Lecture – 17, 18 (Feb 26, March 11, 2014)

DSB-SC (Double Sideband Suppressed carrier)

message signal – $m(t)$.

DSB-SC signal

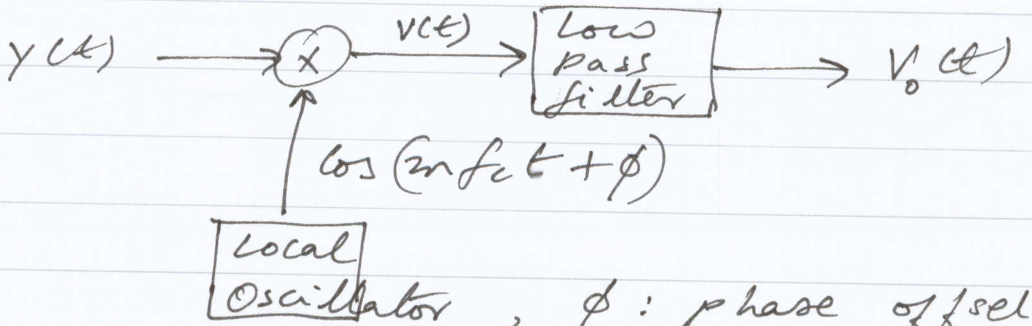
$$s(t) = m(t)\cos 2\pi f_c t. \quad \text{(no carrier signal)}$$

$$\therefore \quad S(f) = \frac{M(f-f_c) + M(f+f_c)}{2}.$$

"Coherent Detection."

Received signal is

$$y(t) = s(t-\tau) = m(t-\tau)\cos(2\pi f_c t - 2\pi f_c \tau)$$



, ϕ : phase offset of local oscillator.

$$V(t) = m(t-\tau)\cos(2\pi f_c t - 2\pi f_c \tau)\cos(2\pi f_c t + \phi)$$

$$= \frac{m(t-\tau)}{2}\left[\cos(2\pi f_c \tau + \phi) + \cos(4\pi f_c t + \phi - 2\pi f_c \tau)\right]$$

Assuming $m(t)$ is bandlimited to $[-W, W]$, and the low pass filter only allows signals in the band $[-W, W]$ to pass through.

This filter therefore rejects the signal

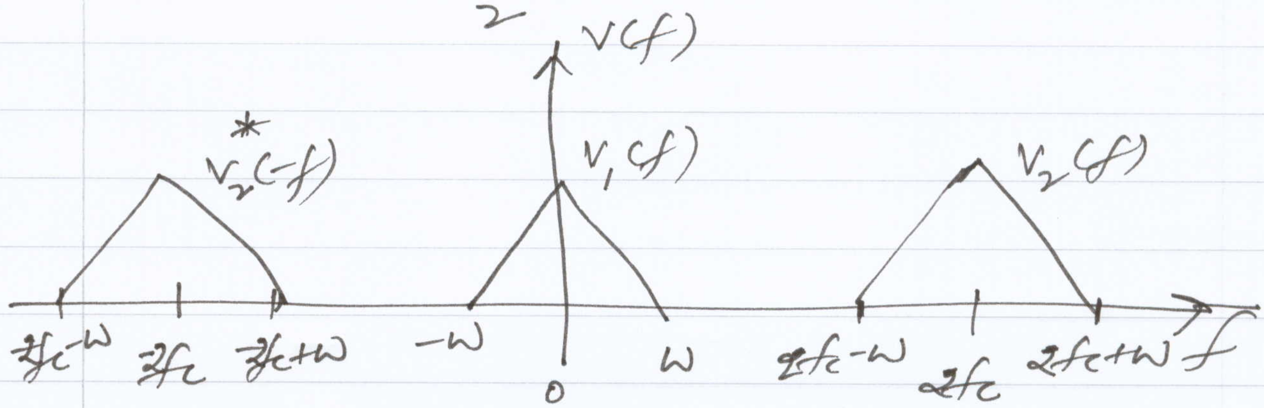
$$\frac{m(t-\tau)}{2}\cos(4\pi f_c t + \phi - 2\pi f_c \tau).$$

