = 1000 \text{ kg/m}^3$. **(15)**

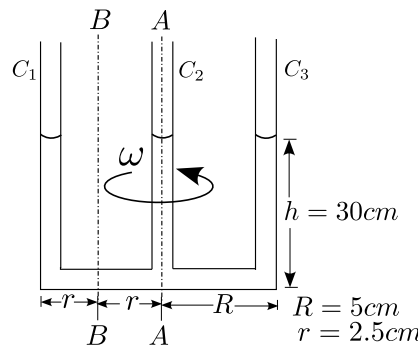


Figure 1: Problem 1.

Answer:- Rigid body rotation with gravity

$$\nabla p = \rho \left(-g \hat{k} + \omega^2 r \hat{e}_r \right) \implies \frac{\partial p}{\partial r} = \rho \omega^2 r, \quad \frac{\partial p}{\partial z} = -\rho g,$$

where r is the horizontal distance from the axis of rotation and z is measured upward. Integrating,

$$p = \rho \left(\frac{1}{2} \omega^2 r^2 - gz \right) + \text{const.}$$

Every free surface is at atmospheric pressure,

$$p(0, h_1) = p(R, h_2) \implies \frac{\omega^2 0^2}{2g} - h_1 = \frac{\omega^2 R^2}{2g} - h_2,$$

where h_1 is height at the axis when axis is AA and h_2 is height in columns C_1 and C_3 . Also, due to conservation of volume,

$$2h_2 + h_1 = 3h \quad (0.1)$$

which yields $h_1 = h - \frac{\omega^2 R^2}{3g}$ and $h_2 = h + \frac{\omega^2 R^2}{6g}$.

When axis is BB , columns C_1 and C_2 are at same radial distance $R/2$ and will have same height (let h_1) and column C_3 at $3R/2$ will have height h_2 . We get the equations

$$h_2 - h_1 = \frac{\omega^2 R^2}{g}, \quad (0.2)$$

$$2h_1 + h_2 = 3h \quad (0.3)$$

which yields $h_1 = h - \frac{\omega^2 R^2}{3g}$ and $h_2 = h + \frac{2\omega^2 R^2}{3g}$.

Numerical values,

$$\omega = \frac{600}{\pi} \text{ rpm} = \frac{600}{\pi} \cdot \frac{2\pi}{60} = 20 \text{ rad/s}, \quad \frac{\omega^2}{2g} = \frac{(20)^2}{2(10)} = 20 \text{ m}^{-1}.$$

When axis is AA ,

$$h_2 = 0.3167 \text{ m}, h_1 = 0.267 \text{ m} \quad (0.4)$$

When axis is BB

$$h_2 = 0.367 \text{ m}, h_1 = 0.267 \text{ m} \quad (0.5)$$

2. Steady flow over a hard non-porous flat plate is shown below. A region close to the flat plate, called the boundary layer, develops where viscous stresses can not be ignored. The starting point of the boundary layer is called the leading edge. The x direction velocity varies in y due to the no-slip condition at the wall and varies in x also. Consider incompressible, 2D, boundary layer flow as shown below. A square fluid particle inside the boundary layer is marked with dashed line at $x = L, y = h(L)/2$. Numerical value of the dimensional constant $K = 1$. **(25)**

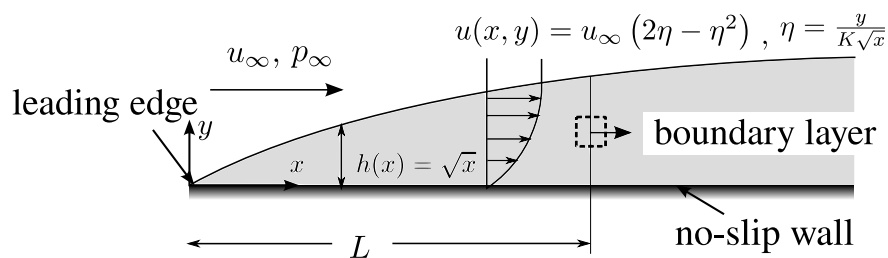


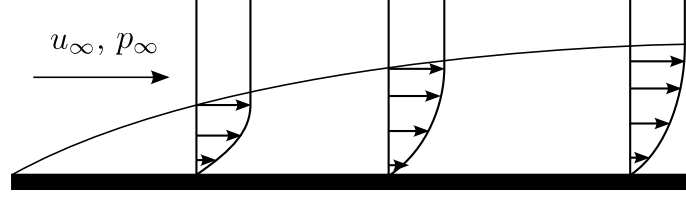
Figure 2: Boundary layer over a flat plate.

