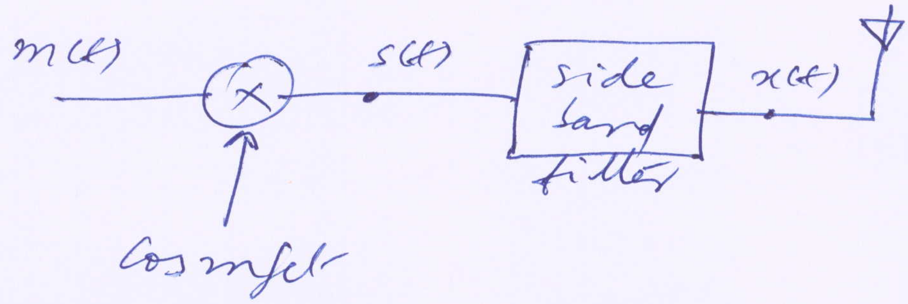
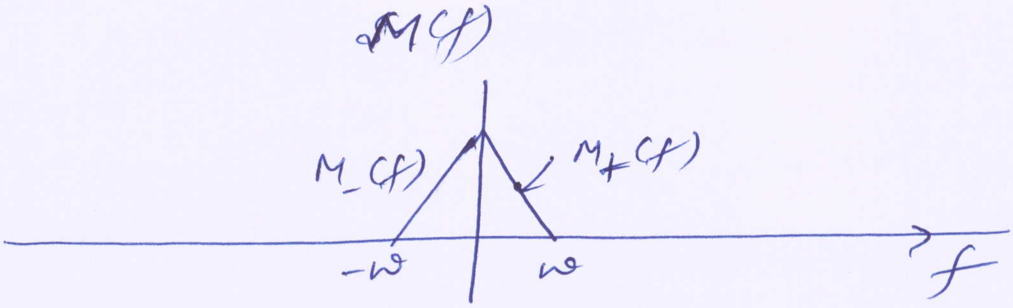
Single Sideband Modulation: (1)

(Synthesis)

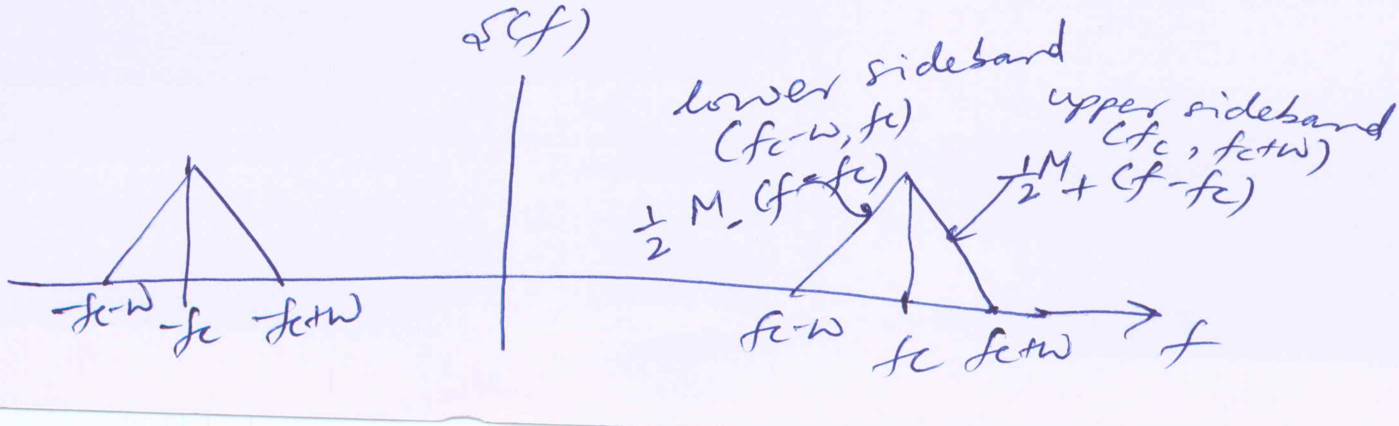




$$M_+(f) = \begin{cases} M(f), & f \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$M_-(f) = \begin{cases} 0, & f \geq 0 \\ M(f), & f < 0 \end{cases}$$

