

September 16, 2026

Maximum score is 100. There are 5 problems in total. Time limit is 120 minutes. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

Useful equations

- Radial component of momentum equation in cylindrical coordinates: $\rho \left(\frac{\partial u_r}{\partial t} + \underline{v} \cdot \nabla u_r - \frac{u_\theta^2}{r} \right) = -\frac{\partial p}{\partial r} + \mu \left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right) + f_r$ for $\underline{v} = u_r \hat{e}_r + u_\theta \hat{e}_\theta + u_z \hat{e}_z$.
- Axial component of momentum equation in cylindrical coordinates: $\rho \left(\frac{\partial u_z}{\partial t} + \underline{v} \cdot \nabla u_z \right) = -\frac{\partial p}{\partial z} + \mu \nabla^2 u_z + f_z$
- $\underline{v} \cdot \nabla = u_r \frac{\partial}{\partial r} + \frac{1}{r} \frac{\partial}{\partial \theta} + u_z \frac{\partial}{\partial z}$
- $\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$.
- Viscous stress components in cylindrical coordinates
 1. $\sigma_{rr} = 2\mu \frac{\partial u_r}{\partial r}$
 2. $\sigma_{\theta\theta} = 2\mu \left(\frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_\theta}{r} \right)$
 3. $\sigma_{zz} = 2\mu \frac{\partial u_z}{\partial z}$
 4. $\sigma_{r\theta} = \mu \left(\frac{1}{r} \frac{\partial u_r}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) \right) = \sigma_{\theta r}$
 5. $\sigma_{\theta z} = \mu \left(\frac{1}{r} \frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z} \right) = \sigma_{z\theta}$
 6. $\sigma_{rz} = \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) = \sigma_{zr}$

1. The tube arrangement shown rotates about axis AA at $600/\pi$ rpm. Determine the levels in columns C_1 , C_2 , C_3 in the new position of equilibrium. If the axis is shifted to BB , what are the levels? The tubes are long enough to prevent spillage and the level h is when the tube is at rest. Take $g = 10 \text{ m}^2/\text{s}$ and $\rho = 1000 \text{ kg/m}^3$. **(15)**

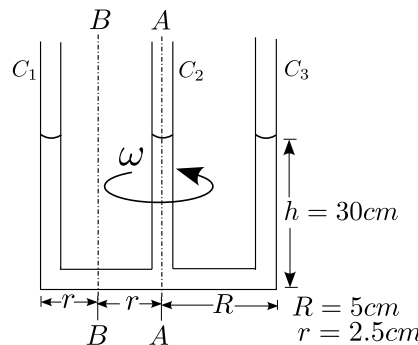


Figure 1: Problem 1.

2. Steady flow over a hard non-porous flat plate is shown below. A region close to the flat plate, called the boundary layer, develops where viscous stresses can not be ignored. The starting point of the boundary layer is called the leading edge. The x direction velocity varies in y due to the no-slip condition at the wall and varies in x also. Consider incompressible, 2D, boundary layer flow as shown below. A square fluid particle inside the boundary layer is marked with dashed line at $x = L, y = h(L)/2$. Numerical value of the dimensional constant $K = 1$. **(25)**

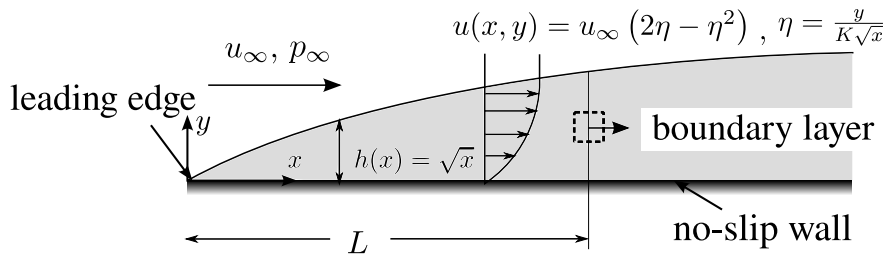


Figure 2: Boundary layer over a flat plate.

- Sketch the x direction velocity profile at some suitably chosen three different x locations. What are the dimensions of the constant K ? **(2)**
 - Using incompressibility condition, derive an expression for the velocity in other direction. Sketch the streamlines (no need to derive the equation of the streamlines). **(4)**
 - Calculate the velocity gradient at the location of the fluid particle. Separate the symmetric and the antisymmetric parts. Also compute the vorticity at the location of the fluid particle. **(6)**
 - Consider two regions. I: $y > h(x)$ (outside the boundary layer) and II: $0 < y < h(x)$ (inside the boundary layer). In which of the regions can we apply Bernoulli's equation? Justify. **(2)**
 - Tabulate velocities of the fluid particle. Assume side length a which obeys $a < \sqrt{L} < L$. Sketch the particle after deformation in some time Δt . **(6)**
 - Assuming viscosity of the fluid as μ , derive an expression of the shear force per unit depth on the section of the flat plate between the fluid particle shown and the leading edge. The section is of length L . **(5)**
3. Consider a viscous film of liquid draining uniformly down the side of a vertical fixed rod of radius a due to gravity g , as shown below. At some distance down the rod the film will approach a terminal or fully developed draining flow of constant outer radius b . Assume incompressible, steady, axisymmetric, and 2D flow. **(30)**

