

AML7870 (2025-2026-II): Mid-term exam

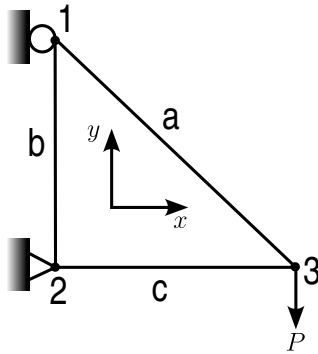
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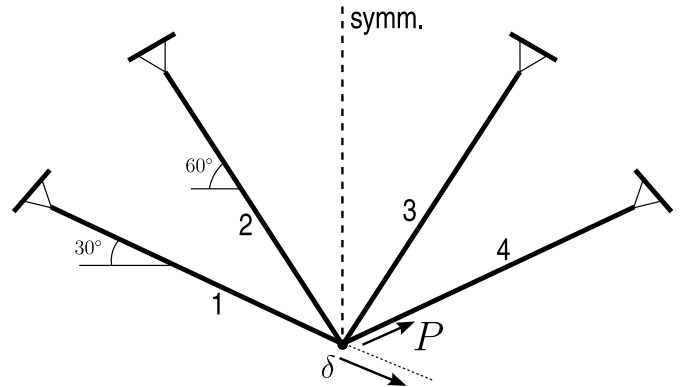
February 20, 2026

Maximum score is 60, time limit is 120 minutes. There are 5 problems in total. Clearly write all your assumptions and steps. No marks will be given for correct answers without any steps.

1. Derive the form of the equality constraint $f(x, y) = 0$ so that it is convex. Both $x, y \in \mathbb{R}$. **(5)**
2. Consider a three-bar truss member as shown. Derive the equilibrium equation for the truss in the form of $\underline{K} \underline{u} = \underline{F}$ where $\underline{u}$ consists of all the displacement degrees of freedom and $\underline{F}$ contains the force components along those degrees of freedom. **(10)**



Problem 2



Problem 3

3. The volume of the four-bar truss needs to be minimized under the constraint that the elongation of bar 1, $\delta \leq \delta_0$. The external force P is positive. All the bars have length ℓ and Young's modulus E . Also, the cross-sectional areas satisfy the relations $A_3 = A_2$ and $A_4 = A_1$. The design variables are the cross-sectional areas A_1 and A_2 . Define $A_0 = P\ell/(10\delta_0 E)$. The bounds on areas are $0.2A_0 \leq A_i \leq 2.5A_0$. Formulate the optimization problem. Clearly write the objective function and constraints in terms of A_1 and A_2 . **(15)**
4. Consider the truss compliance minimization problem as covered in class.
 - (a) Compute the derivative of the compliance $C(\underline{x})$ w.r.t. x_j , the cross-sectional area of the j^{th} member of the truss. **(5)**
 - (b) Using the Lagrangian $\mathcal{L}$ of the compliance minimization problem under the volume constraint $\sum_i x_i \ell_i \leq V_0$, and the box constraints $x_i^{\min} \leq x_i \leq x_i^{\max}$, derive the bounds on stress of bars. **(5)**
 - (c) Use the MMA approximation for the compliance. Formulate the approximate Lagrangian using the volume constraint. Derive the dual function $\phi(\lambda)$ for $\lambda \geq 0$. **(5)**
 - (d) Show that $-\phi(\lambda)$ is a convex function. **(10)**
5. For a quadratic function $f(\underline{x}) = \frac{1}{2} \underline{x}^T \underline{H} \underline{x} - \underline{b}^T \underline{x} + c$, derive the value of the optimum step size α_k at step $k \rightarrow k+1$ defined using,

$$\underline{x}^{k+1} = \underline{x}^k + \alpha_k \underline{d}^k,$$

where $\underline{d}^k$ is the steepest descent direction at $\underline{x}^k$. **(5)**