

August 12, 2026

Maximum score is 40. There are 4 questions in total. Time limit is 60 minutes. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

1. Simplify the following. Assume smooth and differentiable field $\underline{v}$. **(15)**

- (a) $\delta_{ij}\delta_{ij}$ **(1)**
- (b) $\epsilon_{ijk}\epsilon_{ijk}$ **(2)**
- (c) $\nabla \cdot (\nabla r^5)$ for $r = \sqrt{x_i x_i}$ **(4)**
- (d) $\nabla \cdot (\nabla \times \underline{v})$ **(4)**
- (e) $\underline{v} \cdot (\nabla \times \underline{v})$ **(4)**

2. In a 2-D flow field, the velocity field is given by, **(8)**

$$\begin{aligned}\underline{v}(t) &= 4\hat{i}, & 0 \leq t < 4 \text{ s} \\ &4\hat{i} + 4\hat{j}, & 4 \leq t < 8 \text{ s} \\ &4\hat{j}, & 8 \leq t < 12 \text{ s}.\end{aligned}$$

Qualitatively sketch the following showing the origin $(0,0)$ on each graph.

- (a) Streamline passing through $(0,0)$ at $t = 2$ s. **(1)**
- (b) Pathline of a particle which was at $(0,0)$ at $t = 0$, observed at $t = 2$ s. **(1)**
- (c) Streamline passing through the origin at $t = 10$ s. **(2)**
- (d) Pathline of the particle in part 2b observed at $t = 10$ s. **(2)**
- (e) Streakline of particles released from origin from $t = 0$ onwards, observed at $t = 10$ s. **(2)**

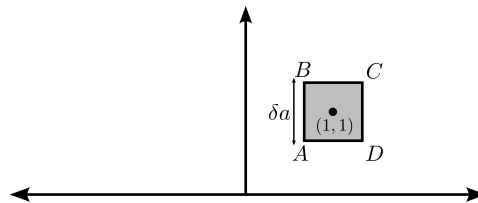


Figure 1: Fluid element $ABCD$.

3. Velocity field in a steady 2-D flow is given by **(13)**,

$$u = x, \quad v = -y.$$

- (a) Find the equation of streamlines. Sketch the streamlines of the flow in the region $-\infty < x < \infty, y > 0$. Show atleast three curves in each quadrant. **(2)**
 - (b) Consider a square shaped fluid element of side length δa centered at $(1, 1)$ at time t , as shown in figure 1. Tabulate the velocities of points $ABCD$. **(4)**
 - (c) Calculate the acceleration of an infinitesimal fluid element when it is at $(1, 1)$. **(2)**
 - (d) Compute the velocity gradient tensor and decompose in symmetric and anti-symmetric parts. What is the rate of bulk strain in this flow? **(3)**
 - (e) Sketch the qualitative shape of the fluid element in part 3b at time $t + \delta t$. Assume small δa . **(2)**
4. If S_{ij} is the symmetric part of velocity gradient tensor $\partial v_i / \partial x_j$, show that $S_{ij} \left(S_{ij} - \frac{1}{3} \nabla \cdot \underline{v} \delta_{ij} \right) \geq 0$. **(4)**