Journal of Structural Engineering
Vol. 50, No. 3, August - Spetember 2023 pp. 1-16 **No. 50-11**

Assessment of one-way shear design method of IS-456:2000

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Received: 07 March 2023; Accepted: 14 November 2023

For decades, the shear carrying capacity of Reinforced Concrete (RC) members has been studied both experimentally and analytically by researchers worldwide. Despite all the efforts by the researchers, the shear resistance mechanisms of RC members remain a complicated and debated phenomenon. As a result, different codes around the world have their shear design provisions based on different experimental and theoretical grounds. The study is categorized into two parts. First, the adequacy of the one-way shear design method of Indian Standard (IS)-456 in predicting the shear strength of RC members without and with stirrups is assessed, and second, the safety level associated with the existing one-way shear design method is examined. The uncertainty associated with the material properties, fabrication factor, and analysis method is considered in this study. The reliability analysis is performed for a combination of dead and imposed load values. The findings of this study exemplify the need for a new shear design method that offers a more uniform level of conservativism for all the factors influencing shear strength, as well as reliability analysis-based calibration for shear strength design with a higher level of uniformity in terms of safety and efficiency.

KEYWORDS: Reinforced concrete; shear design; reliability analysis; IS-456, probabilistic approach; ACI-DAfStb database.

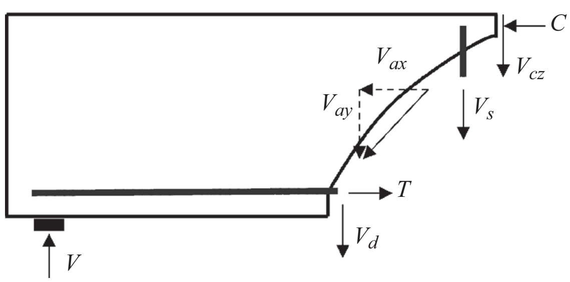
Reinforced Concrete (RC) is one of the most extensively used modern building materials. The design of RC members is based on physical models of behavior and/or test experience. While the flexural design is based on a well-defined theory that considers equilibrium and compatibility, and employs material constitutive laws, the shear design is based on theoretical, semi-empirical, or empirical relationships developed and/or calibrated using the experimental data. The mechanism of shear failure in RC beams is complicated and several factors affect it. These factors include concrete strength, amount of longitudinal reinforcement, coarse aggregate size and type, slenderness ratio, presence of axial force, the overall size of the member, and presence of stirrups (shear reinforcement)[1]. The basic mechanisms of shear transfer are depicted in Fig. 1 and are discussed as follows:

a) Shear force across the compression zone - the shear force across the compression zone is transferred by the uncracked concrete and the degree of the shear strength is limited by the depth of the compression zone. In a slender RC beam with no axial compression, the shear by the uncracked concrete becomes negligible as the depth of the concrete section decreases.
b) Aggregate interlocking caused by the displacement of crack faces- the effect of aggregate interlocking on shear strength is dependent on the size of the aggregate and crack width. Its contribution grows with the increase in aggregate size and decrease in the crack width.
c) Dowel action of longitudinal reinforcement - dowel action is the shear resisted by the longitudinal reinforcement when shear displacement occurs along the cracks. The reinforcement prevents slippage of the concrete sections and increases the shear resistance.
d) Arch action for short and deep members - arch

action is particularly relevant in short-span beams with loads applied near supports. Arch action is attributed to the internal tied arch action, which transfers the load directly to support through concrete struts.

e) Presence of stirrups – in the presence of stirrups, a large part of the shear force is sustained by the stirrups after the occurrence of diagonal cracks. The presence of stirrups ensures a certain level of restraint against the growth of inclined cracks and helps to achieve a more ductile behavior thereby reducing the likelihood of sudden failure.

The mechanism of shear failure in RC without and with stirrups is complicated and difficult to quantify because of the numerous stress redistributions that occur following cracks and these redistributions. Quantification of the shear transfer mechanisms is still the subject of research. The basic mechanisms of shear transfer are given varying degrees of importance by different scholars.



V_{cz} : shear resistance of the uncracked portion of the concrete
V_{ay} : vertical component of the 'interface shear' (aggregate interlock) force V_a
V_d : dowel force in the longitudinal reinforcement (due to dowel action)
V_s : shear force carried by the stirrups, if any

Fig. 1 Basis shear transfer mechanism

Extensive computational and experimental studies have resulted in significant advancements in shear strength models[2-4]. As a result, the shear design method of several codes around the world has been modified taking into account the influence of the key parameters on the shear strength of RC members[5,6]. Models on which shear provisions in most current design standards are based include empirical or semi-empirical truss-based approaches[5], the variable strut inclination Method[7], complex rational models based on modified compression field theory[8] and models based on general stress field approach. The one-way shear design method of the Indian standard code of practice for plain and reinforced concrete[9] was first introduced in its 1978 version and is based on semi-empirical expressions proposed by Rangan[10]. Several drawbacks of the shear design method of IS-456 have been highlighted in the past[11,12]. When the one-way shear design method of IS-456 was developed, there was still disagreement between researchers regarding the shear strength model. At present, a detailed and extensive database of shear test results for RC members has been created and recorded by the joint DAfStb-ACI Committee 445[13,14]. Hence, the first part of this study is to access the adequacy of the one-way shear design method of IS-456 in predicting the shear strength of RC members. For this, the joint DAfStb-ACI Committee 445 database of shear tests on slender RC members without and with stirrups is considered and analyzed. The analysis is based on the methodology proposed in[15].

In structural engineering, a reliability analysis is typically carried out to determine the level of structural safety. In reliability analysis, the reliability index, or the probability of failure (which is the complement of reliability) is used to quantify the level of structural safety. Various studies on reliability have been done in the past focusing on the RC members, such as reliability analysis at serviceability limit state[16-19], reliability analysis at ultimate limit state[20-23], durability reliability analysis of RC members[24-27]. However, only limited studies can be found in the literature assessing the safety level provided by the shear design methods, and none could be found for the shear design method of IS-456. Thus, the second part of this study is focused on finding the safety level provided by the shear design method of IS-456. For this, the uncertainty in the mechanical properties of the material, fabrication parameters, analysis method, and loads are considered. The probabilistic model and its parameters representing these uncertainties are taken from the published literature. The methodology presented in[28] is adopted for developing the probabilistic model to quantify the analysis uncertainty. The reliability indexes are calculated for a practical combination of values of dead and imposed loads and compared to an acceptable value or target value.

The structure of this paper is as follows. Section 2 presents the one-way shear design method of IS-456:2000. Section 3 discusses the database of shear test results created and recorded by the joint DAfStb-ACI Committee 445. In section 4, the adequacy of the one-way shear design method of IS-456 in predicting the shear strength of RC members is accessed. In section 5, the resistance model for the shear strength of RC members is developed. Section 6 presents the results of the reliability analysis. Finally, the paper ends with conclusions, presented in section 7.

