11Q. Write the following space as the solution space of a system of homogeneous linear equations:

$$V = \cancel{\{}\ \text{span}\{(1,2,3,4),\ (1,1,2,2)\}$$

as a subspace of $\mathbb{R}^4$.

<u>Solution:</u> Consider the following matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 2 \end{pmatrix}$$

which is obtained by writing the tuples as rows. Now,

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 2 \end{pmatrix} \overset{R_2 \to R_2 - R_1}{\sim} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -1 & -2 \end{pmatrix}$$

$$\underset{R_1 \to R_1 - 2R_2}{\overset{R_2 \to R_2 \times (-1)}{\sim}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 2 \end{pmatrix}$$

which is row reduced echelon. Then

$$\text{span}\{(1,2,3,4),(1,1,2,2)\}$$

$$= \text{span}\{(1,0,1,0),\ (0,1,1,2)\}$$

$$= \cancel{\text{span}\{a}\ \{a(1,0,1,0) + b(0,1,1,2) : a,b \in \mathbb{R}\}$$

$$= \{(a,\ b,\ a+b,\ 2b) : a,b \in \mathbb{R}\}$$

$$= \{(x,y,z,w) \in \mathbb{R}^4 : z = x+y,\ w = 2y\}$$

$$= \left\{(x,y,z,w) \in \mathbb{R}^4 : \begin{matrix} x+y-z=0 \\ 2y-w=0 \end{matrix}\right\}.$$