Combinatorics and Graph Theory I, Spring 2019, Sheet 7

Each exercise sheet gives 10 points. These points are given for:

- correctness of answers,
- conciseness of arguments, and
- readability of write-ups.

To pass the tutorial and get your credits you need a total of at least 60% of all points from exercise sheets, and 60% of all points from the two quizzes (each gives 10 points).

Please hand in your solution to this sheet at the beginning of the next tutorial on Thursday, April 18th.

Exercise 7.1
Show that the number of times a vertex v appears in Prüfer code of a tree T is $d_T(v) - 1$, where $d_T(v)$ is the degree of the vertex v in T.

Exercise 7.2
What are the Prüfer codes associated wih the following trees.

(a) P_n (a path on n vertices).

(b) $K_{1,n}$ ($K_{1,n}$, also called a *star*, is a complete biparttite graph where the partitions have 1 and n vertices respectively).

Exercise 7.3
Draw the spanning tree for the following Prüfer codes and mark the edges with numbers in the sequence they appear.

(a) $(6, 1, 1, 4)$.

(a) $(1, 7, 2, 2, 2, 2)$

Exercise 7.4
Let G be a connected graph with exactly one cycle. The length of the cycle is r. What is the number of spanning trees of G?

Exercise 7.5
A Dutch Windmill Graph D_n is a graph having n triangles (3-cycles) which exactly one vertex in common. See Figure 1 (on the next page) for an example. How many spanning trees does D_n have? How does the answer change if we replace each trangle with C_k, i.e., a cycle on k vertices? What if we replace the triangles with K_k, i.e., complete graphs of size k?

Exercise 7.6
Show that the number of spanning trees of $K_{2,n}$, i.e., a complete bipartite graph having two vertices on one side, is $n2^{n-1}$.
(Hint: Show and use the fact that in any spanning tree of G, there is a unique path of length 2 between the two vertices of the part having size 2.)

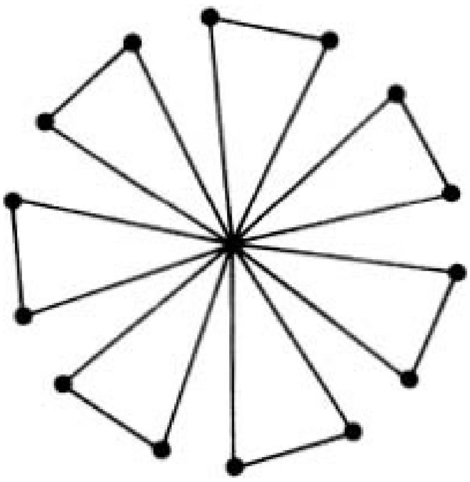

Figure 1: The Dutch Windmill Graph D_7.