ONE-WAY SHEAR DESIGN METHOD OF IS-456:2000

IS-456 suggests an empirical formula considering the grade of concrete and the amount of longitudinal reinforcement for predicting the shear strength of RC members based on the experimental research of Rangan[10]. As per the IS-456, the nominal (a.k.a. design) shear strength. V_N, of an RC member is taken as the sum of the contribution of concrete. V_{cN}, and stirrups, V_{sN}.

$$V_N = V_{cN} + V_{sN} \tag{1}$$

In the above equation, V_{cN} is obtained by using the nominal shear strength of concrete, τ_c, as

$$V_{cN} = \tau_c b_0 d_0 \tag{2}$$

where b_0 and d_0 is the nominal web width and the nominal effective depth of the member, respectively, and τ_c is the shear resistance offered by the concrete. The value of τ_c is obtained with the assumption that whenever the principal tensile stress in the neutral axis of a flexurally cracked member reaches the tensile strength of concrete the diagonal cracking strength in shear is reached. However, it does not represent the actual stress conditions. A semi-empirical formula for τ_c is given by

$$\tau_c = \frac{0.85\sqrt{0.8 f_{ck}}\left(\sqrt{1+5\beta}-1\right)}{6\beta} \tag{3}$$

where,

$$\beta = \max\left(1, \frac{0.8 f_{ck} b_0 d_0}{689 A_{st,0}}\right) = \max\left(1, \frac{0.8 f_{ck}}{6.89 p_{t0}}\right) \tag{4}$$

In the above formula, f_{ck} is the characteristic cube compressive strength of concrete, $A_{st,0}$ and $p_{t,0}=100_{Ast,0}/(b_0 d_0)$ is the nominal area and nominal percentage of longitudinal tension reinforcement, respectively. The factor of 0.85 is a reduction factor similar to the partial safety factor for materials. Originally, the formula for τ_c was derived relating to the characteristic cylinder compressive strength of concrete, f'_c. Later, f'_c was replaced by $0.8 f_{ck}$ for its use in IS-456. IS-456 provides values of τ_c for f_{ck} ranging from 15 MPa to 40 MPa and $p_{t,0}$ ranging from 0.15% to 3%.

Based on several experimental studies conducted, it has been witnessed that shallow members fail at higher loads corresponding to nominal stress in comparison to that applicable for beams of usual proportion. Taking this into consideration, IS-456 proposes an increased shear strength for solid elements which is equal to $k_c \tau_c$, where k_c is a factor having a value ranging between 1.0 to 1.3 and is expressed as

$$k_c = \begin{cases} 1.3 & \text{for } D_0 \leq 150 \text{ mm} \\ 1.6 - 0.002 D_0 & \text{for } 150 < D_0 < 300 \text{ mm} \\ 1.0 & \text{for } D_0 \geq 300 \text{ mm} \end{cases} \tag{5}$$

where D_0 is the nominal overall depth of the member.

A constant 45°angle truss model is used to estimate the nominal shear strength contribution of stirrups. Accordingly, the nominal design shear strength of vertical stirrups, V_{sN}, in Eq. (1) is given by

$$V_{sN} = \frac{0.87 f_{yk} A_{sv,0} d_0}{s_{v,0}} \tag{6}$$

where, f_{yk} is the characteristic yield strength of stirrups, $A_{sv,0}$ is the nominal area of stirrups, and is the nominal spacing of stirrups along the length of member. The factor of 0.87 in Eq. (6) is the inverse of the partial safety factor for steel, which is taken as 1.15. The percentage area of stirrups is calculated as, $p_{v,0} = 100 A_{sv,0} / (s_{v,0} b_0)$.

THE DATABASE FOR ONE-WAY SHEAR

For more than a decade, the joint ACI-ASCE Committee 445, in alliance with the German Committee of Reinforced Concrete (DAfStb) has collected and developed a database of slender beams (a/d > 2.4) that showed a clear shear failure when tested (the joint DAfStb-ACI Committee 445 shear database). For each test carried, the information in the database is compiled in the following main categories: (1) the general information of the test, (2) the fabrication properties of the cross-section, (3) load position and geometry, (4) the reinforcement area and strength, (5) the mechanical

TABLE 1 STATISTICS OF DATABASE PARAMETERS				
Parameter	Without stirrups		With stirrups	
	Min.	Max.	Min.	Max.
Web width, b_0 (mm)	50	3005	64	457.2
Effective depth, d_0 (mm)	57.2	3840	161	1369
Overall depth, D_0 (mm)	76.2	4000	200	1450
Cube strength, $f_{c,cu}$ (MPa)	16.1	173.3	16.73	156.25
Slenderness ratio, a/d	2.4	8.1	2.44	7.10
Longitudinal reinforcement, $p_{t,0}$ (%)	0.1	6.6	0.50	15.61
Max. aggregate diameter (mm)	1.0	51.0	7.0	32.0
Stirrups, $p_{v,0}$ (%)	-	-	0.07	3.87

properties of concrete and (6) the ultimate shear capacity. The database was created to aid researchers and designers in spotting research needs, providing better design code provisions, and making research results more accessible and comparative. The database used in this research includes 946 samples without and 170 samples with stirrups, respectively, subjected to concentrated load(s) and uniformly distributed load. The statistics of the most influential factors in the databases are reported in Table 1, and the distributions of these factors are shown in Figs. (2) and (3). To determine the compressive strength of concrete, the following relationship is used,

$$f_{c,pr} = 0.95 f_{c,cy} = 0.75 f_{c,cu} \tag{7}$$

where $f_{c,pr}$, $f_{c,cy}$ and $f_{c,cu}$ is the uniaxial compressive strength of slender prism (100|100|500 mm), cylinder (150|300 mm), and cube (150 mm), respectively. The prism strength is taken as the in-situ strength. For further details on the criteria for collecting the experimental data and an extensive description of the data, the reader is referred to[13-15].

COMPARISON OF THE TEST RESULTS WITH THE DESIGN APPROACH OF IS-456

Modeling uncertainty exists due to the mathematical model's inability to accurately describe the real physical system (due to assumptions and approximations made) as well as the presence of measurement error in the experimental data. As a result, there is a discrepancy between the expected and experimental values. In the structural reliability framework, this discrepancy is referred to as modeling error, analytical uncertainty, or professional factor. For defining a probabilistic model for the modeling error, the experimentally determined shear forces at failure are compared to predictions made with IS-456 shear design method. The modeling error, γ, indicating the accuracy of the shear design method is expressed as