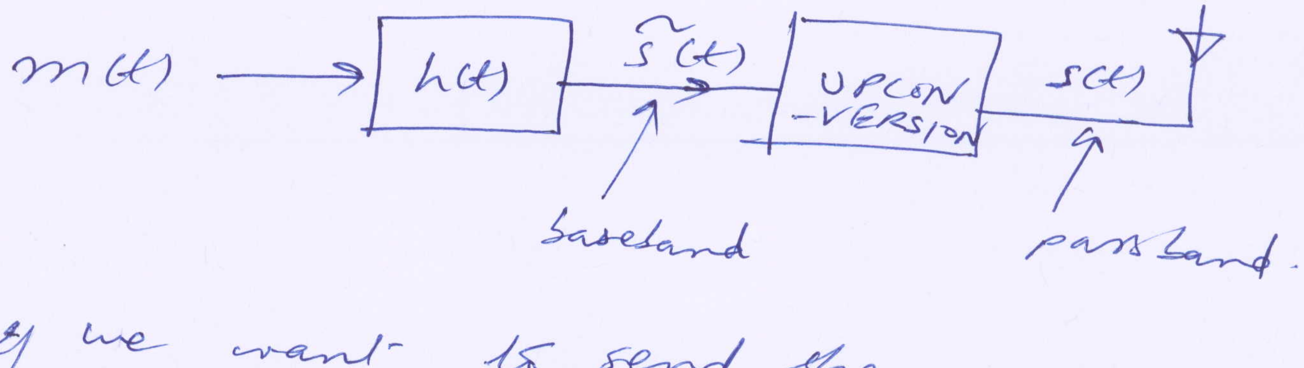
$\therefore \quad M(f) = M_+(f) + M_-(f).$



Since $m(t)$ is real valued,

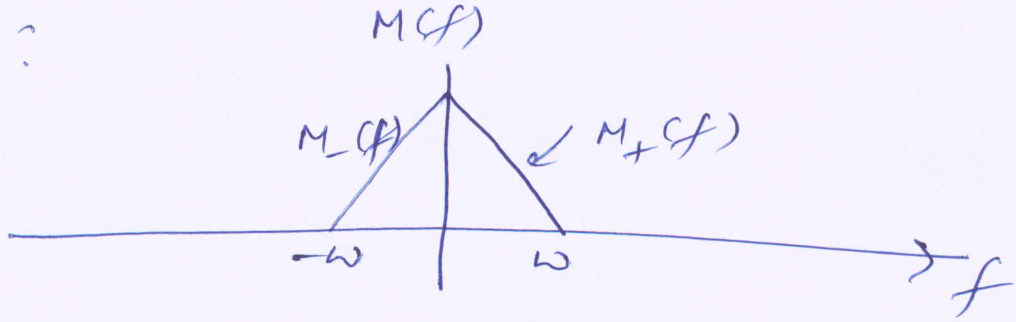
$$M_+(f) = M_-^*(-f), \text{ i.e., } M_+(f)$$

and ~~$M_+(f)$~~ $M_-(f)$ contain the same information. Therefore a spectral efficient way of communication is to only transmit either the upper sideband or the lower sideband of $S(f)$. This can be achieved by firstly filtering out one of the sidebands of $m(t)$ in baseband, followed by upconversion.

