discrete set of frequencies which contribute to the total power have changed from $n/2\pi$ c.p.s. to $n/2T$ c.p.s., $n = 0, 1, 2, \dots$; as shown in Fig. 4.3.

Note that if T is much larger than π the set of frequencies, $(1/2T, 2/2T, 3/2T, \dots)$, which form the above power spectrum is much more "closely packed" then the standard set, $(1/2\pi, 2/2\pi, 3/2\pi, \dots)$, which form the power spectrum for functions with periodicity 2π.

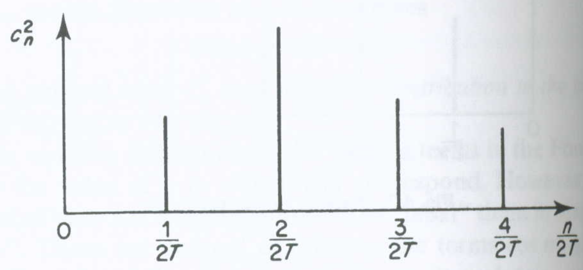

Fig. 4.3. Discrete power spectrum.

4.5 NON-PERIODIC FUNCTIONS; FOURIER INTEGRALS

We now turn our attention to the case of a real valued non-periodic function $X(t)$ (but still deterministic). Since $X(t)$ does not now possess any form of periodic structure we cannot express it in the form of a Fourier series which would be valid for *all t*. However, we can construct a Fourier series which will represent it over a *finite* interval by using the standard trick of defining a new function which is identical to $X(t)$ over a certain interval, but is periodic outside this interval. We thus choose an interval, $(-T, T)$, and define a new function, $X_T^*(t)$, by

$$\begin{aligned} X_T^*(t) &= X(t), \qquad & -T \leq t \leq T, \\ X_T^*(t+2pT) &= X_T^*(t), \qquad & p = 1, 2, 3, \dots. \end{aligned} \tag{4.5.1}$$

Then $X_T^*(t)$ is certainly periodic, with period $2T$, and, subject to the usual regularity conditions, may be expressed in a Fourier series of the form (4.3.1), i.e. we may write,

$$X_T^*(t) = \sum_{n=0}^{\infty} (a_n \cos 2\pi f_n t + b_n \sin 2\pi f_n t), \tag{4.5.2}$$

where, for each n, $f_n = n/2T$, and $b_0 = 0$.