Q

Solve $y'' + 3y' + 2y = r(t)$

where $r(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & t > 1 \end{cases}$

with Initial conditions.

$y(0) = 0$, $y'(0) = 0$

$$(s^2 + 3s + 2)Y = \frac{1}{s} - \frac{e^{-s}}{s}$$

$$Y = \left(\frac{1}{2s} + \frac{1}{2(s+2)} - \frac{1}{s+1} \right)(1 - e^{-s})$$

$$y(t) = \mathcal{L}^{-1}(Y) =$$

$$\left(\frac{1}{2} + \frac{1}{2}e^{-2t} - e^{-t} \right) - \left(\frac{1}{2} + \frac{1}{2}e^{-2(t-1)} - e^{-(t-1)} \right) u(t-1)$$

$$= \begin{cases} \frac{1}{2} + \frac{1}{2}e^{-2t} - e^{-t} & 0 < t < 1 \\ \frac{1}{2}e^{-2t}(1 - e^2) - e^{-t}(1 - e) & \text{if } t > 1 \end{cases}$$