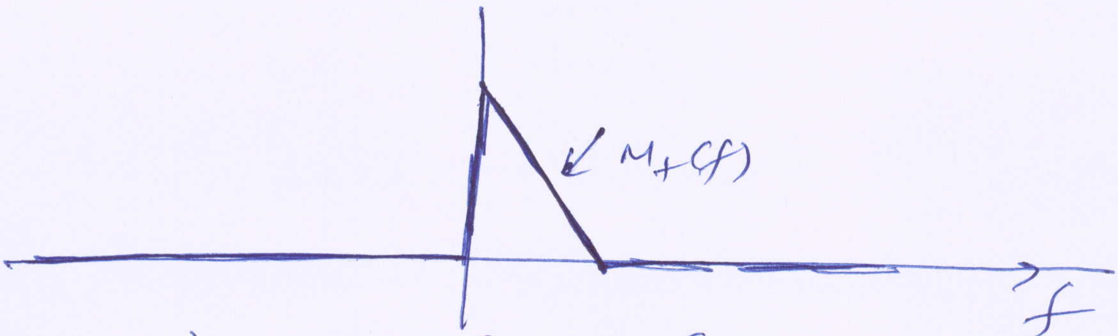


If we want to send the upper side band only we select

$$H(f) = \begin{cases} 1, & f > 0 \\ 1/2, & f = 0 \\ 0, & \text{otherwise} \end{cases}$$



$$\tilde{s}(f) = M(f)\, H(f)$$



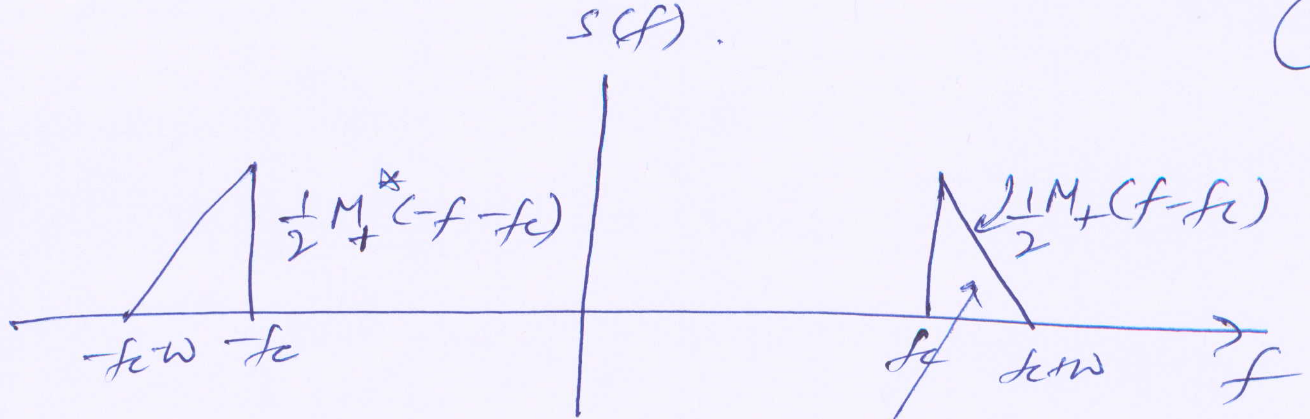
i.e.,
$$\tilde{s}(f) = \begin{cases} M_+(f), & f > 0 \\ \frac{M_+(0)}{2}, & f = 0 \\ 0, & f < 0 \end{cases}$$

Note that $\tilde{s}(t)$ is complex valued since $\tilde{s}(f) \neq \tilde{s}^*(-f)$ (does not satisfy conjugate-symmetry property).

<u>UPCONVERSION:</u>

$$s(t) = \mathrm{Re}\left(\tilde{s}(t)\, e^{j2\pi f_c t}\right)$$
$$= \tilde{s}^I(t) \cos 2\pi f_c t - \tilde{s}^Q(t) \sin 2\pi f_c t$$

where $\tilde{s}(t) = \tilde{s}^I(t) + j\,\tilde{s}^Q(t)$.



only the upper sideband is transmitted.

