Major Exam for EEL306 (II-Sem 2013-14)

Time: 2 Hours **Max. Marks: 35**

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Do not turn over to the next page till you are instructed to do so.

Important Instructions:

1) Write your response in the space provided after each question on this question paper.
2) Show all steps leading to the final answer.
3) There will be partial grading for intermediate steps leading to the final answer.
4) You need not prove/derive any result which has been derived in class.
5) Write legibly and clearly state any assumptions made.
6) Extra sheets for rough work must not be submitted for evaluation.
7) You are allowed to use a sheet of paper containing important formulas.
8) Calculators with clear memory are allowed.
9) Switch off your mobile phone and place it in your bag.
10) Keep your ID cards on the desk for the invigilator to examine.

Student Name: **Student Entry No:**

Student Signature:

Marks Obtained: **Examiner Signature:**

1) (10 Marks)

a) (2 Marks) Evaluate the following integral

$$\int_{-\infty}^{\infty} \frac{\sin(2\pi f_m(t-\tau))}{\tau}\, d\tau.$$

b) (2 Marks) What is the complex envelope of the AM signal $m(t)\sin(2\pi f_c t)$ where $m(t)$ is given to be real-valued and bandlimited to $[-W\,, W]$ with $f_c \gg W$?

c) (2 Marks) What is the output of the envelope detector if its input is $x(t)\sin(2\pi f_c t)$ where $x(t)$ is given to be real-valued and bandlimited to $[-W\,, W]$ with $f_c \gg W$?

d) (2 Marks) Why is the single sideband (SSB) modulation scheme spectrally more efficient than the double sideband suppressed carrier scheme (DSB-SC)?

e) (2 Marks) When is the vestigial sideband transmission scheme used instead of the single sideband scheme ?

Answer:

Answer for question 1

2) (6 Marks) Consider the use of a first-order Phase Locked Loop (PLL) for demodulation a FM signal with message signal $m(t)$.

Consider the message phase with a constant message signal $m(t) = A_m$. Assuming that the message phase starts at time $t = 0$, find an approximate expression for the critical time t_c such that for all time $t > t_c$ the output $v(t) > 0.9 \frac{K_f}{K_{vco}} A_m$. Here K_f and K_{vco} are the frequency sensitivity of the FM modulator and the VCO respectively.

You can assume an ideal synchronization phase before the message phase. You can also assume that the loop parameters are such that the PLL is able to lock successfully, and that the phase error between the input FM signal and the VCO output i.e., $\phi_e(t)$ is small for all $t > 0$ (i.e., $\sin(\phi_e(t)) \approx \phi_e(t)$).

Answer:

Answer for question 2

3) (5 Marks) A wide sense stationary (W.S.S.) real-valued zero-mean white Gaussian random process $N(t)$ has a power spectral density (PSD) $S_N(f) = N_0/2$.

$N(t)$ is firstly passed through a low pass filter (LPF) followed by a differentiator. The LPF has impulse response $h(t)$ and a corresponding frequency response $H(f) = 1$ for $|f| < W$ and zero when $|f| \geq W$. Let $W(t)$ be the output of the LPF. The output $W(t)$ of the LPF is then input to a differentiator whose output is given by

$$Z(t) \;=\; K\,\frac{dW(t)}{dt}$$

where $K > 0$ is a constant.

Argue that $Z(t)$ is also a W.S.S. Gaussian random process and find its PSD. Derive a closed-form expression for $\mathbb{E}[Z^2(t)]$.

Answer:

Answer for question 3

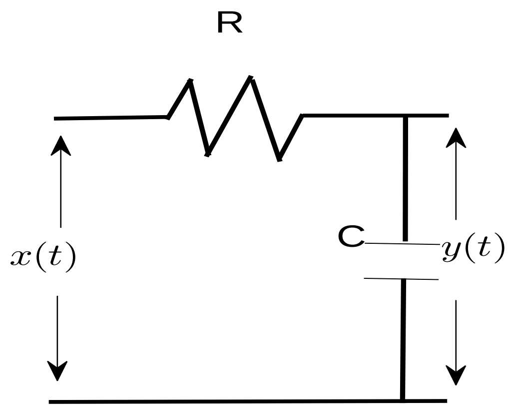

Fig. 1. Low Pass Filter

4) (5 Marks) Let the input to the low pass filter in Fig. 1 be

$$x(t) \;=\; A_c \cos(2\pi f_c t) \,+\, w(t)$$

where $w(t)$ is additive white Gaussian noise having power spectral density $N_0/2$. Derive a closed-form expression for the signal-to-noise ratio at the output of the low pass filter, i.e., $y(t)$.

Answer:

Answer for question 4

5) (5 Marks) (Intermodulation in non-linear receivers)

Let the input signal at the receiver be

$$r(t) \;=\; m(t)\,\cos(2\pi f_c t) \,+\, \cos(2\pi(f_c + 2W)t) \,+\, \cos(2\pi(f_c + 3.5W)t)$$

where $m(t)$ is a real-valued baseband signal bandlimited to $[-W\,,\,W]$ with $f_c \gg W$. Note that the useful signal is $m(t)\cos(2\pi f_c t)$ whereas the other two terms in the summation above can be considered as out-of-band interference. The output of the non-linear receiver is given by

$$y(t) = a\,r(t) \,+\, b\,r^3(t)$$

where a and b are positive constants.

The output $r(t)$ is then passed through a real-valued band-pass filter which rejects all signals outside the bands $[f_c - W\,,\,f_c + W]$ and $[-f_c - W\,,\,-f_c + W]$.

Derive an expression for the output of the band-pass filter. Interpret your result.

Answer:

Answer for question 5

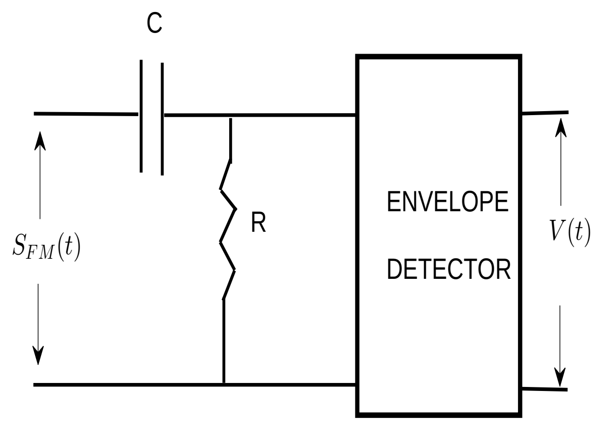

Fig. 2. Using an envelope detector for FM demodulation

6) (4 Marks) (Using an envelope detector for FM demodulation)

Consider a frequency modulated (FM) signal given by

$$S_{FM}(t) \;=\; A_c \cos\left(2\pi f_c t \,+\, 2\pi k_f \int_0^t m(x)\,dx\right)$$

where $m(t)$ is the real-valued baseband message signal band-limited to [-W , W]. The carrier frequency is f_c and it is assumed that $f_c \gg W$.

Fig. 2 describes a circuit to be used for FM demodulation. Using mathematical analysis, describe how this circuit performs FM demodulation. You will need to assume that i) $2\pi fRC \ll 1$ for any f inside the band occupied by $S_{FM}(t)$, ii) the envelope detector does not load the RC-filter before it, iii) $K_f > 0$ is such that $K_f\,|m(t)| \,<\, f_c$ for all t.

Answer:

Answer for question 6

Answer for question 6