<u>Detail:</u>

$v(t) = v_1(t) + v_2(t)$, where

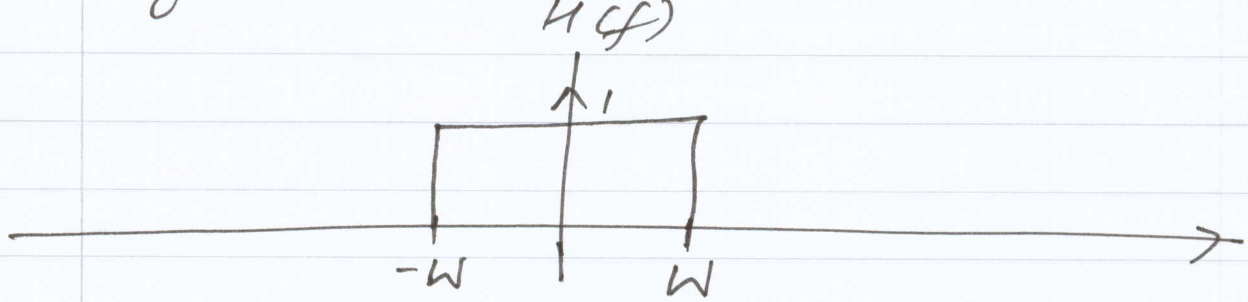
$$v_1(t) = \frac{m(t-\tau)}{2}\cos(2\pi f_c \tau + \phi) \text{ and}$$

$$v_2(t) = \frac{m(t-\tau)}{2}\cos(4\pi f_c t + \phi - 2\pi f_c \tau).$$

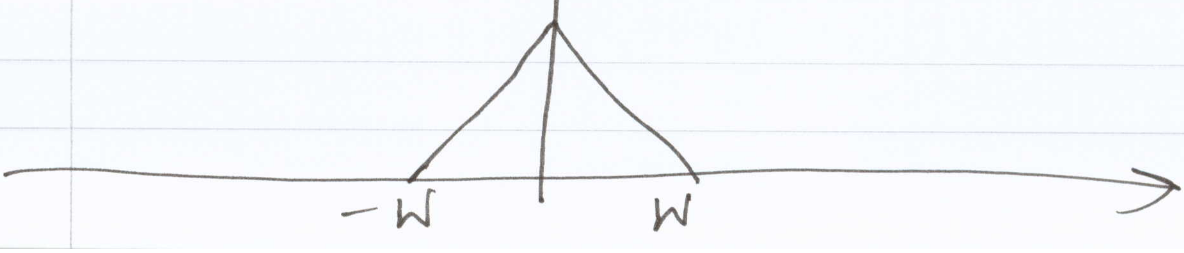


Assuming $2f_c - W$, i.e., $f_c > W$, $V_1(f)$ and $V_2(f)$ donot overlap.

using a low pass filter



$V_0(f) = V_1(f)$



$\therefore$ $V_0(t) = \frac{1}{2} m(t-\tau) \cos(2\pi f_c \tau + \phi)$

If $2\pi f_c \tau + \phi = n\pi/2$, n is an odd integer

then $V_0(t) = 0$.

The unknown phase $(2\pi f_c \tau + \phi)$ affects the received signal to noise ratio.

This problem happens due to the fact that the phase offset of the local oscillator (i.e. ϕ) does not match the phase offset of receiver carrier $\cos(2\pi f_c t - 2\pi f_c \tau)$

Such a receiver is called non-coherent. We next discuss a receiver which can resolve this problem of "non-coherency".

VCO (Voltage Controlled Oscillator)

Instantaneous frequency of the oscillator output

$f_i(t) = f_c + k v(t)$, where $v(t)$ is the input control voltage to the oscillator.