The filter,

$$H(f) = \begin{cases} 1, & f>0 \\ 1/2, & f=0 \\ 0, & f<0 \end{cases}$$

$$h(t) = \frac{\delta(t)}{2} + \frac{1}{j2\pi t}$$

(refer to equation 2.76 on page 38 in the text book)

$$\therefore \tilde{s}(t) = m(t) * h(t)$$

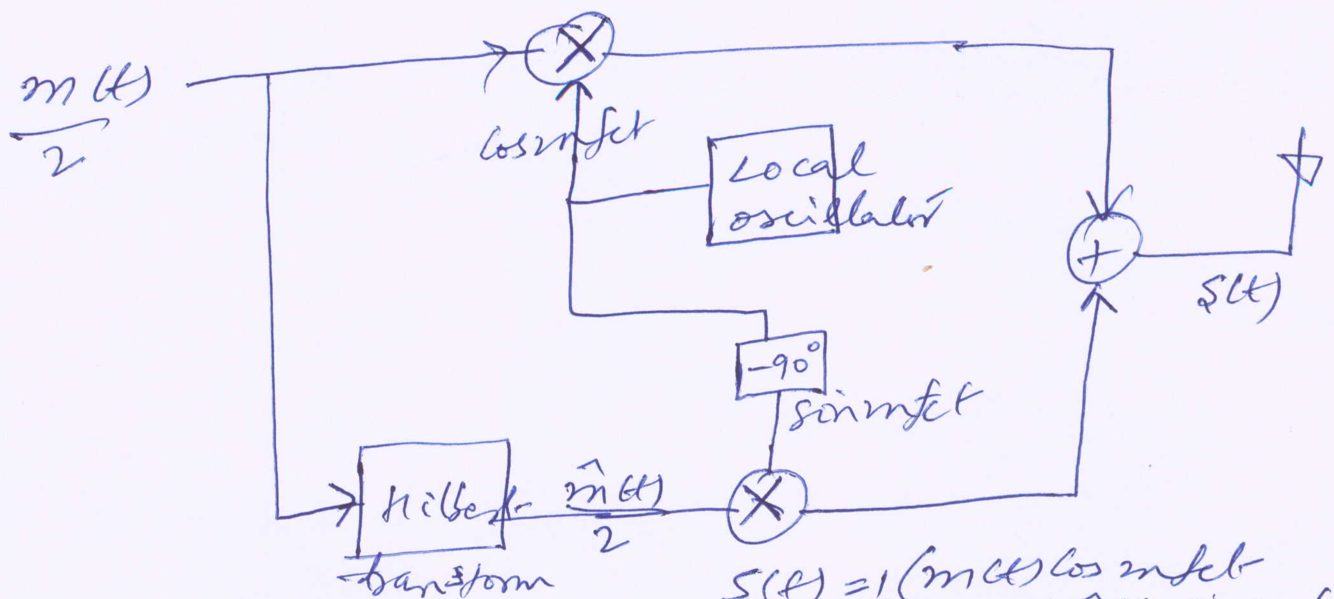
$$= m(t) * \left(\frac{\delta(t)}{2} + \frac{1}{j2\pi t}\right)$$

$$= \frac{m(t)}{2} - \frac{j}{2}\, m(t) * \frac{1}{\pi t}$$

$$= \frac{1}{2}\left(m(t) - j\,\hat{m}(t)\right)$$

where $\hat{m}(t) \triangleq m(t) * \frac{1}{\pi t}$

$$= \int \frac{m(\tau)}{\pi(t-\tau)}\, d\tau$$ is called the Hilbert transform of $m(t)$.



$$s(t) = \frac{1}{2}\left(m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t\right)$$

Synthesis of upper sideband (SSB) using the Hilbert transform.

