AML2020/APL107/APL106 (2026-2027-I): Quiz-4

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Maximum score is 40. There are 2 questions in total. Time limit is 60 minutes. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

1. Consider low Reynolds number viscous flow of a fluid of viscosity μ and density ρ in a slipper pad bearing shown below. The length of the bearing is ℓ, the thickness $h(x)$ varies as $h(x) = h_i + (h_o - h_i)\frac{x}{\ell}$. The width of the slot inside the page is unity. Pressure on both inlet and outlet ends is atmospheric. Consider the origin at the bottom end of the inlet as shown. **(30)**

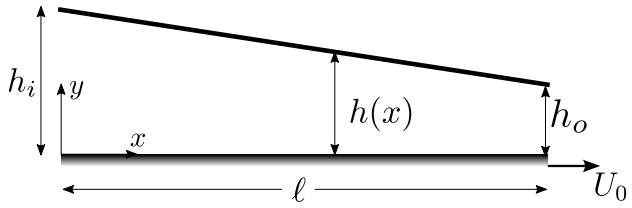


Figure 1: Flow in a slipper-pad bearing

(a) Non-dimensionalize the Navier-Stokes equations and the continuity equation using suitable scales. Derive the conditions under which the x momentum governing equation reduces to the approximate form, **(10)**

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{dp}{dx}.$$

Answer:- Pick the scales $[x] = \ell, [y] = h_i - h_o, [u] = U_0, [v] = V_0$. From continuity, we obtain

$$V_0 = U_0 \left(\frac{h_i - h_o}{\ell}\right) = U_0 \frac{\Delta h}{\ell}$$

The dimensionless continuity equation remains

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0.$$

Taking pressure scale $[p] = \dfrac{\mu U_0 \ell}{\Delta h^2}$, and defining $\epsilon = \dfrac{h_i - h_0}{\ell}$, we get the steady state y momentum equation as

$$\frac{\rho U_0^2 \Delta h}{\ell^2}\left(v^* \frac{\partial v^*}{\partial y^*} + u^* \frac{\partial v^*}{\partial x^*}\right) = -\frac{\mu U_0 \ell}{\Delta h^3}\frac{\partial p^*}{\partial y^*} + \frac{\mu U_0}{\Delta h^2}\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \epsilon^2 \frac{\partial^2 v^*}{\partial x^{*2}}\right) \tag{0.1}$$

which yields

$$Re_h \epsilon^3 \left(v^* \frac{\partial v^*}{\partial y^*} + u^* \frac{\partial v^*}{\partial x^*}\right) = -\frac{\partial p^*}{\partial y^*} + \epsilon\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \epsilon^2 \frac{\partial^2 v^*}{\partial x^{*2}}\right) \tag{0.2}$$

Similarly, for x momentum equation, we obtain

$$Re_h \epsilon \left(v^* \frac{\partial u^*}{\partial y^*} + u^* \frac{\partial u^*}{\partial x^*}\right) = -\frac{\partial p^*}{\partial x^*} + \frac{\partial^2 u^*}{\partial y^{*2}} + \epsilon^2 \frac{\partial^2 u^*}{\partial x^{*2}} \tag{0.3}$$

For $\epsilon \ll 1$ and $\epsilon Re_h \ll 1$, we obtain (in the dimensional form)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{0.4}$$

$$\frac{\partial p}{\partial y} = 0 \tag{0.5}$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}. \tag{0.6}$$

(b) Write down the boundary conditions for u. **(5)**
Answer:- $u(y = 0, x) = U_0, u(y = h(x), x) = 0$ and $v(y = 0, x) = 0, v(y = h(x), x) = 0$.

(c) Solve the approximate form of the x direction momentum equation derived above and determine the pressure distribution in x. Make a sketch of the pressure distribution vs x. **(10)**
Answer:- Integrating, we get

$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2. \tag{0.7}$$

Using the boundary conditions, we get

$$u = -\frac{1}{2\mu}\frac{dp}{dx}\left(yh(x) - y^2\right) + U_0\left(1 - \frac{y}{h(x)}\right) \tag{0.8}$$