$$\gamma = \frac{V_{u,test}}{V_{u,pred}} \tag{8}$$

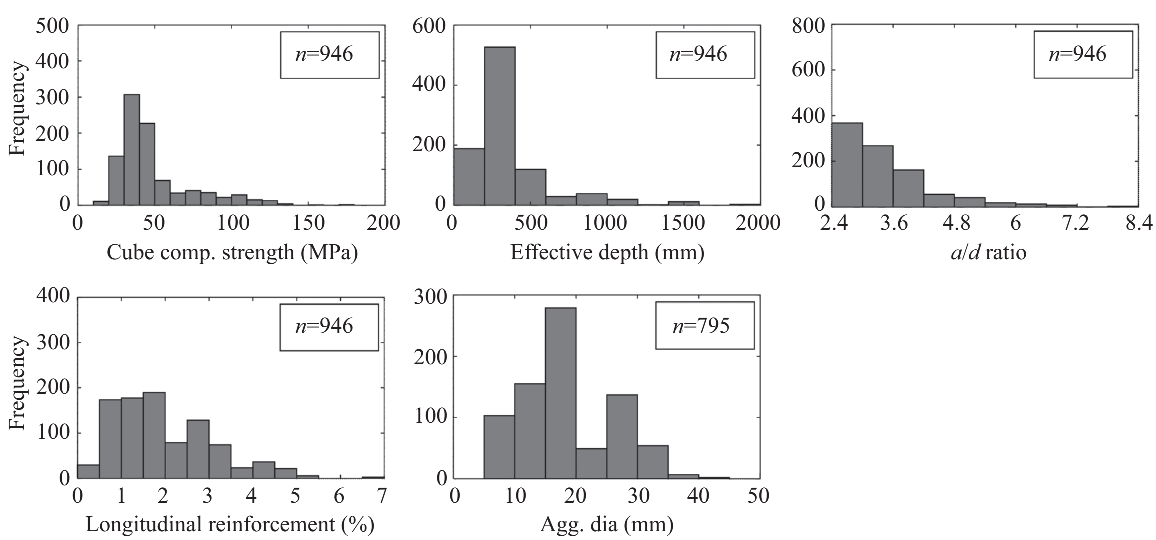


Fig. 2 Distribution of database parameters for members without stirrups

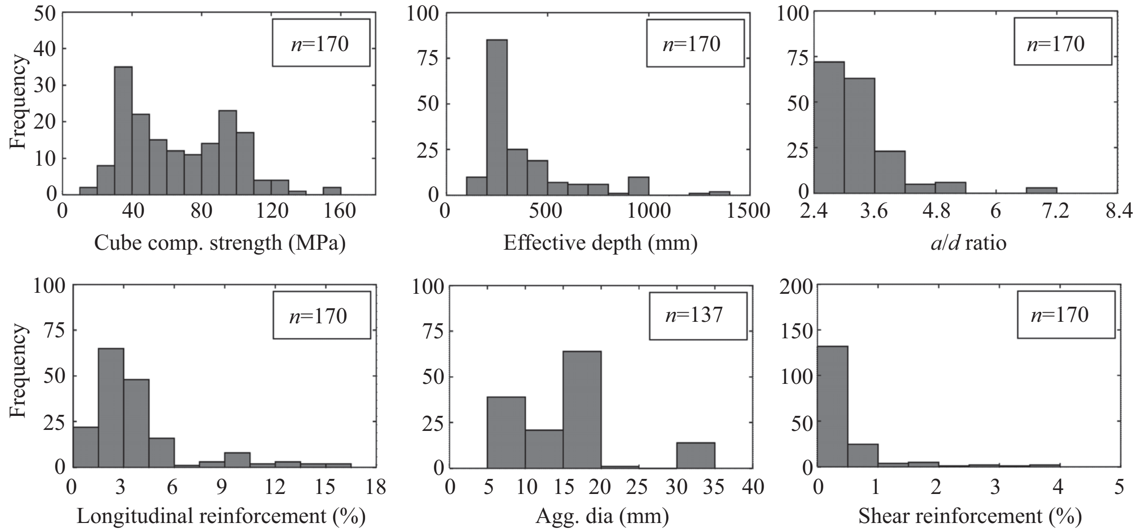
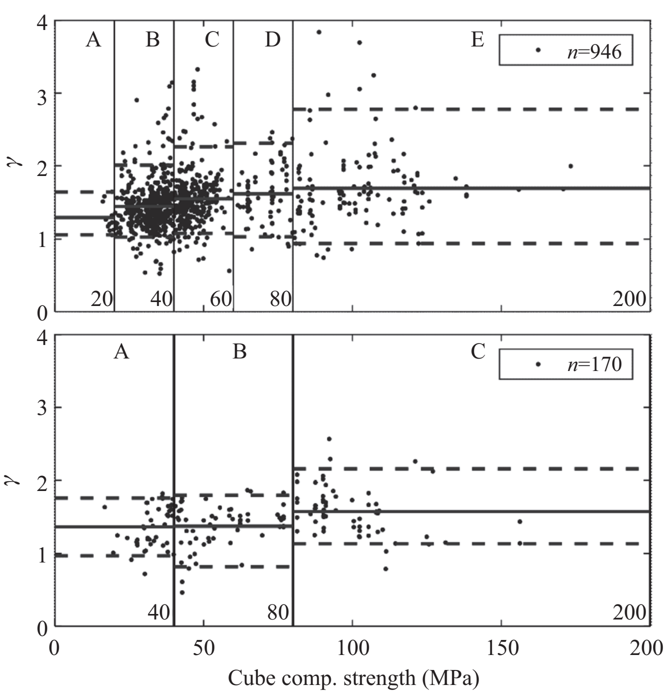


Fig. 3 Distribution of database parameters for members with stirrups

where $V_{u,test}$ is the measured shear force from the test report and $V_{u,pred}$ is the shear force predicted by IS-456 shear design method. Here, in the prediction of the resistance, the measured values of the strength (average cube strength), dimensions, loading, and support conditions reported in the database are used instead of characteristic or nominal values, and the material partial safety factors are set equal to one. To study the dependence of modeling error on different parameters and the quality of the relationship with respect to considering these parameters, the variation of modeling error with respect to the essential parameters is presented in Figs. 4-9. In each diagram, the relevant parameter is subdivided into individual ranges. For each range, the mean, 5%-fractile, and 95%-fractile are shown. Accompanying each of these figures is a table in which the statistical values for the whole dataset and each range are listed. It lists the number of samples, n,



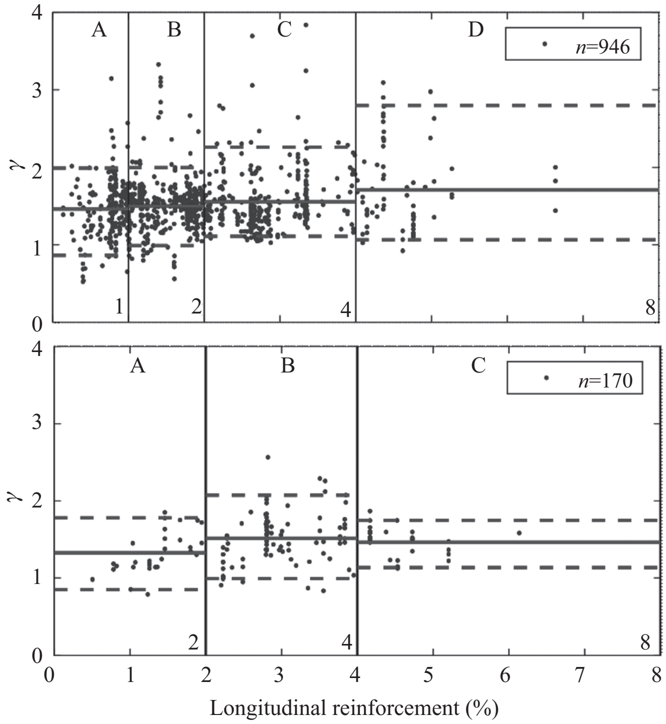
	n	μ	σ	COV	5%	95%
All tests	946	1.5	0.39	0.25	1.03	2.2
A	11	1.29	0.20	0.16	1.06	1.65
B	443	1.45	0.34	0.23	1.03	2.01
C	296	1.55	0.37	0.24	1.08	2.26
D	75	1.62	0.37	0.23	1.03	2.31
E	121	1.70	0.53	0.31	0.94	2.78

	n	μ	σ	COV	5%	95%
All tests	170	1.45	0.32	0.22	0.91	1.97
A	45	1.36	0.28	0.21	0.97	1.76
B	60	1.37	0.29	0.21	0.82	1.80
C	65	1.57	0.32	0.21	1.13	2.16

Fig. 4 Modelling error plotted vs. the cube strength: without stirrups (top) and with stirrups (bottom)

mean, μ, standard deviation, σ, Coefficient of Variation (COV, ratio of standard deviation to mean), 5%-fractile and 95%-fractile.