- (a) Sketch the x direction velocity profile at some suitably chosen three different x locations. What are the dimensions of the constant K ? **(2)**

Answer:- Since η must be dimensionless and $\eta = y/(K\sqrt{x})$, we need $[K\sqrt{x}] = [y] = L$, hence

$$[K] = \frac{L}{L^{1/2}} = L^{1/2} \quad (\text{e.g. m}^{1/2}).$$

Sketch:



- (b) Using incompressibility condition, derive an expression for the velocity in other direction. Sketch the streamlines (no need to derive the equation of the streamlines). **(4)**

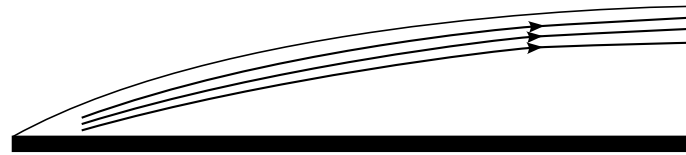
Answer:- 2D incompressibility: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$. First,

$$\frac{\partial u}{\partial x} = u_{\infty}(2 - 2\eta) \frac{\partial \eta}{\partial x} = 2u_{\infty}(1 - \eta) \left(-\frac{\eta}{2x}\right) = -\frac{u_{\infty}}{x} \eta(1 - \eta).$$

Hence $\frac{\partial v}{\partial y} = \frac{u_{\infty}}{x} \eta(1 - \eta)$. Integrating at fixed x (with $dy = K\sqrt{x} d\eta$ and $v = 0$ at the wall):

$$v = \int_0^{\eta} \frac{u_{\infty}}{x} \eta'(1 - \eta') K\sqrt{x} d\eta' = \frac{u_{\infty}K}{\sqrt{x}} \left(\frac{\eta^2}{2} - \frac{\eta^3}{3} \right) \xrightarrow{K=1} \frac{u_{\infty}}{\sqrt{x}} \left(\frac{\eta^2}{2} - \frac{\eta^3}{3} \right).$$

Since $v > 0$ for $0 < \eta \leq 1$, the transverse velocity is directed away from the plate. Notice what happens at $x \rightarrow 0$. The boundary layer becomes infinitesimally thin $h \rightarrow 0$. For larger and larger x , the y direction velocity decreases. Using the given profile, you can only sketch streamlines within the boundary layer. If you have sketched outside the layer, it is fine for now.



- (c) Calculate the velocity gradient at the location of the fluid particle. Separate the symmetric and the antisymmetric parts. Also compute the vorticity at the location of the fluid particle. **(6)** Answer:- At the particle $x = L$, $y = h(L)/2 = \frac{1}{2}\sqrt{L}$, so $\eta = \frac{1}{2}$. The four gradients:

$$\begin{aligned} \frac{\partial u}{\partial x} &= -\frac{u_{\infty}}{x} \eta(1 - \eta) = -\frac{u_{\infty}}{4L}, & \frac{\partial u}{\partial y} &= \frac{2u_{\infty}(1 - \eta)}{K\sqrt{x}} = \frac{u_{\infty}}{K\sqrt{L}} \xrightarrow{K=1} \frac{u_{\infty}}{\sqrt{L}}, \\ \frac{\partial v}{\partial y} &= +\frac{u_{\infty}}{4L}, & \frac{\partial v}{\partial x} &= -\frac{u_{\infty}K}{2} x^{-3/2} \left(\frac{3}{2}\eta^2 - \frac{4}{3}\eta^3 \right) = -\frac{5u_{\infty}K}{48L^{3/2}} \xrightarrow{K=1} -\frac{5u_{\infty}}{48L^{3/2}}. \end{aligned}$$