To get pressure distribution, using continuity equation, we get

$$\frac{d}{dx}\int_0^h u dy = 0 \implies \frac{U_0 h}{2} - \frac{h^3}{12\mu}\frac{dp}{dx} = D_1 \tag{0.9}$$

for some arbitrary constant D_1. Integrating in x, we get

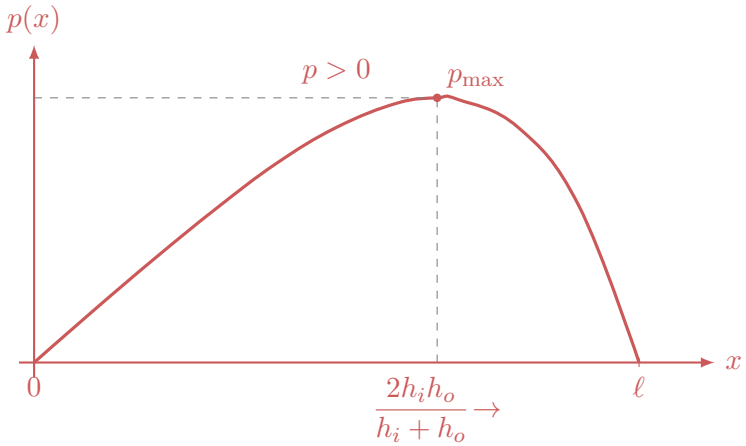
$$p = 6\mu U_0 \int \frac{1}{h^2} dx - 12\mu D_1 \int \frac{1}{h^3} dx + D_2 = 6\mu U_0 \frac{\ell}{\Delta h}\frac{1}{h_i - \frac{\Delta h}{\ell}x} - 6\mu D_1 \frac{\ell}{\Delta h}\frac{1}{\left(h_i - \frac{\Delta h}{\ell}x\right)^2} + D_2. \tag{0.10}$$

Since $p = p_0$ at both $x = 0, \ell$. We can solve for gauge pressure $p - p_0$ which is 0 at both $x = 0, \ell$. We get ,

$$D_1 = \frac{U_0 h_i h_o}{h_i + h_o}, D_2 = -\frac{6\mu U_0 \ell}{\Delta h(h_i + h_o)}, \tag{0.11}$$

which yields for gauge pressure

$$p(x) = \frac{6\mu U_0 \ell (h_i - h)(h - h_o)}{\Delta h(h_i + h_o)h^2}. \tag{0.12}$$



(d) Find the total vertical force on the lower plate. **(5)**
Answer:- Integrating

$$F_y = \int_0^\ell p(x)\, dx. \tag{0.13}$$

downward. Changing variable to h ($dx = -\frac{\ell}{\Delta h} dh$, limits $h_i \to h_o$) and integrating, yields force per unit width

$$F_y = \frac{6\mu U_0\, \ell^2}{(h_i - h_o)^2} \left[\ln \frac{h_i}{h_o} - \frac{2(h_i - h_o)}{h_i + h_o} \right] \tag{0.14}$$

2. In some speed ranges, vortices are shed from the rear of bluff cylinders placed across a flow. The vortices alternately leave the top and bottom of the cylinder causing an alternating force normal to the freestream velocity. The vortex shedding frequency, f, is thought to depend on ρ, d, V, and μ. Use dimensional analysis to develop a functional relationship for f. Vortex shedding occurs in standard air on two cylinders with a diameter ratio of 2. Determine the velocity ratio for dynamic similarity, and the ratio of vortex shedding frequencies. **(10)**

   Answer:- The variables $f(\rho, d, V, \mu)$ involve 5 quantities in 3 dimensions (M, L, T), giving $5 - 3 = 2$ dimensionless groups. Taking ρ, V, d as repeating variables,

$$\Pi_1 = \frac{fd}{V} \text{ (Strouhal number} St), \qquad \Pi_2 = \frac{\rho V d}{\mu} \text{ (Reynolds number} Re), \tag{0.15}$$

   so the functional relationship is

$$\frac{fd}{V} = g\left(\frac{\rho V d}{\mu}\right), \qquad \text{i.e.} \quad St = g(Re). \tag{0.16}$$

   Dynamic similarity (same air, $d_2/d_1 = 2$) requires equal Reynolds number:

$$\frac{\rho V_1 d_1}{\mu} = \frac{\rho V_2 d_2}{\mu} \;\Rightarrow\; \frac{V_1}{V_2} = \frac{d_2}{d_1} = 2. \tag{0.17}$$

   Equal Re makes St equal, so $\frac{f_1 d_1}{V_1} = \frac{f_2 d_2}{V_2}$ and

$$\frac{f_1}{f_2} = \frac{V_1}{V_2}\frac{d_2}{d_1} = 2 \times 2 = 4. \tag{0.18}$$

   The smaller cylinder sheds vortices at four times the frequency of the larger one.