EE-306
(lecture 31st Jan 2014)

Relation between the power spectral density of the input and output random process.

$$X(t) \longrightarrow \boxed{\begin{matrix} LTI \\ h(t) \end{matrix}} \longrightarrow Y(t)$$

We know that if $X(t)$ is w.s.s. (finite power) and $\int |h(t)| < \infty$ (filter is stable), then $Y(t)$ is w.s.s and has finite power.

The next question is, what is the P.S.D of $Y(t)$?

From the previous lecture we know that

$$R_Y(\tau) = \iint h(\tau_1)\, h^*(\tau_2)\, R_X(\tau + \tau_2 - \tau_1)\, d\tau_1\, d\tau_2 \qquad (1)$$

Since P.S.D. is the Fourier transform of the autocorrelation function we have (2)

$$S_Y(f) = \int R_Y(\tau) e^{-j2\pi f\tau} d\tau$$

$$= \int\int\int h(\tau_1) h^*(\tau_2) R_X(\tau_2 - \tau_1 + \tau) e^{-j2\pi f\tau} d\tau_1 d\tau_2 d\tau$$

(using (1))

$$= \int\int\int h(\tau_1) h^*(\tau_2) R_X(\tau_2 - \tau_1 + \tau) e^{-j2\pi f(\tau + \tau_2 - \tau_1)} e^{j2\pi f(\tau_2 - \tau_1)} d\tau_1 d\tau_2 d\tau$$

Next we do the substitution of integration variables (we keep τ_1 and τ_2 same, but we define $\tau' \triangleq \tau + \tau_2 - \tau_2$ instead of τ.

$$= \int_{\tau_1=-\infty}^{\infty}\int_{\tau_2=-\infty}^{\infty} h(\tau_1) h^*(\tau_2) \left[\int R_X(\tau_2 - \tau_1 + \tau) e^{-j2\pi f(\tau + \tau_2 - \tau_1)} d\tau\right] e^{j2\pi f(\tau_2 - \tau_1)} d\tau_1 d\tau_2$$

$$= \int_{\tau_1=-\infty}^{\infty}\int_{\tau_2=-\infty}^{\infty} h(\tau_1) h^*(\tau_2) e^{j2\pi f(\tau_2 - \tau_1)} \left[\int R_X(\tau') e^{-j2\pi f\tau'} d\tau'\right] d\tau_1 d\tau_2$$

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Since $\int R_X(\tau') e^{-j2\pi f\tau'} d\tau' = S_X(f)$, we have

$$S_Y(f) = S_X(f) \iint h(\tau_1) h^*(\tau_2) e^{j2\pi f(\tau_2-\tau_1)} d\tau_1 d\tau_2$$

$$= S_X(f) \int h(\tau_1) e^{-j2\pi f\tau_1} d\tau_1 \int h^*(\tau_2) e^{j2\pi f\tau_2} d\tau_2$$

$$= S_X(f) \quad H(f)\, H^*(f)$$

$$= S_X(f)\, |H(f)|^2 .$$

— Discussion on white noise.

— Discussion on strict stationarity

— Ergodic processes.

white <u>noise</u> $W(t)$

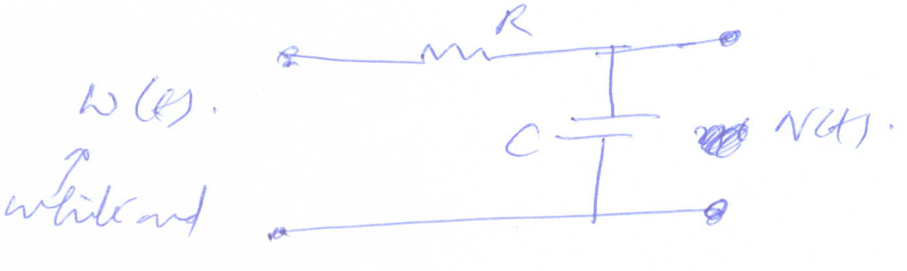
$$S_W(f) = \frac{N_0}{2}$$

$$N_0 = kT$$

$$R_W(\tau) = \frac{N_0}{2}\delta(\tau).$$

- mathematical model for situations where the noise process bandwidth >> signal bandwidth.

- do <u>Example 5-15</u> in book.



<u>W.S.S.</u>, find $R_N(\tau) = ?!$

$$H(f) = \frac{1}{1 + j2\pi fRC}$$

$$S_N(f) = \frac{N_0/2}{1 + (2\pi fRC)^2} \rightleftharpoons \frac{N_0}{4RC} e^{\frac{-|\tau|}{RC}}$$

(use $e(-a|t|) \rightleftharpoons \dfrac{2a}{a^2 + (2\pi f)^2}$.