

Tutorial 1 : Mathematical preliminaries (Answers)

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1 Solutions

1. Using divergence theorem, show that,

$$V = \frac{1}{3} \int_S \vec{n} \cdot \vec{r} dS, \quad (1.0.1)$$

where S is the surface enclosing the volume V . Using above equation, find the volume of,

- a rectangular parallelopiped with sides a, b and c
- a right circular cone with height H and base radius R
- a sphere of radius R

Solution:- See hand-written solutions

2. Evaluate the surface integral $\iint_S \vec{F} \cdot \vec{n} dS$ where $\vec{F} = e^x \hat{i} + e^y \hat{j} + e^z \hat{k}$ and S is the surface of the cube defined by $|x| \leq 1, |y| \leq 1$, and $|z| \leq 1$.

Solution:-

$$\iint_S \vec{F} \cdot \vec{n} dS = 12(e - e^{-1}).$$

3. Show that area enclosed by a curve C is

$$A = \frac{1}{2} \oint_C (\vec{r} \times \vec{t}) \cdot \hat{k} dl. \quad (1.0.2)$$

Solution:-

4. A piston is vibrating in a resonating tube of cross-sectional area S . The location of the piston is given by $x_L = L + \ell \sin \omega t$. The other end of the tube is fixed at $x = 0$. Density of the gas is varying with a different rate as $\rho = \rho_0 + r \cos \omega t$ which is imposed by some external agent. Using Leibniz formula, compute the rate of change of mass of the cylinder.

Solution:- Rate of change of mass of the gas,

$$\frac{dm}{dt} = \rho S(\omega \ell \cos \omega t) - S(\omega r \sin \omega t)x_L.$$

5. Consider a 2D flow in a layer whose thickness is $\delta(x) = \sqrt{x}$ and flow speed is $u(y) = ky$. Calculate the volume flux given by $\int_0^{\delta(x)} u dy$ and its rate of change with x .

Solution:- Rate of change of volume flux Q with respect to x ,

$$\frac{dQ}{dx} = \frac{k}{2}.$$

6. Suppose a $\partial f / \partial x = e^x$ is known for a real valued function $f(x, y)$

- Show that you can write $f(x, y) = \int \frac{\partial f}{\partial x} dx + g(y) + C$ for a finite valued smooth f with x and y as independent variables.
- Solve for $f(x, y)$ in the above equation given that $f(x = 0, y) = 5 + \sin(y)$.

Solution:-

(b)

$$f(x, y) = e^x + \sin(y) + 4$$

7. For a function $z(x, y)$, show the following identity,

$$\left. \frac{\partial z}{\partial x} \right|_y \left. \frac{\partial x}{\partial y} \right|_z \left. \frac{\partial y}{\partial z} \right|_x = -1. \quad (1.0.3)$$

8. Consider a real square matrix A and its decomposition in diagonal and deviatoric parts, D and T respectively.

- Show that if A is symmetric, T is symmetric.
- Consider that these matrices are 3×3 . If all diagonal components of D are p , then compute $\text{tr}(T)$ in terms of $\text{tr}(A)$.
- Consider a column vector v and product Av . Discuss the effect of D and T on v separately.

Solution:- (b)

$$\text{tr}(T) = \text{tr}(A) - 3p$$

(c) The diagonal matrix D will scale the column vector v and the deviatoric part will change the orientation of the vector v .