Using the well known relations,

$$\cos 2\pi f_n t = \frac{1}{2}[\exp(2\pi i f_n t) + \exp(-2\pi i f_n t)],$$

$$\sin 2\pi f_n t = \frac{1}{2i}[\exp(2\pi i f_n t) - \exp(-2\pi i f_n t)],$$

we may rewrite (4.5.2) in the more convenient "complex exponential" form,

$$X_T^*(t) = \sum_{n=-\infty}^{\infty} A_n \, e^{2\pi i f_n t}, \tag{4.5.3}$$

where

$$A_n = \begin{cases} \tfrac{1}{2}(a_n - ib_n), & n \geqslant 1 \\ \tfrac{1}{2}a_0, & n = 0 \\ \tfrac{1}{2}(a_{|n|} + ib_{|n|}), & n \leqslant -1. \end{cases} \tag{4.5.4}$$

Substituting the expressions (4.3.2) for a_n, b_n, into (4.5.4) we have,

$$A_n = \frac{1}{2T}\int_{-T}^{T} X_T^*(t)\, e^{-2\pi i f_n t}\, dt$$

$$= \frac{1}{2T}\int_{-T}^{T} X(t)\, e^{-2\pi i f_n t}\, dt, \tag{4.5.5}$$

since $X(t)$ and $X_T^*(t)$ are identical over interval $(-T, T)$. Hence, for $-T \leqslant t \leqslant T$, we may write,

$$X(t) \equiv X_T^*(t) = \sum_{n=-\infty}^{\infty} \left(\int_{-T}^{T} X(t)\, e^{-2\pi i f_n t}\, dt \right) e^{2\pi i f_n t}\, \delta f_n, \tag{4.5.6}$$

where we have written

$$\delta f_n = f_n - f_{n-1} = 1/2T.$$

Now consider the limiting situation as we let $T \to \infty$. In this case $\delta f_n \to 0$, i.e. the discrete set of frequency points, $(\dots f_{-2}, f_{-1}, f_0, f_1, f_2, \dots)$ becomes a *continuous* set of points. The summation on the right-hand side of (4.5.6) will then become an *integral*, so that as $T \to \infty$ we obtain formally, for all t,

$$X(t) = \int_{-\infty}^{\infty} p(f)\, e^{2\pi i f t}\, df, \tag{4.5.7}$$

where

$$p(f) = \int_{-\infty}^{\infty} X(t)\, e^{-2\pi i f t}\, dt, \tag{4.5.8}$$