Velocity-gradient tensor $\nabla \underline{v} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}:$

$$\nabla \underline{v} = \begin{pmatrix} -\frac{u_{\infty}}{4L} & \frac{u_{\infty}}{\sqrt{L}} \\ -\frac{5u_{\infty}}{48L^{3/2}} & \frac{u_{\infty}}{4L} \end{pmatrix}.$$

Symmetric part $\underline{\underline{S}} = \frac{1}{2}(\nabla \underline{v} + \nabla \underline{v}^T)$ and antisymmetric part $\underline{\underline{\Omega}} = \frac{1}{2}(\nabla \underline{v} - \nabla \underline{v}^T)$:

$$\underline{\underline{S}} = \begin{pmatrix} -\frac{u_\infty}{4L} & s \\ s & \frac{u_\infty}{4L} \end{pmatrix}, \quad s = \frac{1}{2} \left(\frac{u_\infty}{\sqrt{L}} - \frac{5u_\infty}{48L^{3/2}} \right); \quad \underline{\underline{\Omega}} = \begin{pmatrix} 0 & w \\ -w & 0 \end{pmatrix}, \quad w = \frac{1}{2} \left(\frac{u_\infty}{\sqrt{L}} + \frac{5u_\infty}{48L^{3/2}} \right).$$

Vorticity (out-of-plane component):

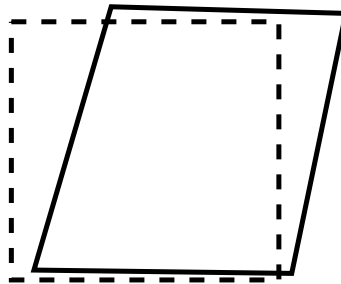
$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = - \left(\frac{u_\infty}{\sqrt{L}} + \frac{5u_\infty}{48L^{3/2}} \right).$$

It is negative (clockwise).

- (d) Consider two regions. I: $y > h(x)$ (outside the boundary layer) and II: $0 < y < h(x)$ (inside the boundary layer). In which of the regions can we apply Bernoulli's equation? Justify. **(2)** Answer:- Region I only. Outside the boundary layer the flow is the uniform free stream ($u = u_\infty, v = 0$): it is effectively inviscid and irrotational ($\omega_z = 0$), so Bernoulli's equation holds (indeed between any two points). Inside the boundary layer (Region II) viscous stresses are important and $\omega_z \neq 0$; mechanical energy is dissipated, so Bernoulli's equation cannot be applied.
- (e) Tabulate velocities of the fluid particle. Assume side length a which obeys $a < \sqrt{L} < L$. Sketch the particle after deformation in some time Δt . **(6)** Answer:- Centre ($\eta = \frac{1}{2}$): $u_c = u_\infty(2 \cdot \frac{1}{2} - \frac{1}{4}) = \frac{3}{4}u_\infty$, $v_c = \frac{u_\infty}{\sqrt{L}}(\frac{1}{8} - \frac{1}{24}) = \frac{u_\infty}{12\sqrt{L}}$. Using a linear (Taylor) expansion $u \approx u_c + \frac{\partial u}{\partial x} \Delta x + \frac{\partial u}{\partial y} \Delta y$ etc. with $\Delta x, \Delta y = \pm a/2$, the corner velocities are:

Corner ($\Delta x, \Delta y$)	u	v
Top-right ($+\frac{a}{2}, +\frac{a}{2}$)	$\frac{3}{4}u_\infty - \frac{u_\infty a}{8L} + \frac{u_\infty a}{2\sqrt{L}}$	$\frac{u_\infty}{12\sqrt{L}} + \frac{u_\infty a}{8L} - \frac{5u_\infty a}{96L^{3/2}}$
Top-left ($-\frac{a}{2}, +\frac{a}{2}$)	$\frac{3}{4}u_\infty + \frac{u_\infty a}{8L} + \frac{u_\infty a}{2\sqrt{L}}$	$\frac{u_\infty}{12\sqrt{L}} + \frac{u_\infty a}{8L} + \frac{5u_\infty a}{96L^{3/2}}$
Bottom-right ($+\frac{a}{2}, -\frac{a}{2}$)	$\frac{3}{4}u_\infty - \frac{u_\infty a}{8L} - \frac{u_\infty a}{2\sqrt{L}}$	$\frac{u_\infty}{12\sqrt{L}} - \frac{u_\infty a}{8L} - \frac{5u_\infty a}{96L^{3/2}}$
Bottom-left ($-\frac{a}{2}, -\frac{a}{2}$)	$\frac{3}{4}u_\infty + \frac{u_\infty a}{8L} - \frac{u_\infty a}{2\sqrt{L}}$	$\frac{u_\infty}{12\sqrt{L}} - \frac{u_\infty a}{8L} + \frac{5u_\infty a}{96L^{3/2}}$

