

AML2020/APL107/APL106 (2026-2027-I): Quiz-3 solutions

Department of Applied Mechanics, IIT Delhi

October 23, 2026

Maximum score is 40. There are 2 questions in total. Time limit is 60 minutes. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

1. Constant volume flow rate Q of fluid of density ρ flows between parallel circular plates of radius R and gap d , rotating at a angular velocity ω (both plates are rotating). The flow between the plates is purely radial in the reference frame of the plates. Compute the torque (along with its direction) required to keep the assembly rotating at a constant angular velocity ω . Assume inviscid flow between the plates. **(20)**

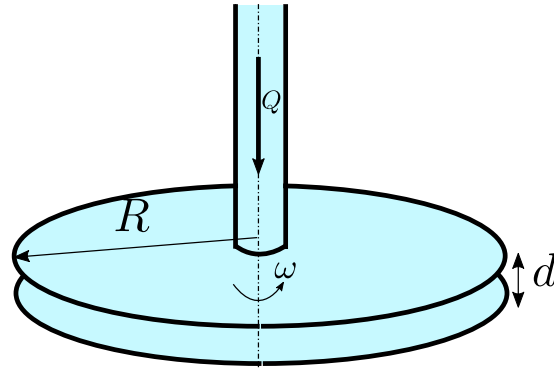


Figure 1: Rotating discs

Answer:- For the control volume shown by the dashed lines, applying integral version of the mass conservation equation,

$$\frac{\partial}{\partial t} \int_V \rho dV + \oint_{\partial V} \rho \underline{v} \cdot \hat{n} dS = 0. \quad (0.1)$$

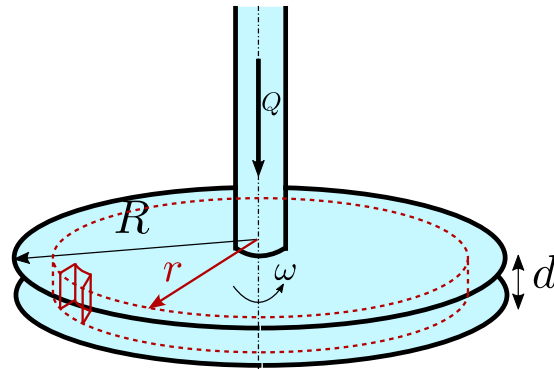
For steady flow,

$$Q + (-u_r 2\pi r d) = 0, \quad (0.2)$$

which yields the radial velocity field u_r in the frame of reference of the control volume as,

$$u_r = \frac{Q}{2\pi r d}. \quad (0.3)$$

For a rotating control volume, applying the angular momentum conservation,



$$\frac{\partial}{\partial t} \int_V \underline{r} \times \rho \underline{v} dV + \oint_{\partial V} \underline{r} \times \rho \underline{v} (\underline{v} \cdot \hat{n}) dS = - \oint_{\partial V} \underline{r} \times p \hat{n} dS + \oint_{\partial V} \underline{r} \times \underline{\tau} \cdot \hat{n} dS + \underline{M} - \int_V \underline{r} \times \rho \underline{a}_{rel} dV, \quad (0.4)$$

where $\underline{a}_{rel}$ is due to velocity and acceleration being measured in the non-inertial frame of reference. Applying the above equation on the small volume shown in the figure (and assuming inviscid flow),

$$\oint_{\partial V} \underline{r} \times \rho \underline{v} (\underline{v} \cdot \hat{n}) dS = 0 \text{ because } \underline{r} \times \underline{v} = 0, \quad (0.5)$$

$$\oint_{\partial V} \underline{r} \times \hat{n} p dS = 0 \text{ because } \underline{r} \times \hat{n} = 0, \quad (0.6)$$

for curved surfaces. For the flat surfaces, pressure forces cancel out. Finally,

$$\underline{M} = \int_V \underline{r} \times \rho \underline{a}_{rel} dV = \int_V \rho \underline{r} \times (2\underline{\Omega} \times \underline{v}) dV. \quad (0.7)$$

