Q Solve

$y_1' = 2y_1 + y_2$

$y_2' = 4y_1 + 2y_2 + 64t\,u(t-1)$

$y_1(0) = 2$

$y_2(0) = 0$

Taking L.T. $sY_1 = 2 + 2Y_1 + Y_2$

$sY_2 = 4Y_1 + 2Y_2 + 64e^{-s}\left(\frac{1}{s^2} + \frac{1}{s}\right)$

$$Y_1(s) = 2\left[\frac{1}{s-4} - \frac{2}{s(s-4)} + \frac{32e^{-s}}{s-4}\left(\frac{1}{s^2} + \frac{1}{s^3}\right)\right]$$

Now $\frac{1}{s-4} - \frac{2}{s(s-4)} = \frac{1}{2(s-4)} + \frac{1}{2s}$

$$\frac{64}{\cancel{s}}\left(\frac{1}{s^2(s-4)} + \frac{1}{s^3(s-4)}\right) = -\frac{5}{s} - \frac{20}{s^2} - \frac{16}{s^3} + \frac{5}{s-4}$$

So $\mathcal{L}^{-1}\left(e^{-s}\left(\frac{-5}{s} - \frac{20}{s^2} - \frac{16}{s^3} + \frac{5}{s-4}\right)\right)$

$= \left(-5 - 20(t-1) - 8(t-1)^2 + 5e^{4(t-1)}\right)u(t-1)$

Hence $y_1(t) = 1 + e^{4t} + u(t-1)\left[-8t^2 - 4t + 7 + 5e^{4(t-1)}\right]$

Similarly

$y_2(t) = -2 + 2e^{4t} + u(t-1)\left[16t^2 - 8t - 18 + 10e^{4(t-1)}\right].$