Combinatorics and Graph Theory I, Spring 2019, Sheet 11

Each exercise sheet gives 10 points. These points are given for:

- correctness of answers,
- conciseness of arguments, and
- readability of write-ups.

To pass the tutorial and get your credits you need a total of at least 60% of all points from exercise sheets, and 60% of all points from the two quizzes (each gives 10 points).

Please hand in your solution to this sheet at the beginning of the next tutorial on Thursday, May 23rd.

Exercise 11.1
Show that any graph on n vertices has two vertices which have the same degree.

Exercise 11.2

(a) Show that if $n \geq (r-1)(s-1)(t-1)+1$, then any sequence of n real numbers must contain either a strictly increasing subsequence of length r, a strictly decreasing subsequence of length s, or a constant subsequence of length t.

(a) Show also that the result of (a) is best possible, i.e., construct a sequence of $(r-1)(s-1)(t-1)$ real numbers with no strictly increasing subsequence of length r, no strictly decreasing subsequence of length s, and no constant subsequence of length t.

Exercise 11.3
Let T be a tree on $k \geq 2$ vertices and let G be a graph such that $\delta(G) \geq k-1$. Prove that G contains a copy of T, i.e., there is a subgraph of G isomorphic to T.

Exercise 11.4
Show that Checksum, as defined in the lecture, is a $(n, n-1, 2)_q$-code for a prime number q.

Exercise 11.5
Show that if C is a q-ary block code of length n and minimum distance d, then $|C| \leq q^{n-d+1}$.