The controlling difference is $\partial u / \partial y > 0$: the top edge moves faster in x than the bottom edge. *Sketch*: after Δt the square shears into a parallelogram whose top edge is displaced further downstream (top-leading), with a slight vertical stretch ($\partial v / \partial y > 0$) and slight horizontal compression ($\partial u / \partial x < 0$); the net rotation is clockwise (negative vorticity).



- (f) Assuming viscosity of the fluid as μ , derive an expression of the shear force per unit depth on the section of the flat plate between the fluid particle shown and the leading edge. The section is of length L . **(5)**

Answer:- The wall shear stress is $\tau_w(x) = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}$. From (c) with $\eta = 0$,

$$\left. \frac{\partial u}{\partial y} \right|_{y=0} = \frac{2u_\infty}{K\sqrt{x}} \Rightarrow \tau_w(x) = \frac{2\mu u_\infty}{K\sqrt{x}}.$$

Shear force per unit depth on $0 \leq x \leq L$:

$$F = \int_0^L \tau_w dx = \frac{2\mu u_\infty}{K} \int_0^L x^{-1/2} dx = \frac{2\mu u_\infty}{K} 2\sqrt{L} = \frac{4\mu u_\infty \sqrt{L}}{K} \xrightarrow{K=1} 4\mu u_\infty \sqrt{L},$$

acting on the plate in the $+x$ direction (drag).

3. Consider a viscous film of liquid draining uniformly down the side of a vertical fixed rod of radius a due to gravity g , as shown below. At some distance down the rod the film will approach a terminal or fully developed draining flow of constant outer radius b . Assume incompressible, steady, axisymmetric, and 2D flow. **(30)**

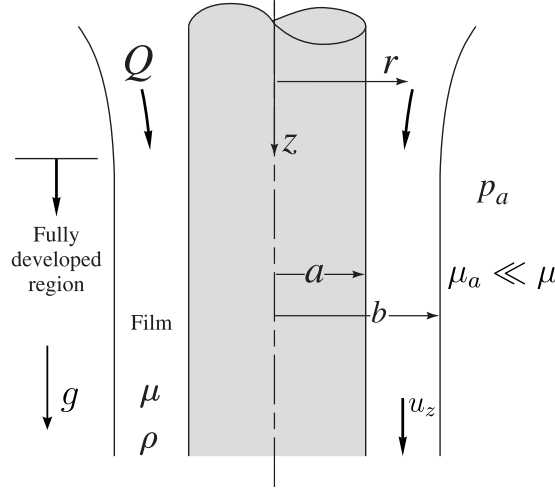


Figure 3: Viscous film of liquid. The gray shaded portion is the fixed vertical rod.

- (a) Simplify the continuity and momentum equations to derive an ODE for u_z . **(10)**

Answer:- Take z positive downward (direction of g and of the flow).

Assumptions:

- steady $\frac{\partial}{\partial t} = 0$
- axisymmetric $\frac{\partial}{\partial \theta} = 0$
- 2D flow $\underline{v} = u_z \hat{k} + u_r \hat{e}_r$
- fully developed $\frac{\partial v}{\partial z} = 0$

Continuity: $\frac{1}{r} \frac{\partial(r u_r)}{\partial r} + \frac{\partial u_z}{\partial z} = 0 \implies r u_r = \text{const.}$ Since $u_r = 0$ at $r = a$, $u_r = 0$ everywhere.

z-momentum: show all terms vanish with the following simplified equation

$$\frac{\mu}{r} \frac{d}{dr} \left(r \frac{du_z}{dr} \right) + \rho g - \frac{dp}{dz} = 0.$$

Note that r and θ momentum equations give $\partial p / \partial r = 0$ and $\partial p / \partial \theta = 0$, with latter one known from axisymmetric assumption as well.