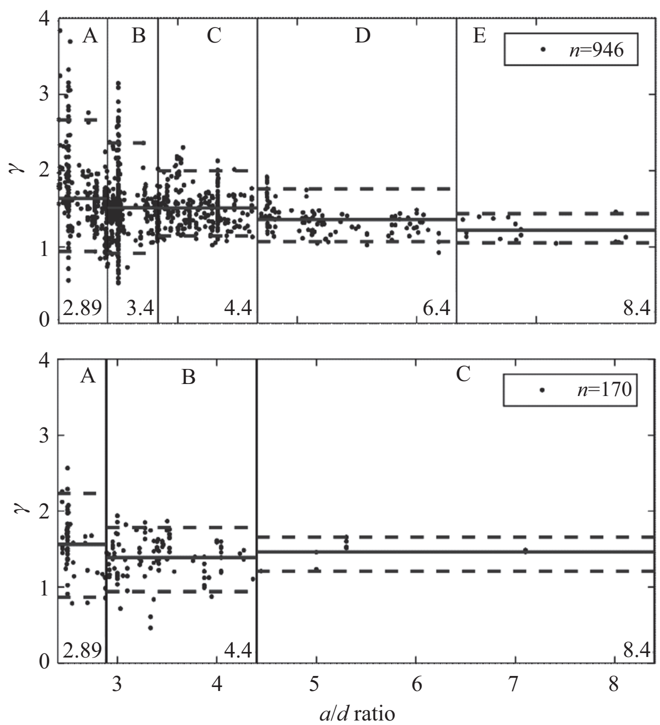
The modeling error is displayed against the cube strength in Fig. 4. For members without stirrups, the mean of the modeling error increases with higher



	n	μ	σ	COV	5%	95%
All tests	946	1.5	0.39	0.25	1.03	2.2
A	204	1.46	0.37	0.25	0.87	1.99
B	370	1.50	0.34	0.23	0.99	2.00
C	304	1.55	0.40	0.26	1.11	2.26
D	68	1.71	0.55	0.32	1.06	2.80

	n	μ	σ	COV	5%	95%
All tests	170	1.45	0.32	0.22	0.91	1.97
A	29	1.33	0.28	0.21	0.85	1.78
B	92	1.52	0.33	0.21	0.99	2.08
C	30	1.46	0.20	0.13	1.14	1.75

Fig. 5 Modelling error plotted vs. the percentage longitudinal reinforcement: without stirrups (top) and with stirrups (bottom)



	n	μ	σ	COV	5%	95%
All tests	946	1.5	0.39	0.25	1.03	2.2
A	271	1.64	0.49	0.30	0.94	2.67
B	299	1.51	0.41	0.27	0.92	2.36
C	246	1.51	0.25	0.17	1.15	2.00
D	113	1.36	0.20	0.14	1.07	1.76
E	17	1.22	0.14	0.11	1.05	1.44

	n	μ	Σ	COV	5%	95%
All tests	170	1.45	0.32	0.22	0.91	1.97
A	54	1.56	0.38	0.24	0.87	2.23
B	59	1.38	0.30	0.22	0.77	1.84
C	57	1.42	0.22	0.16	1.01	1.75

Fig. 6 Modelling error plotted vs. the slenderness ratio, a/d: without stirrups (top) and with stirrups (bottom)

cube strength. For members with stirrups, increased cube strength causes an increase in the mean of the modeling error, but the increase is small compared to members without stirrups. This shows that the IS-456 shear design approach, especially for members without stirrups, does not adequately capture the influence of compressive strength. Furthermore, the variability of the modeling error for members without stirrups, represented by COV values, is higher.

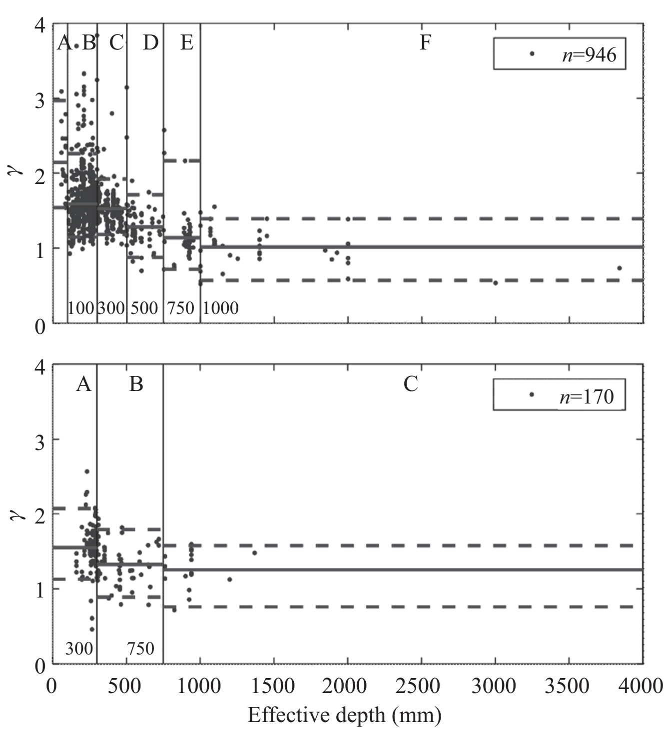
Figure 5 shows the modeling error plotted vs. the percentage longitudinal reinforcement. The mean values increase with an increased percentage of longitudinal reinforcement for members without stirrups. This suggests that the influence of percentage longitudinal reinforcement is not well captured by the IS-456 shear design method; nonetheless, the increase is not substantial. For members with stirrups, one cannot draw a meaningful conclusion probably because of a limited number of test data. Note that the beams with high reinforcement ratios are likewise ones with high-strength concrete.

Figure 6 shows the modeling error plotted vs. the slenderness a/d. The mean and COV values decrease with increased a/d both for members without and with stirrups, suggesting that the influence of a/d is not well captured by the IS-456 shear design method. For members with stirrups, a decrease in COV values with increased a/d is observed.

Figure 7 shows the modeling error plotted vs. the effective depth. A steady and considerable decrease in the mean value of modeling error is observed with increasing effective depth both for members without and with stirrups, demonstrating that the size effect is not well captured by the IS-456 shear design method. It is seen that for members without stirrups, the IS-456 shear design method on average is overly conservative with a smaller depth ($d_0 < 100$ mm) and is unsafe with a larger depth ($d_0 > 750$ mm). For members with stirrups, the decrease in modeling error with increased effective depth is small as compared with members without stirrups.

Figure 8 shows the modeling error plotted vs. the percentage stirrups. A minor variation in the mean and COV values of the modeling error is observed.

