EEL-306.

(1)

VESTISIAL SIDEBAND MODULATION (VSB)

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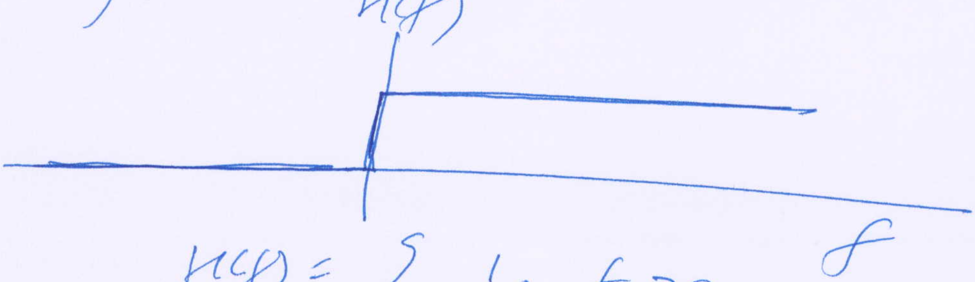
SSB is difficult to realize practically if the message signal m(t) has energy content ~~around~~ at low frequencies. this is due to the fact of very high implementation complexity of filters having very sharp (narrow) transition band at $f=0$.

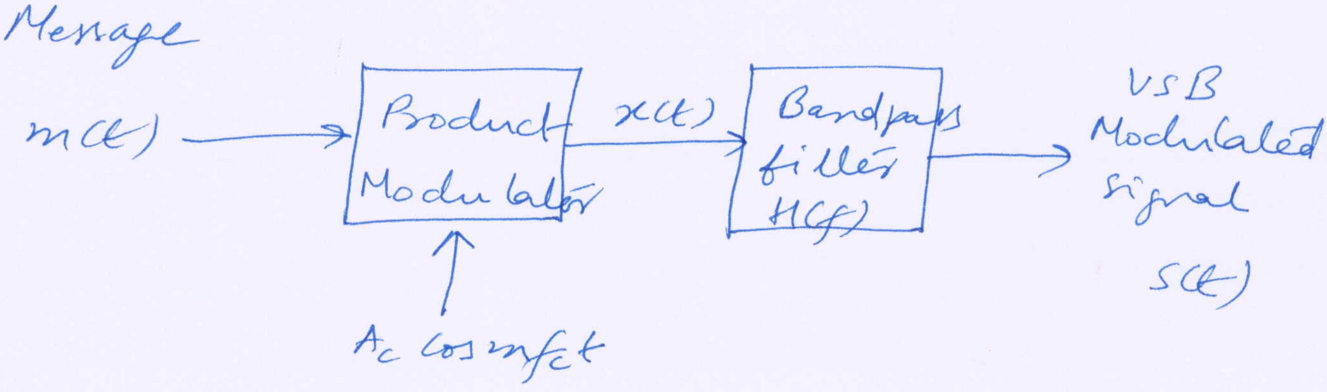


$$H(f) = \begin{cases} 1, & f>0 \\ \frac{1}{2}, & f=0 \\ 0, & f<0 \end{cases}$$

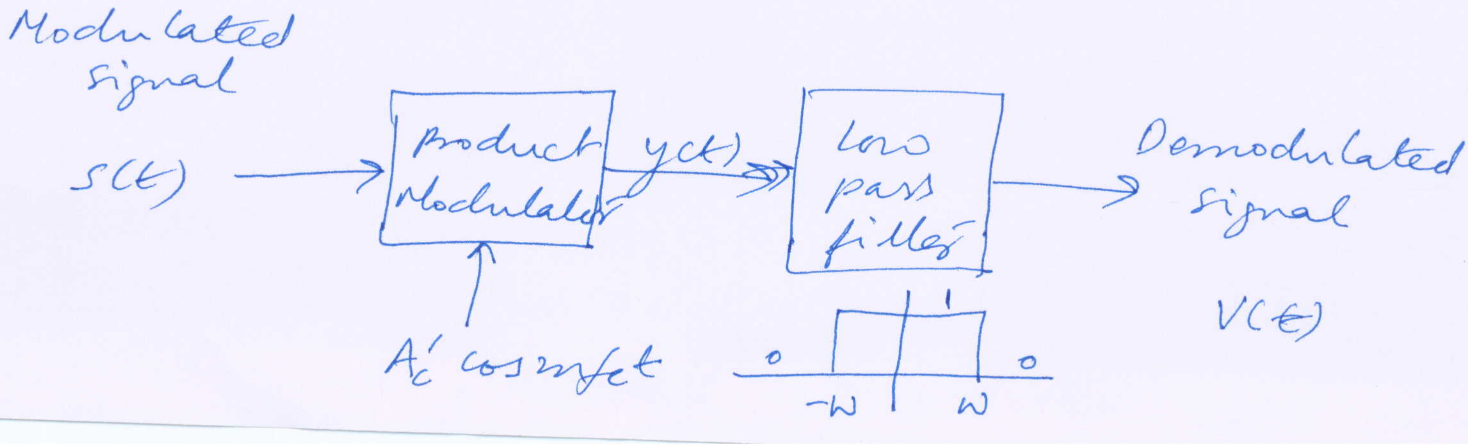
To realize H(f) for ~~transmitted~~ ing the upper side band requires the design of a filter with a very narrow transition band (i.e., the region where the frequency response goes from 0 to 1).

Instead of designing (trying to design) very narrow transition band filters, can we design filters which allow some part of the other sideband ("vestige") to also be communicated in a controlled manner such that it does not impact the detection of the message signal at the coherent receiver.

Transmitter



COHERENT RECEIVER.