- (b) State all the appropriate boundary conditions and solve for u_z . **(10)**

Answer:- *Boundary conditions:* (i) no slip at the rod, $u_z(a) = 0$; (ii) the outer surface $r = b$ is an interface with air with $\mu_a \ll \mu$, hence

$$\sigma_{rz} = \mu \frac{\partial u_z}{\partial r} = \mu_a \frac{\partial u_{z,a}}{\partial r} \quad (0.6)$$

at $r = b$. Since $\mu_a \ll \mu$, we can use free-shear condition $\frac{du_z}{dr} = 0$ at $r = b$. Additionally,

$$p = p_a \quad (0.7)$$

at $r = b$. Since $\frac{\partial p}{\partial r} = 0$, $p = p_a$ everywhere inside the layer. Ignoring any pressure gradient in air, we can say $p = p_a = \text{const.}$. This yields,

$$\frac{\mu}{r} \frac{d}{dr} \left(r \frac{du_z}{dr} \right) + \rho g = 0 \quad (0.8)$$

If you do not make this assumption, you can carry on with a constant $\frac{dp}{dz} = \frac{dp_a}{dz} = \rho_a g$ (assuming negligible velocity of air) which will change the equation to

$$\frac{\mu}{r} \frac{d}{dr} \left(r \frac{du_z}{dr} \right) + (\rho - \rho_a)g = 0 \quad (0.9)$$

where $\rho_a g$ is bouyancy term.

Integrating the ODE twice:

$$\frac{d}{dr} \left(r \frac{du_z}{dr} \right) = -\frac{\rho g}{\mu} r \Rightarrow r \frac{du_z}{dr} = -\frac{\rho g}{2\mu} r^2 + C_1 \Rightarrow \frac{du_z}{dr} = -\frac{\rho g}{2\mu} r + \frac{C_1}{r}.$$

BC (ii): $0 = -\frac{\rho g}{2\mu} b + \frac{C_1}{b} \Rightarrow C_1 = \frac{\rho g b^2}{2\mu}$, so $\frac{du_z}{dr} = \frac{\rho g}{2\mu} \left(\frac{b^2}{r} - r \right)$. Integrating and using BC (i) $u_z(a) = 0$:

$$u_z(r) = \frac{\rho g}{4\mu} \left[2b^2 \ln \frac{r}{a} - (r^2 - a^2) \right].$$

The velocity is zero at the rod and maximum at the free surface $r = b$.

(c) How does the film radius b relate to the flow rate Q ? **(5)**

Answer:- $Q = \int_a^b u_z(r) 2\pi r dr$. With u_z from (b),

$$\begin{aligned} Q &= \frac{2\pi \rho g}{4\mu} \int_a^b \left[2b^2 \ln \frac{r}{a} - r^2 + a^2 \right] r dr \\ &= \frac{\pi \rho g}{2\mu} \left[b^4 \ln \frac{b}{a} - \frac{3}{4} b^4 + a^2 b^2 - \frac{a^4}{4} \right], \end{aligned}$$

i.e.

$$Q = \frac{\pi \rho g}{8\mu} \left[4b^4 \ln \frac{b}{a} - 3b^4 + 4a^2 b^2 - a^4 \right].$$

This is an implicit relation: for a prescribed Q (and a) it fixes the terminal film radius b (the right side increases monotonically with b for $b > a$).