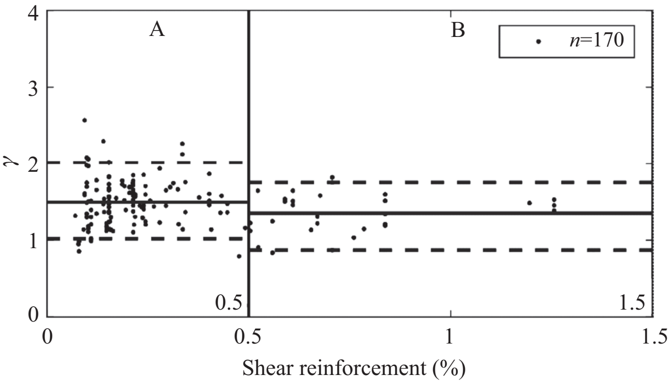
The plots presented in this section show that the influence of the factors affecting the shear strength of RC members is not well captured by the IS-456 shear design method. This suggests that a new shear design



	n	μ	σ	COV	5%	95%
All tests	946	1.5	0.39	0.25	1.03	2.2
A	18	2.14	0.45	0.21	1.54	2.97
B	620	1.59	0.37	0.23	1.15	2.26
C	166	1.53	0.28	0.18	1.18	1.92
D	55	1.28	0.24	0.19	0.88	1.71
E	50	1.14	0.36	0.32	0.72	2.16
F	37	1.01	0.23	0.23	0.57	1.39

	n	μ	σ	COV	5%	95%
All tests	170	1.45	0.32	0.22	0.91	1.97
A	96	1.55	0.31	0.20	1.13	2.07
B	58	1.33	0.27	0.21	0.89	1.79
C	16	1.26	0.25	0.20	0.76	1.58

Fig. 7 Modelling error plotted vs. the effective depth: without stirrups (top) and with stirrups (bottom)



	n	μ	σ	COV	5%	95%
All tests	170	1.45	0.32	0.22	0.91	1.97
A	131	1.50	0.30	0.20	1.02	2.02
B	29	1.36	0.26	0.19	0.87	1.76

Fig. 8 Modelling error plotted vs. the percentage area of stirrups

method is needed that exhibits a more uniform level of conservativism across a full range of the factors influencing the shear strength.

RESISTANCE MODEL

The three major sources that contribute to the uncertainty in predicting the shear resistance are uncertainty in the material properties, uncertainty in the fabrication parameters, and uncertainty associated with the methods of analysis. Due to these uncertainties, the shear resistance differs from their nominal capacity, which is calculated based on nominal values. The resistance ratio, λ_V, defined as the ratio of actual strength, V_u, and nominal strength, V_N, can be expressed as

$$\lambda_V = \frac{V_u}{V_N} = \frac{V_{u,MF}}{V_N} \times \frac{V_u}{V_{u,MF}} = \lambda_{V,MF} \times \lambda_{V,P} \quad (9)$$

where M, F, and P represent the three major uncertainties described above. In the above equation, the first term, $\lambda_{V,MF} = V_{u,MF}/V_N$, holds for the uncertainty in the material and fabrication properties, and the second term $\lambda_{V,P} = V_u / V_{u,MF}$ holds for the accuracy of the shear design method in estimating the actual shear strength. For reliability analysis, the probabilistic model for λ_V is required. For this, uncertainty in $\lambda_{V,MF}$ and $\lambda_{V,P}$ is computed individually and then integrated by simulating the possible outcomes using the crude Monte Carlo Simulation (MCS) method.

Uncertainty in material mechanical properties

Over the years, various researchers have studied the variability in concrete compressive strength, and most of them have concluded that the variability may be quantified using the normal or lognormal distribution[29-31]. A normal distribution (truncated at zero) has been assumed in this study to quantify the variability in the concrete compressive strength. The uncertainty in the concrete compressive cube strength, $f_{c,cu}$, is specified in terms of f_{ck} as

$$f_{c,cu} = \lambda_{f_{c,cu}} f_{ck} \quad (10)$$

where $\lambda_{fc,cu}$ is a normal random variable. The bias values (ratio of mean to the characteristic value) and COV for different grades of concrete are taken from the study done by Ranganathan[32]. These values are consistent with data available in the recent literature2[9-31]. For the yield strength of reinforcement steel, a few statistical distribution types have been put forward by the researchers like normal[31], log-normal[33], and beta distributions[34]. As for concrete compressive strength, the truncated normal distribution has been assumed for the yield strength of reinforcement steel. The yield strength of reinforcement, f_y, is defined as

$$f_y = \lambda_{f_y} f_{yk} \quad (11)$$

where λ_{fy} is a normal random variable. The statistical parameters of λ_{fy} are taken from the literature[32]. The statistical parameters representing the uncertainty in the material mechanical properties are summarized in Table 2.

Uncertainty in fabrication parameters

The imperfection in the dimension of RC members is captured by the fabrication uncertainty. In RC members, the fabrication uncertainty is caused by shifting from the nominal values of the cross-sectional dimensions and shape, reinforcing bars position, ties and the stirrups, and the members alignment. Variability

TABLE 2
PROBABILISTIC CHARACTERISTICS OF RANDOM VARIABLES.

Parameters	Uncertain variables	Nominal parameters	Distribution type	Bias	COV
Source of uncertainty: material mechanical properties[26]					
Compressive strength of concrete, $f_{c,cu}$	λ_{fc}	f_{ck}	Normal	1.21	0.10
Yield strength of steel, f_y	λ_{fy}	f_{yk}	Normal	1.10	0.06
Source of uncertainty: fabrication parameters[30]					
Width, b	λ_b	b_0	Normal	1.01	0.04
Cover, c	λ_c	c_0	Normal	1.00	0.04
Effective depth, beams, d_{Beam}	$\lambda_{Beam,\,d}$	d_0	Normal	0.99	0.04
Effective depth, slabs, d_{Slab}	$\lambda_{slab,\,d}$	d_0	Normal	0.92	0.12
Area of longitudinal reinf., A_{st}	λ_{Ast}	$A_{st,0}$	Normal	1.00	0.02
Area of stirrups, A_{sv}	λ_{Asv}	$A_{sv,0}$	Normal	1.00	0.02
Spacing of stirrups, S_v	λ_s	$S_{v,0}$	Normal	1.00	0.04
Source of uncertainty: loads[26]					
Dead load, DL	λ_{DL}	DL_N	Normal	1.05	0.10
Imposed load for office buildings, IL	λ_{IL}	IL_N	Type 1 extreme	0.62	0.25

in the dimensions of RC members is generally not significant. However, fabrication uncertainty can have a big impact on the overall reliability analysis response in some cases. For example, the effective depth of RC slabs is often relatively small and is subject to considerable uncertainty. Based on the several studies done by the researchers, the majority of them suggested normal distribution to model the statistical variation of dimensions of the structural members[18,31,35]. In this study, the width of the section b, the effective depth d, the cover to reinforcement c, the tensile reinforcement area A_{st}, the stirrups area A_{sv}, and the spacing of stirrups S_v, have been assumed as uncertain variables. For these variables, the statistical models are taken from the literature[36]. The mean and variations are considered to follow an independent normal distribution, given by

$$\begin{aligned} b &= \lambda_b b_0 \\ d &= \lambda_d d_0 \\ c &= \lambda_c c_0 \\ A_{st} &= \lambda_{Ast} A_{st,0} \\ A_{sv} &= \lambda_{Asv} A_{sv,0} \\ s_v &= \lambda_{sv} s_{v,0} \end{aligned} \tag{12}$$

where λ_b, λ_d, λ_c, λ_{Ast}, λ_{Asv} and λ_{sv} are independent random variables, and the subscript '0' denotes the nominal values. The statistical parameters representing the uncertainty in the fabrication parameters are also summarized in Table 2.

