Combinatorics and Graph Theory I, Spring 2019, Sheet 8

Each exercise sheet gives 10 points. These points are given for:

- correctness of answers,
- conciseness of arguments, and
- readability of write-ups.

To pass the tutorial and get your credits you need a total of at least 60% of all points from exercise sheets, and 60% of all points from the two quizzes (each gives 10 points).

Please hand in your solution to this sheet at the beginning of the next tutorial on Thursday, April 25th.

Exercise 8.1
Prove the Cauchy–Schwarz inequality using arithmetic mean-geometrich mean (AM-GM) inequality.

Exercise 8.2
For natural numbers k and n, determine all values of natural numbers $a_1, \ldots, a_k$ satisfying $\sum_{i=1}^{k} a_i = n$ such that the product $\prod_{i=1}^{k} a_i$ is maximized.

Exercise 8.3
A complete k-partite graph $K(V_1, V_2, \ldots, V_k)$ on a vertex set V is determined by a partition $V_1, \ldots, V_k$ of the set V, in which edges are pairs $\{x, y\}$ of vertices such that x and y lie in different classes of the partition. Formally, $K(V1, \ldots, V_k) = (V, E)$, where $\{x, y\} \in E$ if and only if $x \neq y$ and $|\{x, y\} \cap V_i| \leq 1$ for all $i = 1, \ldots, k$. Using part the earlier exercise, prove that the maximum number of edges of a complete k-partite graph on a given vertex set corresponds to a partition with almost equal parts, i.e., one with $||V_i| - |V_j|| \leq 1$ for all i, j. How many edges are there in such a graph $K(V_1, \ldots, V_k)$?

Exercise 8.4
Prove that for any $t \geq 2$, the maximum number of edges of a graph on n vertices containing no $K_{2,t}$ as a subgraph is at most

$$\frac{1}{2}\big((\sqrt{t-1})n^{3/2} + n\big).$$