<u>On the Hilbert transform.</u>

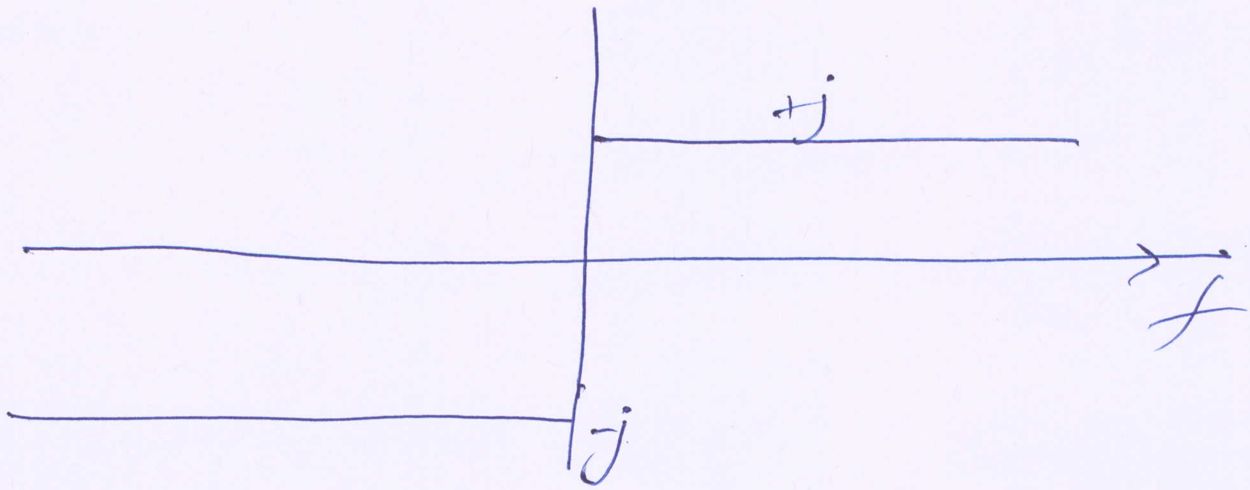
$$h_p(t) = \frac{1}{\pi t}.$$

$$m(t) \longrightarrow \int \frac{m(\tau)}{\pi(t-\tau)}\, d\tau$$

$$H_p(f) = \int_{-\infty}^{\infty} h_p(t)\, e^{+j2\pi f t}\, dt$$

$$= \begin{cases} +j, & f > 0 \\ 0, & f = 0 \\ -j, & f < 0 \end{cases}$$

$H_p(f)$.



$\therefore$ The hilbert filter is essentially a phase shifter of $+\pi/2$ for $f>0$ and $-\frac{\pi}{2}$ for $f<0$.

eg: $m(t) = \cos 2\pi f_m t \longrightarrow \boxed{\text{hilbert } \frac{1}{\pi t}} \longrightarrow\ ??\ \hat{m}(t)$

$$\hat{m}(t) = \int \frac{\cos 2\pi f_m \tau}{\pi(t-\tau)}\, d\tau .$$

$$\hat{M}(f) = M(f)\, H_p(f) = \left[\frac{\delta(f-f_m)}{2} + \frac{\delta(f+f_m)}{2}\right] H_p(f)$$

$$= \frac{j\,\delta(f-f_m)}{2} - \frac{j\,\delta(f+f_m)}{2}$$

$\therefore \hat{m}(t)$ = Inverse Fourier Transform of $\hat{M}(f)$

$$= \frac{j e^{j2\pi f_m t} - j e^{-j2\pi f_m t}}{2}$$

$$= -\sin 2\pi f_m t = \cos(2\pi f_m t + 90^\circ)$$

Coherent demodulation of SSB signals.

1. ~~Pha~~ Synchronization phase.

The transmitter sends a pilot carrier $\cos 2\pi f_c t$ for a duration of t_p seconds, during which the phase locked loop (PLL) at the receiver locks to the phase of the carrier $+ 90°$.

∴ At the end of the synch. phase if the carrier wave is $\cos(2\pi f_c t + \phi)$ then the output of the VCO of the PLL is $-\sin(2\pi f_c t + \phi)$.

E.g. when the received carrier is $\cos 2\pi f_c (t-\tau)$, the VCO o/p after lock is $-\sin(2\pi f_c t - 2\pi f_c \tau)$.

2. Coherent demodulation in message phase.

Message phase.

TX signal : $s(t) = m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t$.

Rx signal: $r(t) = A s(t-\tau)$ channel delay

amplitude loss

$\therefore r(t) = A\left(m(t-\tau)\cos(2\pi f_c t - 2\pi f_c \tau) + \hat{m}(t-\tau)\sin(2\pi f_c t - 2\pi f_c \tau)\right)$.

The VCO o/p after phase lock will be

$$-A_{VCO}\sin(2\pi f_c t - 2\pi f_c \tau)$$