Instantaneous phase

$$\phi_{vco}(t) = 2\pi \int_0^t f_i(t)\,dt + \phi_{vco}(0)$$

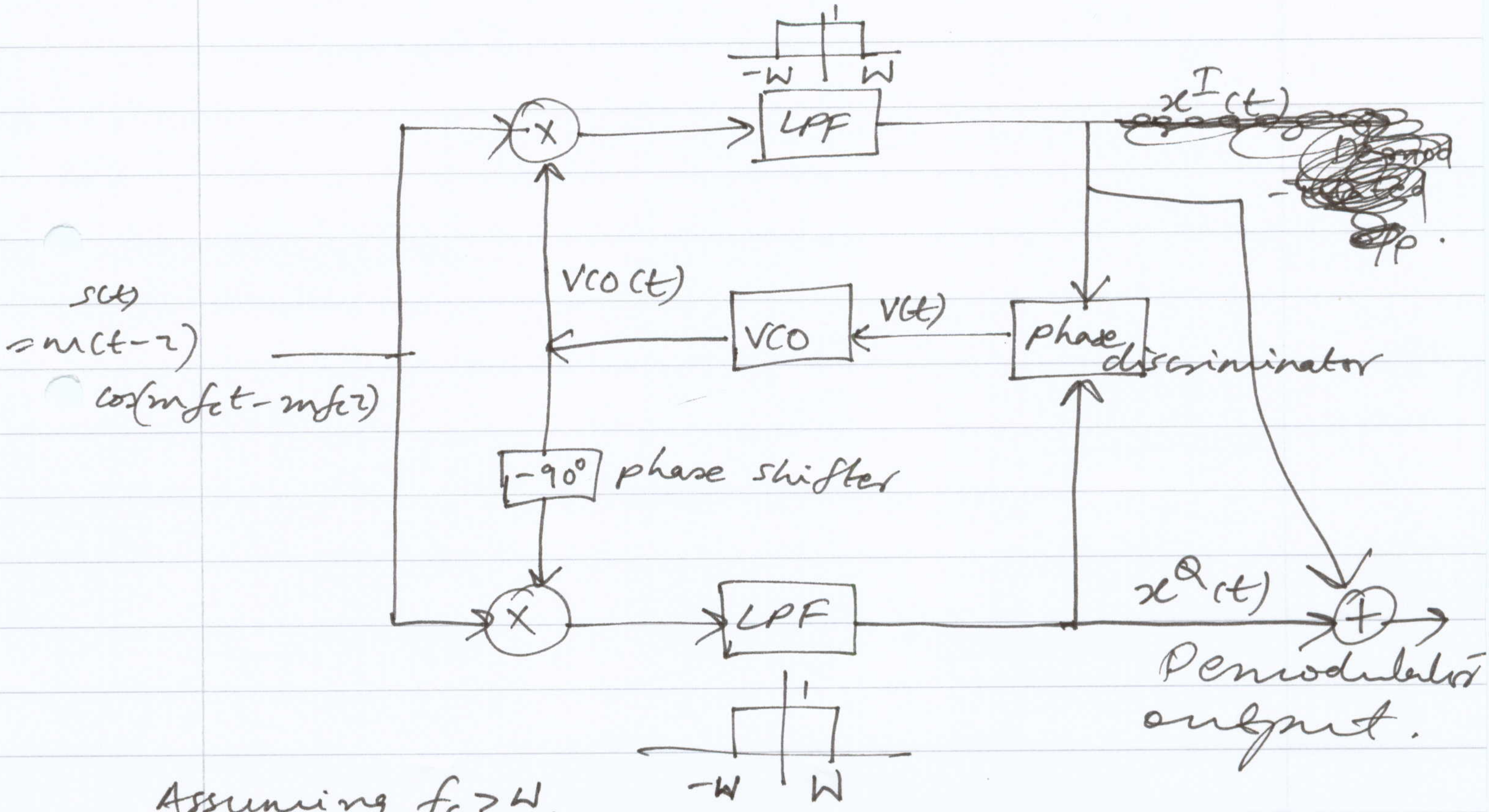
$$= 2\pi f_c t + 2\pi k \int_0^t v(t)\,dt + \phi_{vco}(0)$$

$\cancel{VCO}$: $VCO(t) = A_{VCO}\cos\left(\phi_{VCO}(t)\right)$

$$VCO(t) = A_{VCO}\cos\left(2\pi f_c t + \phi_{VCO}(0) + 2\pi k\int_0^t v(t)\,dt\right)$$

(COSTAS RECEIVER.).

A COHERENT RECEIVER.



Assuming $f_c > W$, we get

$$x^I(t) = \frac{A_{VCO}}{2}\, m(t-\tau)\cos\phi_e(t) \quad \text{and}$$

$$x^Q(t) = \frac{A_{VCO}}{2}\, m(t-\tau)\sin\phi_e(t)$$

where

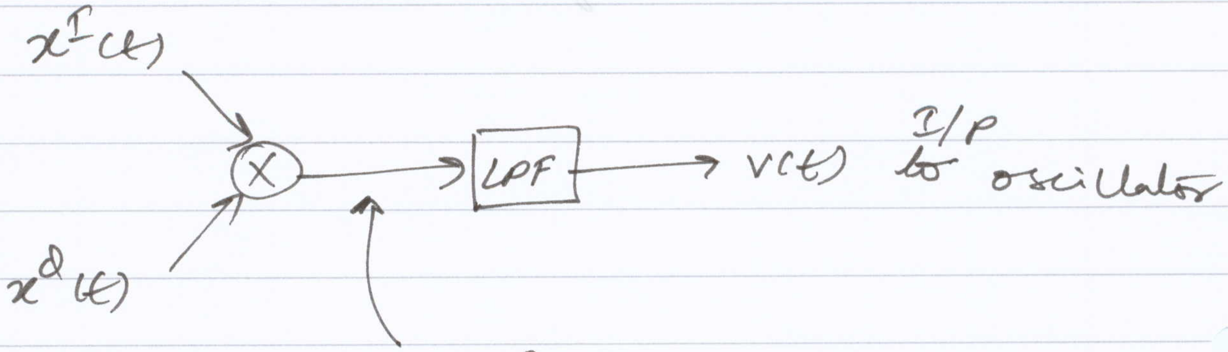
$$\phi_e(t) \triangleq 2\pi f_c \tau + \phi_{VCO}(0) + 2\pi k\int_0^t v(t)\,dt.$$

