Q.17 Consider the IVP $y\,dy = x\,dx$, $y(0)=\beta$. Find all possible $\beta \in \mathbb{R}$ for which the IVP has (a) a unique soln (b) More than one solution (c) no solutions.

Solⁿ: By solving the ODE, $y^2 = x^2 + C$. Using the initial condition $\beta^2 = C$. Hence the solns are given by $y^2 = \beta^2 + x^2$. Thus $y = \pm\sqrt{\beta^2 + x^2}$.

If $\beta = 0$, $y = x$ as well as $y = -x$ are solutions of the given IVP. If $\beta \neq 0$, then $y = \sqrt{\beta^2 + x^2}$ when $\beta > 0$ (Observe that $y = -\sqrt{\beta^2+x^2}$ is not a solⁿ).

If $\beta < 0$, $y = -\sqrt{\beta^2 + x^2}$ is a solution, & $y = \sqrt{\beta^2+x^2}$ is not a solution.

Q.18 Find all the curves in the XY-plane whose tangents pass through (a, b).

Solution: The equation of the tangent to the curve $y = y(x)$, which passes (a, b) is

$$\frac{y-b}{x-a} = \frac{dy}{dx}. \quad \text{Solving } \frac{dy}{y-b} = \frac{dx}{x-a} \Leftrightarrow \ln(y-b) = \ln(x-a) + \ln\lambda$$

$$\Rightarrow \underline{y - b = \lambda(x-a)}$$ which is a straight line.