For $\underline{\Omega} = \omega \hat{k}$ and $\underline{v} = u_r \hat{r}$,

$$\underline{M} = 2\rho\omega d \int_0^R \int_0^{2\pi} r \frac{Q}{2\pi r d} r d\theta dr \hat{k}, \quad (0.8)$$

which yields,

$$\underline{M} = \rho\omega QR^2 \hat{k} \quad (0.9)$$

2. Geometry of the inlet section of a wind-tunnel needs to be analyzed. The inlet section contains a fan of area A and a nozzle-shaped duct with inlet area A_1 and outlet area A_2 . Inlet and outlet of this section are labelled as 1 and 2 in the figure. Sections just before and after the fan are labelled as a and b. Assume density of air as ρ . **(20)**

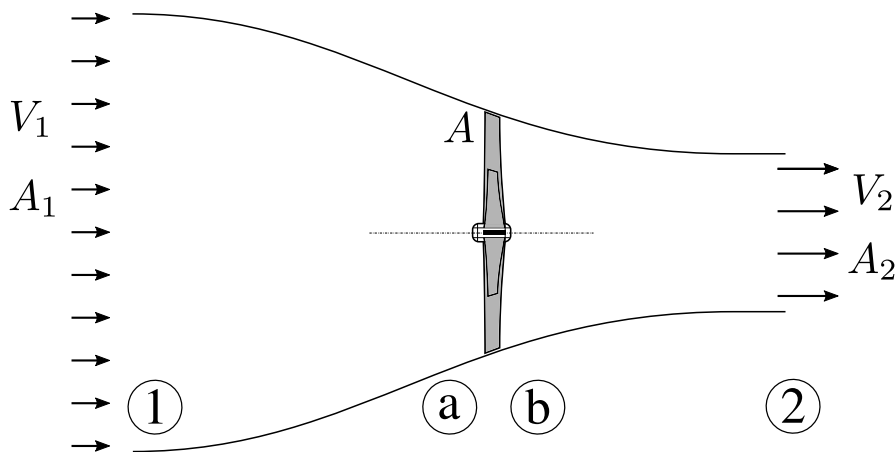


Figure 2: A wind-tunnel fan in the inlet section of a wind-tunnel.

- (a) Estimate the force on the fan in terms of given variables and given mass flow rate $\dot{m}$, draw a sketch of a suitable control volume. Assume that points 1 and 2 are far away from fan and atmospheric pressure is p_0 . Show the boundaries of the control volume clearly. **(5)**

- (b) Is Bernoulli's equation in the form,

$$\frac{p}{\rho} + \frac{v^2}{2} + gz = \text{const.}$$

applicable between point pairs 1&2, a&b, 1&a and b&2? **(2)**

- (c) Plot shown in figure below shows the variation of the gauge pressure along the flow stream. Sketch a suitable control volume and compute the force calculated in the above part in terms of p_a and p_b . Using previous parts' answers, compute the flow speed V at the fan location in terms of V_1 and V_2 . Assume points a and b to be very close (fan is very thin) **(3)**

- (d) Apply the energy equation across the appropriate control volume. Simplify the terms with suitable assumptions. Calculate the power supplied by the fan to the flow stream. **(10)**

Answer:-

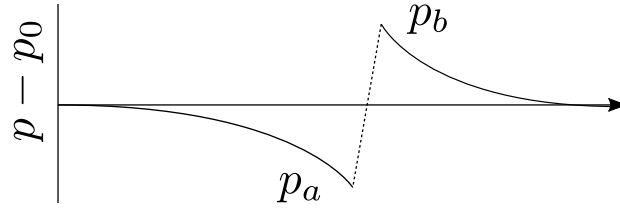
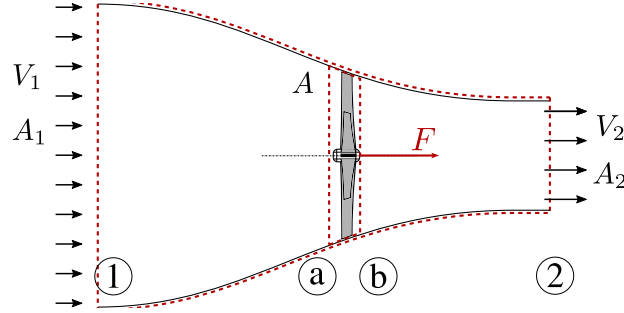


Figure 3: Pressure distribution in a wind-tunnel inlet section



- (a) A suitable control volume for calculating the force on the fan is shown. Points 1 and 2 are far away. Hence $p_1 = p_0$ and $p_2 = p_0$. From continuity equation $V_1 A_1 = V_2 A_2$. From momentum equation,