(d) Compute the total force per unit length on the vertical rod due to the flow. Clearly specify the contributing shear stress components **(5)**

Answer:- The outer surface as a unit normal $\hat{e}_r$. The traction $\underline{t}$ becomes

$$t_i = T_{ji} n_j = -p n_i + \sigma_{ji} n_j \quad (0.10)$$

Pressure only contributes to the normal component of the traction. When this is integrated over the surface, this will vanish (check for yourself). So the only viscous stress traction remains. Since $n_\theta = n_z = 0$ and $n_r = 1$,

$$\sigma_{ji} n_j = \sigma_{rr} \hat{e}_r + \sigma_{r\theta} \hat{e}_\theta + \sigma_{rz} \hat{e}_z \quad (0.11)$$

Only $\sigma_{rz} \neq 0$. We get

$$\underline{t} = \mu \frac{du_z}{dr} \bigg|_{r=a} \hat{e}_z = \mu \frac{\rho g}{2\mu} \left(\frac{b^2}{a} - a \right) \hat{e}_z \quad (0.12)$$

Force per unit z length is

$$\frac{dF}{dz} = \int \underline{t} d\theta = 2\pi a \mu \frac{\rho g}{2\mu} \left(\frac{b^2}{a} - a \right) \hat{e}_z = \pi \rho g (b^2 - a^2) \hat{e}_z. \quad (0.13)$$

Note: this equals the weight per unit length of the film, $\rho g \cdot \pi(b^2 - a^2)$ — in fully developed flow the rod's shear supports the entire weight of the film, since the free surface is stress-free and $\partial p / \partial z = 0$. If you accounted for bouyancy, the force will be slightly reduced with $\rho - \rho_a$ instead of just ρ .

4. Two fluid layers exist between two plates as shown below. The bottom plate is fixed and the top plate is moving with a constant speed V_0 . The viscosity, density, and layer height of both the fluids are shown in the figure. Assume no pressure gradient in x direction. (20)

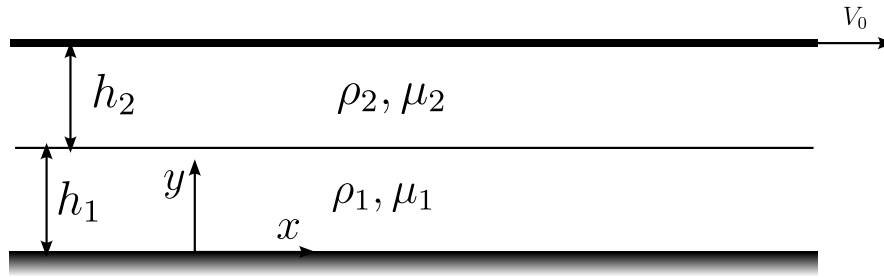


Figure 4: Two layer Couette flow

- (a) Make suitable assumptions. Simplify the continuity and momentum equations as a result of these assumptions. (10) **Answer:-** *Assumptions:* steady, incompressible, fully developed, unidirectional flow $\underline{v} = u(y)\hat{i}$ with $v = w = 0$; $\partial/\partial x = 0$ for velocity; no imposed pressure gradient ($\partial p/\partial x = 0$); Ignore gravity.

Continuity: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial u}{\partial x} = 0$, so $u = u(y)$. x -momentum in each layer $i = 1, 2$ reduces to

$$0 = -\frac{\partial p}{\partial x} + \mu_i \frac{d^2 u_i}{dy^2} \Rightarrow \frac{d^2 u_i}{dy^2} = 0,$$

so each layer has a *linear* profile: $u_1 = A_1 y + B_1$ ($0 < y < h_1$), $u_2 = A_2 y + B_2$ ($h_1 < y < h_1 + h_2$). The y -momentum equation just gives $\partial p/\partial y = 0$.

- (b) Calculate the force per unit area required to keep the top plate moving. (5)

Answer:- *Boundary conditions:* $u_1(0) = 0 \Rightarrow B_1 = 0$; velocity continuity at the interface $u_1(h_1) = u_2(h_1)$; shear-stress continuity $\mu_1 \frac{du_1}{dy} = \mu_2 \frac{du_2}{dy}$ (i.e. $\mu_1 A_1 = \mu_2 A_2$); top plate $u_2(h_1 + h_2) = V_0$. Solving,

$$A_1 = \frac{\mu_2 V_0}{\mu_2 h_1 + \mu_1 h_2}, \quad A_2 = \frac{\mu_1}{\mu_2} A_1.$$

With no pressure gradient the shear stress is the *same* in both layers, $\tau = \mu_1 A_1$. The force per unit area on the top plate is

$$\frac{F}{A} = \tau = \frac{\mu_1 \mu_2 V_0}{\mu_1 h_2 + \mu_2 h_1} = \frac{V_0}{\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}}.$$