Uncertainty in analysis

As stated earlier, modeling uncertainty exists due to the mathematical model's inability to accurately describe the real physical system. In this section, the modeling uncertainty is characterized and quantified for reliability analysis of RC members in shear. It is observed in the previous section that the factors that influence the modeling error include concrete strength, effective depth, slenderness, size of aggregate, percentage longitudinal reinforcement, and the presence of stirrups. However, modeling error is most significantly influenced by the effective depth. Hence, for further analysis, the database is divided into three size categories based on the overall depth: that is, $D_0 \leq 300$ mm (representing common-sized slabs or foundations), 300 mm $< D_0 <$ 750 mm (representing common-sized beams or thick slabs or foundations), and $D_0 \geq 750$ mm (representing large transfer beams or thick foundations). The D_0 = 300 boundary value is selected because as per the IS-456, it is the limiting depth where the size effect influences the predicted strength. The D_0 = 750 mm boundary value is selected because it is the limiting value for IS-456 at which side face reinforcement is required to control the crack widths of relatively deep members.

Figures 9 and 10 shows the normal probability plot of samples of modeling error for members without and with stirrups, respectively, for the three sizes categorized above. The normal probability plot provides a linear plot if the data follows a normal distribution and deviates from this line otherwise. Clearly, the modeling error samples do not follow a normal distribution. The focus of reliability analysis is to determine the failure probability of a structural member by computing the probability of overload and understrength. As a result, the lower tail, exhibiting the least value of the modeling error is vital. Thus, as in[28], for members without stirrups, Fig. 9 shows the normal distribution fitted for the modeling error for the three size categories mainly focused on fitting the lowest 25% of the data. In the case of members with stirrups, because of the availability of a limited number of test data, the normal distribution is fitted to the entire data as shown in Fig. 10. In Figs. 9 and 10, the dashed vertical line indicates the boundary between the unsafe and safe predictions. The estimated mean and COV value of the modeling error for the fitted distribution are also shown in the plots. It is seen that for members without stirrups, the shear design method as per IS-456 is highly unconservative for larger depths (D_0 > 750 mm), indicated by a smaller value of mean and larger value of COV of modeling error.

The uncertainty in modeling error estimated in the form of COV values for the above-mentioned three categories includes uncertainty from two sources; the first is the uncertainty arising because of the

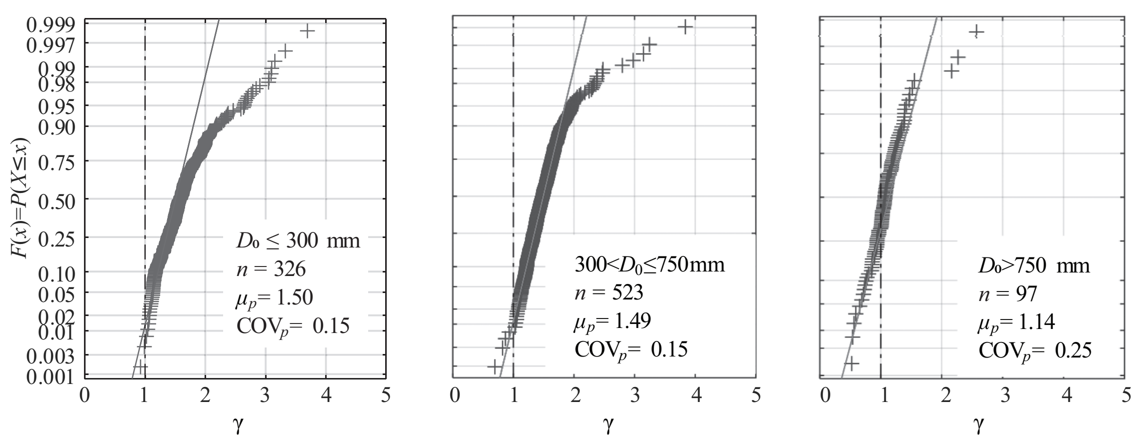


Fig. 9 Normal distribution fits for the modelling error for members without stirrups for the three size categories

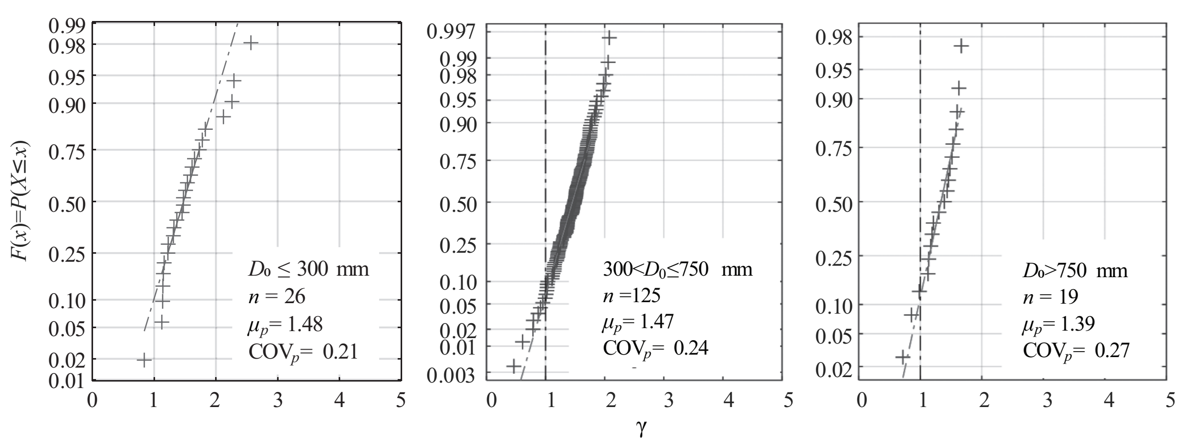


Fig. 10 Normal distribution fits for the modelling error for members without stirrups for the three size categories

unpredictable, random nature of the system under study. The second is the uncertainty arising because of the imprecisions in the sensors, readings, failure definition, uncertainty in the estimated material properties, errors present during the process of testing and measurement, and because of the disparity in the actual and the reported dimensions. Therefore, the variability in the model error should be estimated based on the distribution of γ after discounting for the uncertainty arising from the second source mentioned earlier. This is called laboratory variability. Similar to Eq. (9), γ can be rewritten as

$$\gamma = \lambda_{V,P} \times (\lambda_{V,MFT})_{Lab} \tag{13}$$

where $(\lambda_{V,MFT})_{Lab}$ denotes the laboratory variability arising from factors such as fabrication tolerance, F', material control, M', and the adopted testing procedures, T'. Considering the laboratory uncertainty to be uncorrelated with the model uncertainty, based on the first order estimate the COV of γ is given as

$$\mathrm{COV}_{\gamma}^{2} = \mathrm{COV}_{\lambda_{V,P}}^{2} + \mathrm{COV}_{Lab}^{2} \tag{14}$$

where $\mathrm{COV}_{\lambda V,P}$ represents the variability in the model error after discounting for the uncertainty arising from laboratory variability, and COV_{Lab} represents the variability because of laboratory variability. In[28], using distinct test results with more than three replicates from the database, two levels of laboratory variability are identified. These are COV_{Lab} = 0.06 and 0.10, and are adopted in this study. $(\lambda_{V,MFT})_{Lab}$ is taken equal to one (assuming statistical fluctuation in either direction in the measurement data). The statistical parameters of $\lambda_{V,P}$ after discounting the COV_{Lab} are presented in Tables 3.

