Minor-II Exam for EEL306 (II-Sem 2013-14)

Time: 1 Hour **Max. Marks: 25**

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Do not turn over to the next page till you are instructed to do so.

Important Instructions:

1) Write your response in the space provided after each question on this question paper.
2) Show all steps leading to the final answer.
3) There will be partial grading for intermediate steps leading to the final answer.
4) You need not prove/derive any result which has been derived in class.
5) Write legibly and clearly state any assumptions made.
6) Extra sheets for rough work must not be submitted for evaluation.
7) You are allowed to use a sheet of paper containing important formulas.
8) Calculators with clear memory are allowed.
9) Switch off your mobile phone and place it in your bag.
10) Keep your ID cards on the desk for the invigilator to examine.

Student Name: **Student Entry No:**

Student Signature:

Marks Obtained: **Examiner Signature:**

1) (7 Marks) A zero mean real-valued strictly stationary white Gaussian random process $N(t)$ is input to a linear time-invariant filter having real-valued impulse response $h(t)$. The output of the filter is denoted by $X(t)$. Assume the power spectral density of $N(t)$ to be $S_N(f) = N_0/2$.

   a) (4 Marks) Derive an expression for the autocorrelation function $R_X(\tau) \triangleq \mathbb{E}[X(t)X(t-\tau)]$ of the output random process $X(t)$.

   b) (3 Marks) Assuming that $h(t) = 2B\, Sinc(2Bt)$, for what values of $(t_2 - t_1)$ are $X(t_1)$ and $X(t_2)$ statistically independent? ($Sinc(x) \triangleq \frac{\sin(\pi x)}{\pi x}$)

Answer:

Answer for question 1

2) (5 Marks) A strictly stationary real-valued Gaussian random process $X(t)$ has a mean value of $\mathbb{E}[X(t)] = 2$ and power spectral density $S_X(f)$.

Show that $\int_{-\infty}^{\infty} S_X(f)\, df \;\geq 4$.

Determine the probability density function of a random variable obtained by observing $X(t)$ at some time t_k.

Answer:

3) (9 Marks) Consider the DSB-SC transmission of a real-valued baseband message $m(t)$ ($m(t)$ is band-limited to $[-B\,,\,B]$). The transmitted DSB-SC signal is given by $s(t) = m(t)\sin(2\pi f_c t)$. Let the received passband signal be $r(t) = s(t-\tau)$ (τ is the constant propagation delay). Assume a coherent receiver which uses a phase locked loop (PLL) to acquire the phase of the received carrier during the synchronization phase (the transmitted signal during the synchronization phase is $s(t) = \sin(2\pi f_c t)$).

   a) (1 Mark) Give an expression for the output of the voltage controlled oscillator (inside the ideal PLL) at the end of the synchronization phase. (no need to derive it)

   b) (4 Marks) Draw the schematic/block diagram for the coherent demodulator which uses the carrier signal acquired during the synchronization phase for demodulation.

   c) (3 Marks) Describe the demodulation process using mathematical equations.

   d) (1 Mark) For a given B, what is the lowest possible carrier frequency f_c for which $m(t)$ can be recovered perfectly by the coherent demodulator ?

Answer:

Answer for question 3

Answer for question 3

4) (4 Marks) Consider a real-valued baseband message signal $m(t)$ band-limited to $[-W\ ,\ W]$. Let $c(t)$ be a periodic square wave having period T_p i.e., $c(t) = c(t+T_p)$. Further it is given that $c(t) = 1$ for $0 \leq t < T_p/2$ and $c(t) = -1$ for $T_p/2 \leq t < T_p$.

The product signal $x(t) = m(t)c(t)$ is passed through an ideal real-valued bandpass filter whose passband is $[f_p - W\ ,\ f_p + W]$ where $f_p \stackrel{\Delta}{=} \frac{1}{T_p}$ (the frequency response of this filter is 1 for frequencies in the passband and is zero elsewhere). Let the output of this filter be $s(t)$.

Assuming $f_p > 2W$, derive an expression for $s(t)$. Describe the significance of your result.

Answer:

Answer for question 4