$$\therefore \frac{d\phi_e(t)}{dt} = 2\pi k\, v(t).$$

We would ideally like the demodulated o/p

$$\left(x^Q(t) + x^I(t)\right) = \frac{A_{VCO}}{2}\, m(t-\tau),$$

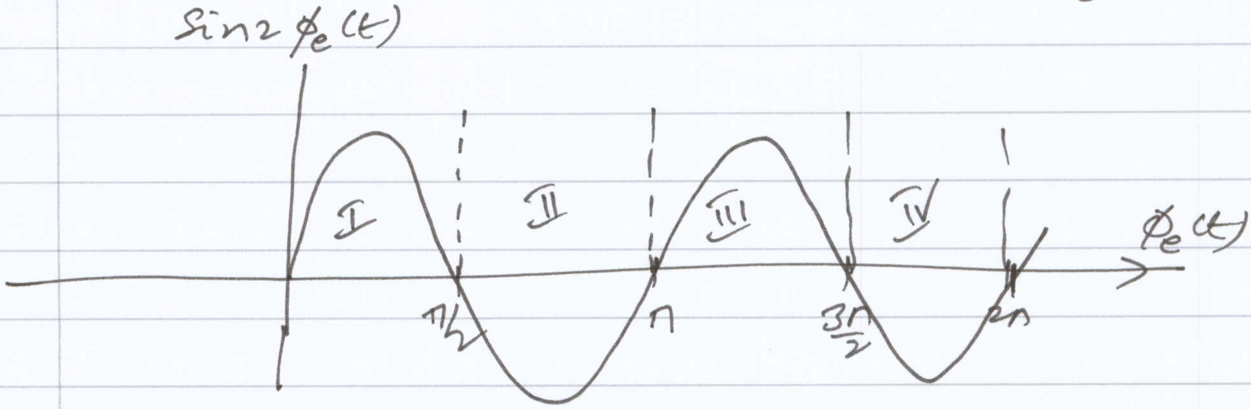
phase discriminator.



$$= \frac{A_{vco}^2}{4} m^2(t-\tau) \sin\phi_e(t) \cos\phi_e(t)$$

$$= \frac{A_{vco}^2}{8} m^2(t-\tau) \sin 2\phi_e(t).$$

$$v(t) = \frac{A_{vco}^2}{8} c \sin 2\phi_e(t)$$ where $c>0$ is the D.C. component of $m^2(t-\tau)$.



As $t \to \infty$, what are the possible values for $\phi_e(t)$.

since $$\frac{d\phi_e(t)}{dt} = 2\pi k v(t),$$

equilibrium point (after which $\phi_e(t)$ and $v(t)$ donot change) are clearly $\phi_e(t) = 0, \pi/2, \pi, 3\pi/2$ (modulo 2π)

This is because for each of these values of $\phi_e(t)$, $\sin 2\phi_e(t) = 0$ and therefore

$$\frac{d\phi_e(t)}{dt} = 2\pi k v(t)$$

$$= 2\pi k \cdot \frac{A_{V_{CO}}^2}{8} c \sin 2\phi_e(t)$$

$= 0$, i.e; $\phi_e(t)$ ~~stabilizes~~. does not change.

But which of these four locations is a stable equilibrium point?

If $\phi_e(t)$ is in region I (see the figure) then $\sin 2\phi_e(t) > 0$, i.e, $v(t) > 0$ and therefore the oscillator phase increases resulting in $\frac{d\phi_e(t)}{dt} > 0$ (since $\frac{d\phi_e(t)}{dt} = 2\pi k v(t)$)

$\therefore$ $\phi_e(t)$ increases till it exceeds $\pi/2$ (i.e it enters region II). In region II, $\sin 2\phi_e(t) < 0 \Rightarrow v(t) < 0 \Rightarrow \frac{d\phi_e(t)}{dt} < 0$,

and therefore $\phi_e(t)$ reverts back to $\pi/2$.

It can shown (by similar arguments) that $\phi_e(t) = \pi/2$ and $\phi_e(t) = 3\pi/2$ are the two stable ~~points~~ equilibrium points.

For both $\phi_e(t) = \pi/2$ and $\phi_e(t) = 3\pi/2$,

$$x^I(t) = \frac{A_{vco}}{2} m(t-\tau) \cos\phi_e(t) = 0$$, and

$$x^Q(t) = \frac{A_{vco}}{2} m(t-\tau) \sin\phi_e(t)$$
$$= \pm \frac{A_{vco}}{2} m(t-\tau)$$

therefore the demodulator o/p

has a sign ambiguity.