Resistance statistics

In this study, for reliability analysis: for beams, the range of f_{ck} is varied from 20 to 40 MPa in intervals of 5 MPa, the range for b_0 is varied from 100 mm to 1000 mm in intervals of 100 mm, the range for d_0 is varied from 100 mm to 1000 mm in intervals of 50 mm, c_0 is varied from 20 mm to 70 mm in an interval of 5 mm, $p_{t,0}$ is varied from 0.15% to 4% in steps of 0.1%, and for members with stirrups $p_{v,0}$ is varied from 0.10% (corresponding to a minimum percentage of stirrups) to 1.1% (corresponding to a maximum percentage of stirrups) in steps of 0.1%; and for slabs, a 1000 mm wide segment is considered with f_{ck} varied from 20 to 40 MPa in steps of 5 MPa, d_0 varied from 100 mm to 300 mm in steps of 25 mm, c_0 varied from 20 mm to 30 mm in steps of 5 mm, and $p_{t,0}$ varied from 0% to 2% in steps of 0.1%. For a given sample of material

TABLE 3
STATISTICAL PARAMETERS OF $\lambda_{V,P}$

Assumed laboratory uncertainty	Size category	μ_γ	COV_γ	$\mu_{\lambda_{V,P}}$	$\mathrm{COV}_{\lambda_{V,P}}$
Members without stirrups					
COV_{Lab} = 0.06	$D_0 \le 300$	1.50	0.15	1.50	0.13
	300 mm < D_0 < 750 mm	1.49	0.15	1.49	0.13
	$D_0 \ge 750$ mm	1.14	0.25	1.14	0.24
COV_{Lab} = 0.10	$D_0 \le 300$	1.50	0.15	1.50	0.11
	300 mm < D_0 < 750 mm	1.49	0.15	1.49	0.11
	$D_0 \ge 750$ mm	1.14	0.25	1.14	0.23
Members with stirrups					
COV_{Lab} = 0.06	$D_0 \le 300$	1.48	0.21	1.48	0.20
	300 mm < D_0 < 750 mm	1.47	0.24	1.47	0.23
	$D_0 \ge 750$ mm	1.39	0.28	1.39	0.27
COV_{Lab} = 0.10	$D_0 \le 300$	1.48	0.21	1.48	0.18
	300 mm < D_0 < 750 mm	1.47	0.24	1.47	0.21
	$D_0 \ge 750$ mm	1.39	0.28	1.39	0.26

mechanical properties and fabrication parameters, $\lambda_{V,MF}$ is estimated by evaluating Eqs. (1)-(6) with characteristic and nominal values replaced with simulated values, and material partial safety factors are set equal to one. $\lambda_{V,P}$ is simulated by samples from a normal distribution with statistical parameters provided in Table 3. Finally, λ_V is simulated by applying Eq. (9).

RELIABILITY ANALYSIS

For reliability analysis, the limit state of shear is herein expressed as the difference between the resistance and load effects using the following equation,

$$g_s = V_u - (DL + IL) \tag{15}$$

where V_u represents the shear strength, DL represents the dead load effect, and IL represents the imposed load effect. The above equation can be brought to a simpler form by dividing it by DL_N as

$$g'_s = \left(\frac{V_N}{DL_N}\right)\left(\frac{V_u}{V_N}\right) - \left(\frac{DL}{DL_N}\right) - \left(\frac{IL_N}{DL_N}\right)\left(\frac{IL}{IL_N}\right) \tag{16}$$

As specified by the IS 875: Part 1[37], the relationship between nominal resistance and the nominal load effects is given by

$$V_{uN} = \gamma_D DL_N + \gamma_L IL_N \tag{17}$$

where γ_D = 1.5 and γ_L = 1.5. From Eq. (17), the limit state equation can be rewritten as

$$\begin{aligned} g'_s &= \left(\gamma_D + \gamma_L \frac{IL_N}{DL_N}\right)\left(\frac{V_u}{V_N}\right) - \left(\frac{DL}{DL_N}\right) - \left(\frac{IL_N}{DL_N}\right)\left(\frac{IL}{IL_N}\right) \\ &= \alpha_1 \lambda_V - \lambda_{DL} - \alpha_3 \lambda_{IL} \end{aligned} \tag{18}$$

where $\alpha_1 = \left(\gamma_D + \gamma_L \frac{IL_N}{DL_N}\right)$ and $\alpha_3 = IL_N / DL_N$ are the equation constants, and $\lambda_V = V_u / V_N, \lambda_{DL} = DL / DL_N$ and $\lambda_{LL} = IL / IL_N$ are the random variables that represent the uncertainty in the resistance, dead load effect, and imposed load effect, respectively.

Reliability is generally defined in terms of reliability index, β, or in terms of probability of failure, P_F. The relationship between β and P_F is given by

$$P_F = P\left(g'_s < 0\right) = \Phi\left(-\beta\right) \tag{19}$$

where Φ is the Cumulative Distribution Function (CDF) of the standard normal distribution. Several methods are available in the literature to estimate the reliability indices which include the first order reliability method, advanced second moment method, asymptotic Laplace expansions, and simulation-based methods. The crude MCS method with number of sample equal to 10^8 is used to evaluate the reliability indices for the limit state of shear in this study. This is because the evaluation of the limit state function in Eq. (18) is computationally inexpensive.

Reliability analysis results

In this study, the reliability analysis for members without and with stirrups for the three size categories and two different values assumed for COV_{Lab} is conducted at the limit state of shear. The load effects include the combined effects of dead load and imposed load. For the load components, the statistical parameters have been taken from the literature and are summarized[32,38] in Table 2. The RC sections were subjected to different ratios of IL_N / DL_N ratios varying from 0.25 to 2.0. The chosen range of IL_N / DL_N ratio shows the expected load combination for the real-life situation. Figure 11 shows the estimated reliability indices as a function of IL_N / DL_N ratio for the two different assumed values of laboratory variability in shear strength measurements. It is observed that the reliability indices remain confined in a relatively narrow band with varying IL_N / DL_N ratio for a given value of COV_{Lab}. Table 4 summarizes the findings of the reliability analysis. Table 4 also includes the desired reliability index, β_T, for a reference period of 50 years for different structural members taken from[39]. The desired reliability index is a function of several factors, e.g., past structural performance, engineering experience, estimated construction cost, and consequences of failure. A flexural failure is a ductile failure mode, thus, β_T = 3.5 is recommended for the flexural strength limit state. Shear failure of a beam without shear reinforcing is extremely brittle, thus, β_T = 3.5-4.0 is recommended for the shear failure of beams without stirrups. The shear failure of a beam with stirrups is less brittle than the shear failure of a beam without web reinforcement; therefore β_T = 3.5 is recommended. There is a significant amount of load sharing in one-way slabs, and the system reliability is substantially higher than for a 1000 mm wide segment. Moreover, the shear failure is likely to influence a relatively small portion of the structure, without a global collapse of the members. Therefore, the target reliability for shear in one-way slabs is taken as β_T = 2.50.

