

Q. Solve the following system.

$$y' = \begin{pmatrix} -1 & -4 & 2 \\ 2 & 5 & -1 \\ 2 & 2 & 2 \end{pmatrix} y, \text{ where } y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Soln: The eigenvalues of A are 0, 3, 3

Corresponding to $\lambda = 0$, we get an eigenvector $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

For $\lambda = 3$, the eigenspace is two dimensional and $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\}$ forms a basis for the eigenspace.

(The calculations are left to the students).

In this example, we see that we get a complete set of eigenvectors (ie. the matrix is diagonalizable). This is the easier case when eigenvalues are repeated.

The general soln. is given by

$$y = c_1 e^{0t} \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + c_3 e^{3t} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

ie.
$$y = c_1 \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + c_3 e^{3t} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$