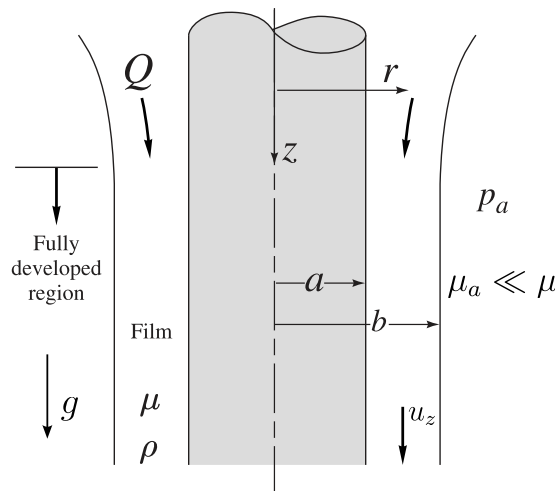


Figure 3: Viscous film of liquid. The gray shaded portion is the fixed vertical rod.

- Simplify the continuity and momentum equations to derive an ODE for u_z . **(10)**
- State all the appropriate boundary conditions and solve for u_z . **(10)**
- How does the film radius b relate to the flow rate Q ? **(5)**

- (d) Compute the total force per unit length on the vertical rod due to the flow. Clearly specify the contributing shear stress components **(5)**
4. Two fluid layers exist between two plates as shown below. The bottom plate is fixed and the top plate is moving with a constant speed V_0 . The viscosity, density, and layer height of both the fluids are shown in the figure. Assume no pressure gradient in x direction. **(20)**

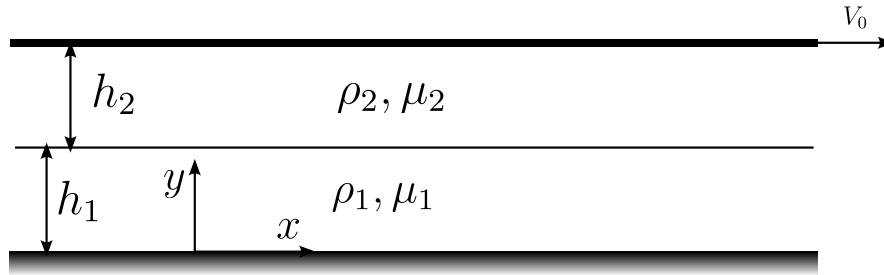


Figure 4: Two layer Couette flow

- (a) Make suitable assumptions. Simplify the continuity and momentum equations as a result of these assumptions. **(10)**
- (b) Calculate the force per unit area required to keep the top plate moving. **(5)**
- (c) If the two fluids are immiscible with a surface tension coefficient σ , what is the force per unit area required to move the top plate at speed V_0 ? **(5)**
5. Consider inviscid, steady, incompressible flow of a fluid with density ρ being injected between two circular discs of radius R , separated by a gap a , through a circular hole of diameter a in the upper disc at volume flow rate Q . **(10)**

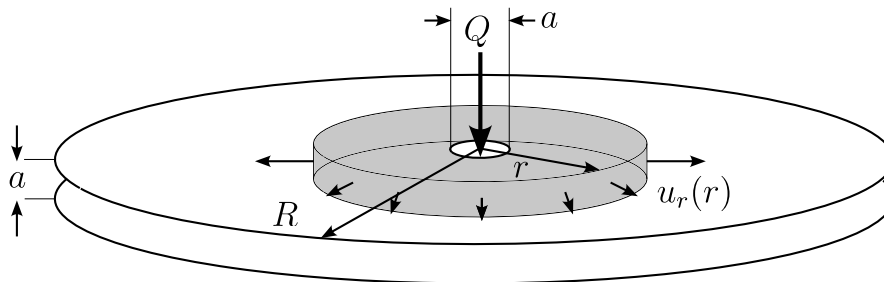


Figure 5: Inviscid flow between discs. The gray shaded portion is the fluid flowing between the discs at some instant.

- (a) Find the radial velocity $u_r(r)$. **(2)**
- (b) Can the Bernoulli's equation be used under the assumptions given? If no, justify. If yes, sketch streamlines. **(3)**
- (c) Find the pressure distribution and calculate the force required to hold the upper disc in place. **(5)**