

September 9, 2026

Maximum score is 40. There are 4 questions in total. Time limit is 60 minutes. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

1. A sealed container of radius  $R$ , filled with water of density  $\rho_w$ , is rotating about its vertical axis at a fixed angular speed of  $\Omega$ . A very small spherical cork of radius  $a \ll R$  and density  $\rho_c < \rho_w$  is attached by a thread of length  $L < R$  to the outer bottom edge of the container as shown. The container, water, and cork rotate like a rigid body. **(15)**

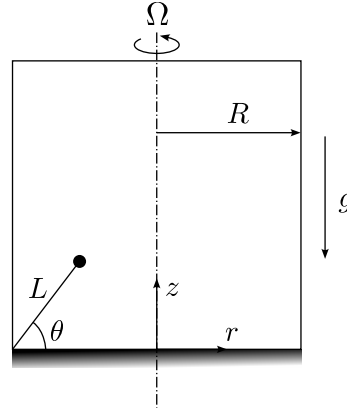


Figure 1: Rotating sealed container

- (a) Find an expression for  $\nabla p$ , where  $p$  is the pressure inside the container. **(8)**
  - (b) Estimate the hydrostatic force on the cork and the angle  $\theta$  at steady state. **(7)**
2. A spherical thrust bearing is shown. The gap between the spherical member and the housing is constant and equal to  $d$ . Take the oil between the bearing and housing to have viscosity  $\mu$ . Take help of the spherical geometry figure shown on the right.

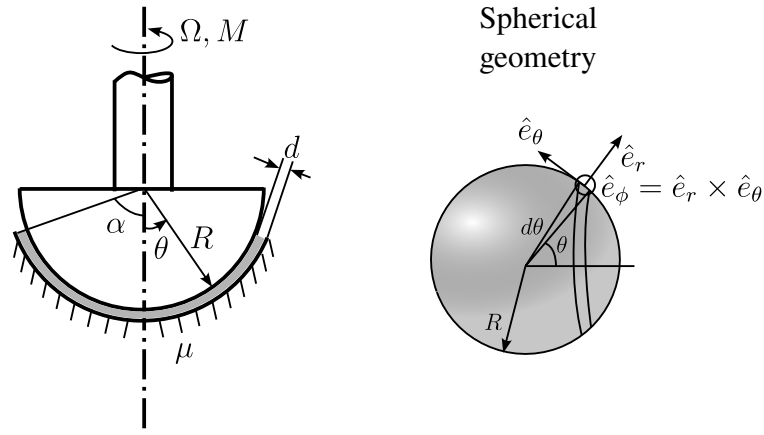


Figure 2: Rotating spherical bearing

- (a) Assume uni-directional flow with velocity varying linearly as  $Ar + B$ . Using appropriate boundary conditions, derive  $A$  and  $B$ . Specify the non-zero component of velocity **(3)**
- (b) What is the unit normal of the upper surface? Use that to identify the stress tensor components contributing to the surface traction. Compute the traction vector **(5)**
- (c) Integrate the torque due to viscous shear stress to obtain a relation between  $M$ ,  $\mu$ ,  $\Omega$ , and geometric parameters. **(7)**

3. Water-air interface near a plane wall makes a shape  $\eta(x)$  as shown. Ignore air density. Density of water is  $\rho$  and surface tension coefficient of the interface is  $\sigma$ . Wetting angle at  $x = 0$  is known to be  $\theta$ . For  $x \rightarrow \infty$ , a horizontal surface is observed.
- (a) Using  $-\frac{1}{2}\nabla \cdot \hat{n}$  expression for the mean curvature, assuming planar shape in depthwise direction, derive an expression for the mean curvature. **(5)**
- (b) Assuming small slope, approximate the mean curvature expression above. Compute the maximum height  $h$  at the wall. **(5)**

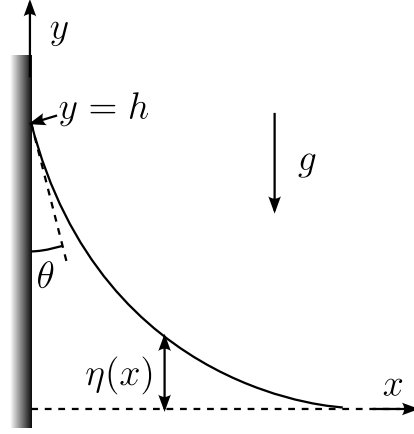


Figure 3: Air water interface

## Useful equations

- Radial component of momentum equation in cylindrical coordinates:  $\rho \left( \frac{\partial u_r}{\partial t} + \underline{v} \cdot \nabla u_r - \frac{u_\theta^2}{r} \right) = -\frac{\partial p}{\partial r} + \mu \left( \nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right) + f_r$  for  $\underline{v} = u_r \hat{e}_r + u_\theta \hat{e}_\theta + u_z \hat{e}_z$ .
- Viscous stress components in spherical coordinates
  1.  $\sigma_{rr} = 2\mu \frac{\partial u_r}{\partial r}$
  2.  $\sigma_{\theta\theta} = 2\mu \left( \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_\theta}{r} \right)$
  3.  $\sigma_{\phi\phi} = 2\mu \left( \frac{1}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r} + \frac{u_\theta \cot \theta}{r} \right)$
  4.  $\sigma_{\theta\phi} = \mu \left( \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \left( \frac{u_\phi}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial u_\theta}{\partial \phi} \right) = \sigma_{\phi\theta}$
  5.  $\sigma_{\phi r} = \mu \left( \frac{1}{r \sin \theta} \frac{\partial u_r}{\partial \phi} + r \frac{\partial}{\partial r} \left( \frac{u_\phi}{r} \right) \right) = \sigma_{r\phi}$
  6.  $\sigma_{r\theta} = \mu \left( r \frac{\partial}{\partial r} \left( \frac{u_\theta}{r} \right) + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right) = \sigma_{\theta r}$