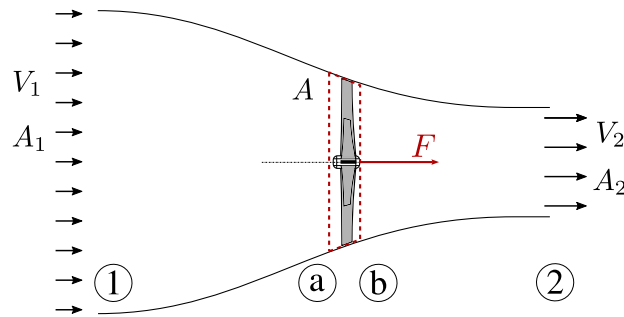
$$\frac{d}{dt} \int_V \rho \underline{v} dV + \oint_{\partial V} \rho \underline{v} (\underline{v} \cdot \hat{n}) dS = - \oint_{\partial V} p \hat{n} dS + \oint_{\partial V} \underline{\tau} \cdot \hat{n} dS + \underline{F}. \quad (0.10)$$

Assuming uni-directional flow and inviscid flow, we can drop viscous traction forces. Also, since pressure is p_0 throughout, pressure forces cancel out. Assuming steady flow,

$$\rho V_1 A_1 (V_2 - V_1) = \dot{m} (V_2 - V_1) = F. \quad (0.11)$$

- (b) i. 1&2 $\rightarrow$ No
 ii. a&b $\rightarrow$ No
 iii. 1&a $\rightarrow$ Yes
 iv. b&2 $\rightarrow$ Yes

- (c) Applying the integral equation for linear momentum,



$$\dot{m} (V_b - V_a) + (p_b - p_a) A = F \quad (0.12)$$

Since a and b are close by, $V_a \approx V_b$.

$$F = (p_b - p_a) A \quad (0.13)$$

Applying Bernoulli's equation between 1&a and b&2

$$p_0 + \frac{\rho V_1^2}{2} = p_a + \frac{\rho V_a^2}{2}, \quad (0.14)$$

$$p_b + \frac{\rho V_b^2}{2} = p_0 + \frac{\rho V_2^2}{2}, \quad (0.15)$$

which yields,

$$F = (p_b - p_a)A = \rho A \frac{(V_2^2 - V_1^2)}{2}. \quad (0.16)$$

Using $\dot{m} = \rho A_1 V_1 = \rho A V$, we obtain,

$$\rho A \frac{(V_2^2 - V_1^2)}{2} = \rho A V (V_2 - V_1) \implies V = \frac{V_1 + V_2}{2}. \quad (0.17)$$

(d) Applying the energy equation (using inviscid flow approximation),

$$\frac{d}{dt} \int_V \rho e_T dV + \oint_{\partial V} \rho \left(e_T + \frac{p}{\rho} \right) \underline{v} \cdot \hat{n} dS = \dot{W}_{fan} + \dot{Q}. \quad (0.18)$$

Assuming steady flow, no heat transfer, constant temperature,

$$\dot{W}_{fan} = \dot{m} \left(\frac{V_2^2 - V_1^2}{2} \right). \quad (0.19)$$

Useful equations

- In a general accelerating and rotating rigid reference frame for a control volume CV with surface CS,

$$\frac{d}{dt} \int_{CV} \rho \underline{v} dV = - \oint_{CS} \rho \underline{v} (\underline{v} - \underline{w}) \cdot \hat{n} dS + \underline{F} - \int_{CV} \rho \underline{a}_{rel} dV,$$

where $\underline{F}$ includes external, viscous, and pressure forces, $\underline{w}$ is the velocity of the control surface in that reference frame, $\underline{v}$ is the material velocity in that reference frame, and the relative acceleration $\underline{a}_{rel}$ is ,

$$\underline{a}_{rel} = \underline{a}_{CV} + 2\dot{\underline{\Omega}} \times \underline{v} + \dot{\underline{\Omega}} \times \underline{r} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r})$$

where $\underline{\Omega}$ and $\dot{\underline{\Omega}}$ are the angular velocity and angular acceleration of the reference frame.

- Similar equation can be derived for moment (torque) by considering appropriate cross-products with $\underline{r}$.