provided, of course, that both the above infinite integrals exist. The function, $p(f)$, is then called the *Fourier transform* of $X(t)$, and (4.5.7) is called the *Fourier integral* representation of $X(t)$. In fact, since the relationship between $X(t)$ and $p(f)$ is symmetrical, $X(t)$ and $p(f)$ are sometimes called a "*Fourier pair*". Alternatively, (4.5.8) may be written as

$$\begin{aligned} p(f) &= \int_{-\infty}^{\infty} X(t)\{\cos 2\pi ft - i \sin 2\pi ft)\, dt \\ &= g(f) - ik(f), \end{aligned} \tag{4.5.8'}$$

say, and then (4.5.7) may be written in the form,

$$X(t) = \int_{-\infty}^{\infty} \{g(f) \cos 2\pi ft + k(f) \sin 2\pi ft\}\, df, \tag{4.5.7)'}$$

remembering that $X(t)$ is real valued. If $X(t)$ is an even function (i.e. if $X(t) = X(-t)$, all t), then clearly $k(f) \equiv 0$, and in place of (4.5.7)′ we have the *cosine transform*,

$$X(t) = \int_{-\infty}^{\infty} g(f) \cos 2\pi ft\, df.$$

Similarly, if $X(t)$ is an odd function (i.e. $X(t) = -X(-t)$, all t), we have a *sine transform*,

$$X(t) = \int_{-\infty}^{\infty} \{-k(f)\} \sin 2\pi ft\, dt.$$

(Well known examples of functions and their Fourier transforms are given in Stuart (1961), Kufner and Kadlec (1971).)

When we recall our previous discussion on the existence of Fourier series it becomes apparent that establishing the existence of the Fourier integral representations (4.5.7) as the limiting form of (4.5.6) is a step which requires some caution. In order for (4.5.7), (4.5.8), to be meaningful we require, at the very least, that the function $X(t)$ should be such that $p(f)$ exists for all f. A sufficient condition for this is that $X(t)$ *be absolutely* integrable over the infinite interval, $(-\infty, \infty)$, i.e. that

$$\int_{-\infty}^{\infty} |X(t)|\, dt < \infty. \tag{4.5.9}$$

It then follows trivially that,

$$|p(f)| < \int_{-\infty}^{\infty} |X(t)|\, |\exp(-2ift)|\, dt = \int_{-\infty}^{\infty} |X(t)|\, dt < \infty, \qquad \text{all } f.$$