- (c) If the two fluids are immiscible with a surface tension coefficient σ , what is the force per unit area required to move the top plate at speed V_0 ? (5)

Answer:- The interface here is flat and parallel to the plates (planar, zero curvature). Hence $p_1 - p_2 = 2\sigma H$ gives $p_1 = p_2$ since $H = 0$. Irrespective, the shear force is unchanged, the required force per unit area is the same as in (b):

$$\frac{F}{A} = \frac{\mu_1 \mu_2 V_0}{\mu_1 h_2 + \mu_2 h_1} \quad (\text{unaffected by } \sigma).$$

The answer would change if σ was varying in x (see class notes on surface tension force).

5. Consider inviscid, steady, incompressible flow of a fluid with density ρ being injected between two circular discs of radius R , separated by a gap a , through a circular hole of diameter a in the upper disc at volume flow rate Q . (10)

- (a) Find the radial velocity $u_r(r)$. (2)

Answer:- The same flow rate Q crosses every cylindrical surface of radius r and height equal to the gap a : $Q = u_r(2\pi r a)$. Hence

$$u_r(r) = \frac{Q}{2\pi a r} \quad \left(\frac{a}{2} \leq r \leq R\right).$$

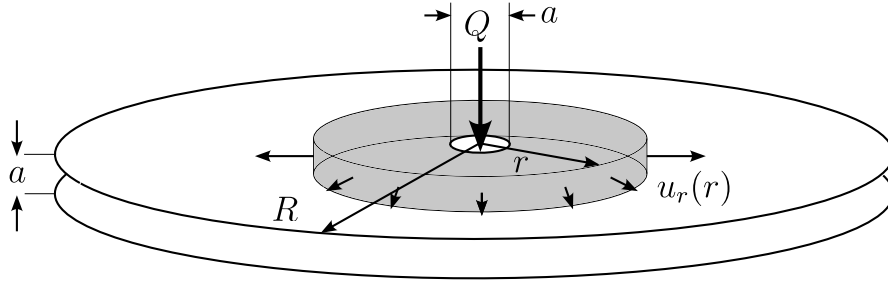


Figure 5: Inviscid flow between discs. The gray shaded portion is the fluid flowing between the discs at some instant.

- (b) Can the Bernoulli's equation be used under the assumptions given? If no, justify. If yes, sketch streamlines. **(3)** Answer:- Yes. The flow is inviscid, steady and incompressible, so Bernoulli's equation applies between any two points on streamlines. *Streamlines*: straight radial lines directed outward from the central hole. Later on in the course, we can show that actually we can apply Bernoulli's equation between any two points in this particular problem.
- (c) Find the pressure distribution and calculate the force required to hold the upper disc in place. **(5)** Answer:- Apply Bernoulli between radius r and the rim $r = R$ (where the fluid discharges to atmosphere, $p = p_{\text{atm}}$):

$$p(r) + \frac{1}{2}\rho u_r^2(r) = p_{\text{atm}} + \frac{1}{2}\rho u_r^2(R) \Rightarrow p(r) = p_{\text{atm}} + \frac{\rho Q^2}{8\pi^2 a^2} \left(\frac{1}{R^2} - \frac{1}{r^2} \right).$$

Since $1/r^2 > 1/R^2$ for $r < R$, the gap pressure is *below* atmospheric everywhere between the discs (suction), so the discs are drawn together. The net force from the (gauge) pressure on the wetted underside of the upper disc, from the hole edge $r = a/2$ to $r = R$, is

$$F = \int_{a/2}^R [p_{\text{atm}} - p(r)] 2\pi r dr = \frac{\rho Q^2}{4\pi a^2} \left[\ln \frac{2R}{a} - \frac{1}{2} + \frac{a^2}{8R^2} \right].$$

Thus

$$F = \frac{\rho Q^2}{4\pi a^2} \left[\ln \frac{2R}{a} - \frac{1}{2} + \frac{a^2}{8R^2} \right]$$

is the (attractive) suction force pulling the discs together. Upward force of this magnitude is required for keeping the disc in place.