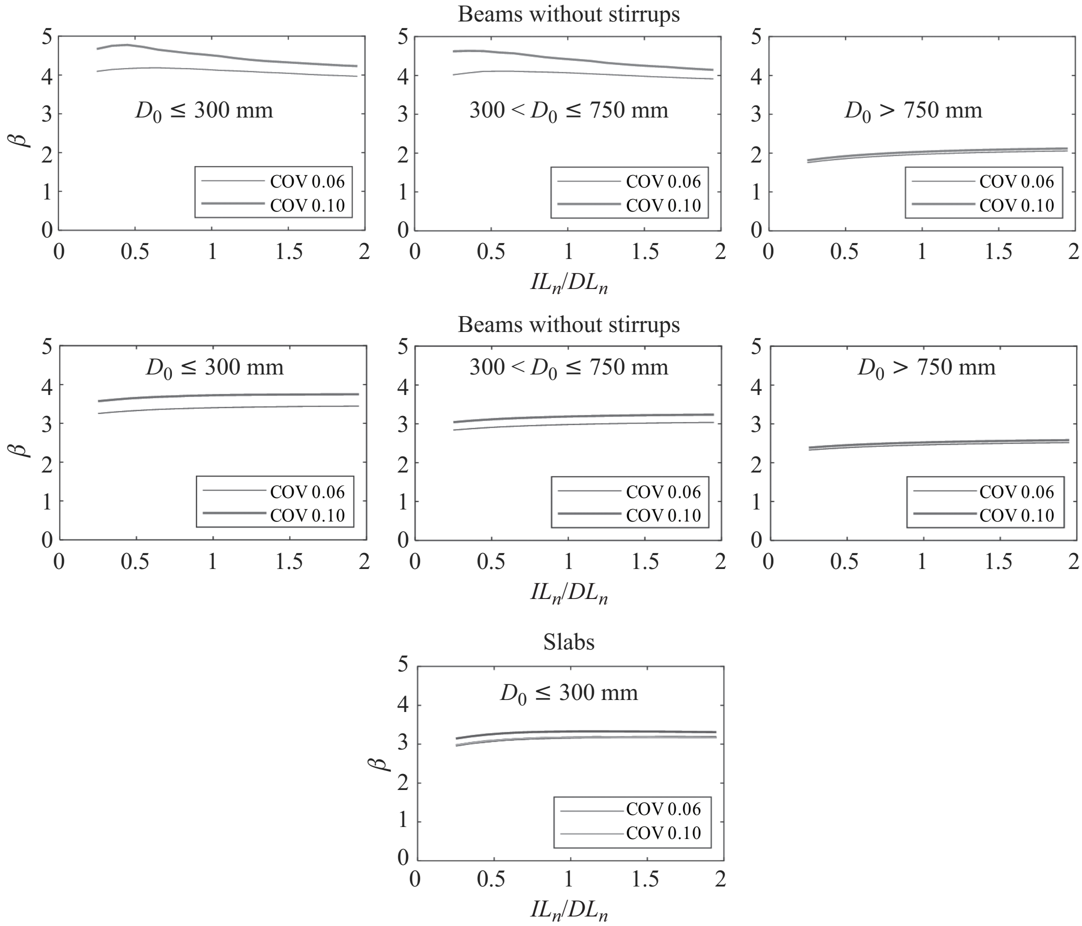


Fig. 11 Estimated reliability indices (COV indicates values of COV_{Lab})

TABLE 4

ESTIMATED AND TARGET RELIABILITY INDICES FOR THE LIMIT STATE OF SHEAR

Structural member		β range						Target β_T[39]
		Min.	Max.	Avg.	Min.	Max.	Avg.	
COV_{Lab}		0.06			0.10			
	Slab	3.00	3.21	3.17	3.14	3.33	3.30	2.50
Without stirrups	Beam: $D_0 \leq 300$ mm	3.96	4.18	4.09	4.22	4.77	4.48	3.50-4.00
	Beam: $300 < D_0 \leq 750$ mm	3.91	4.10	4.02	4.14	4.63	4.39	
	Beam: $D_0 > 750$ mm	1.75	2.05	1.96	1.81	2.12	2.02	
With stirrups	Beam: $D_0 \leq 300$ mm	3.25	3.44	3.34	3.57	3.75	3.70	3.50
	Beam: $300 < D_0 \leq 750$ mm	2.84	3.03	3.00	3.04	3.24	3.18	
	Beam: $D_0 > 750$ mm	2.32	2.52	2.45	2.38	2.6	2.51	

The results of this study show that except for beams deeper than 750 mm, the level of safety in terms of reliability indices provided by the IS-456 one-way shear design method is close to the desired level of safety, and the safety level offered for beams without stirrups is substantially higher than that for beams with stirrups. The level of safety provided for beams deeper than 750 mm is significantly low, which is undesirable. The level of safety provided for one-way slabs is smaller as compared to that for beams. This is because of considerable uncertainty in the effective depth in the case of slabs. However, the level of safety provided is significantly high, which is inefficient.

CONCLUSIONS

The main contribution of this study is the assessment of the IS-456 one-way shear design method. The adequacy of the present method in predicting the shear strength of reinforced concrete and providing the desired level of safety is assessed. The results of this study support the following conclusions:

- The plots presented in earlier show that the influence of the factors affecting the shear strength of RC members is not well captured by the IS-456 shear design method. A new shear design method is needed that exhibits a more uniform level of conservativism across a full range of the factors influencing the shear strength.
- The reliability analysis results presented in Table 4 suggest that the IS-456 shear design method is not appropriately calibrated for the limit state of shear. A reliability analysis-based calibration is warranted for a higher level of uniformity in safety and efficiency.

It should be noted that the results of the study are dependent on a) the parameter ranges in the experimental database used to assess model error; b) the probabilistic model used to represent the uncertainty in material mechanical properties and fabrication parameters and loads; and c) the parameter ranges chosen in the reliability analysis. The accuracy of these calculations needs to be validated in future studies. More research is needed to assess the reliability of RC beams and slabs that incorporate the interaction of flexure, shear, and torsion. This interaction becomes significant at the failure stage as the longitudinal reinforcement plays an important role in the shear and torsion-resisting mechanisms. Future research may also focus on the reliability analysis of RC members under earthquake load and wind load combinations.

ACKNOWLEDGMENT

The author is highly grateful to Prof. Dr.-Ing. Birol Fitik (https://www.hft-stuttgart.de/p/birol-fitik) of Stuttgart University for sharing the updated shear test database. The database forms the basis for developing this research paper. The authors would like to express their gratitude to the Indian Institute of Technology Delhi, India, for providing them with a New Faculty Seed Grant.

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(Discussion on this article must reach the editor before February 29, 2024)