

Q Solve $y(t) + \int_0^t y(z) \cosh(t-z) dz = t + e^t$

This can be written as

$$y(t) + y(t) \cosh(t) = t + e^t$$

Take L.T. we get

$$Y(s) + Y(s) \left(\frac{s}{s^2-1} \right) Y(s) = \frac{1}{s^2} + \frac{1}{s-1}$$

$$s, \quad Y(s) = \frac{1}{s^2} + \frac{1}{s}$$

$$y(t) = \mathcal{L}^{-1}(Y(s)) = t + 1.$$