③

$x(t) = A_c m(t) \cos 2\pi f_c t$

$\therefore X(f) = \frac{A_c}{2} \left(M(f+f_c) + M(f-f_c) \right)$

and

$S(f) = H(f)\, X(f)$

$= \frac{A_c}{2} H(f) \left(M(f+f_c) + M(f-f_c) \right)$ ——— ①

At the coherent receiver, we assume synchronous phase and frequency and therefore

$y(t) = A_c' s(t) \cos 2\pi f_c t$

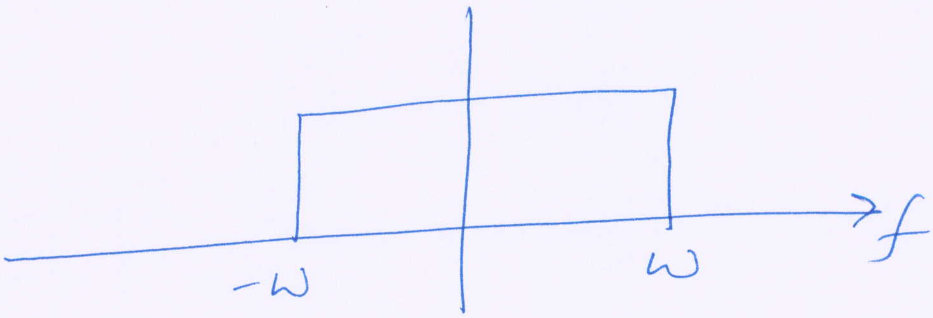
$\therefore Y(f) = \frac{A_c'}{2} \left[S(f-f_c) + S(f+f_c) \right]$

using ① we get

$$Y(f) = \frac{A_c' A_c}{\cancel{2}\, 4} \Big[H(f-f_c) M(f) + H(f-f_c) M(f-2f_c) + H(f+f_c) M(f+2f_c) + H(f+f_c) M(f) \Big]$$

$$= \frac{A_c A_c'}{4} M(f) \left[H(f-f_c) + H(f+f_c) \right] + \frac{A_c A_c'}{4} \left[H(f-f_c) M(f-2f_c) + H(f+f_c) M(f+2f_c) \right]$$ ——— ②

The low pass filter in the coherent receiver has response



In (2), the term $M(f-2f_c)$ has power in the band $(2f_c-W, 2f_c+W)$ (assuming that $M(f)$ is bandlimited to $[-W, W]$). If $f_c > W$, then the bands $(2f_c-W, 2f_c+W)$ and $(-W, W)$ are disjoint

$\therefore$ $M(f-2f_c)$ is filtered out at the receiver.

Similarly the term $M(f+2f_c)$ is also filtered out.

$\therefore$ ~~At~~ the output of the low pass filter at the coherent receiver is given by

$$V(f) = \begin{cases} \frac{A_c A_c'}{4} M(f) \left[H(f+f_c) + H(f-f_c) \right], & |f| < W \\ 0, & \text{otherwise} \end{cases}$$

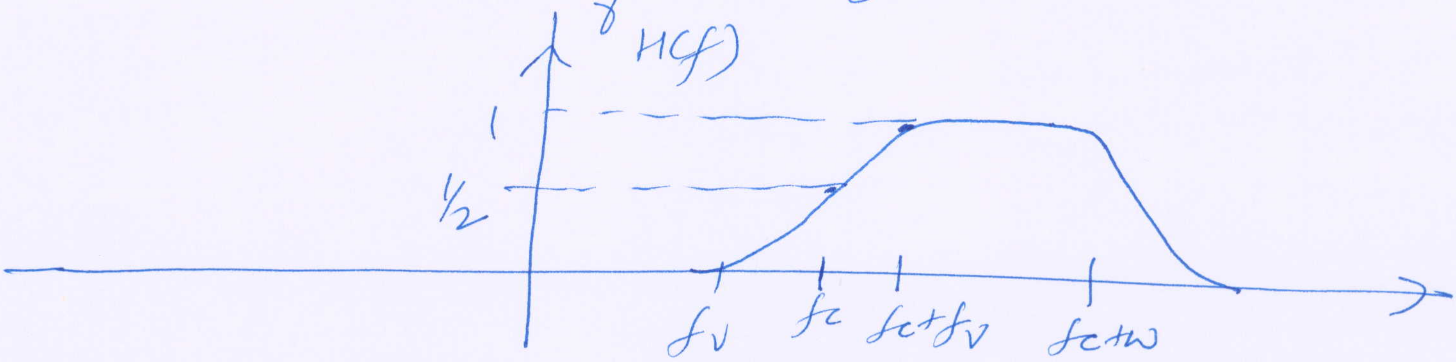
——— (3)

∴ To recover $m(t)$ at the receiver, one could impose the condition that

$$H(f-f_c) + H(f+f_c) = 1 \leftarrow \text{(some constant) which does not vary with } f\text{)}$$

for all $-W < f < W$. — (4)

For upper sideband transmission, this condition can be satisfied by a filter $H(f)$ of the form



letting $H(f)$ to be real-valued, we have $H(f) = H^*(-f) = H(-f)$ since $H(f)$ represents a band-pass filter whose impulse response is real-valued. ~~using the fact that~~ Imposing the constraint that $H(f) = H(-f)$ in (4) we get

$$H(f_c - f) + H(f_c + f) = 1$$

for all $-W < f < W$ — (5)

$\therefore$ From 5 it follows that $H(f)$ exhibits <u>odd symmetry</u> around $f = f_c$, i.e., for any f s.t. $|f| < W$, the sum of $H(f_c - f)$ and $H(f_c + f)$ is a constant.

~~5 implies that~~

putting $f = 0$ in 5 gives us $H(f_c) = \frac{1}{2}$.

Note that 5 does not impose any constraint/restriction on the shape of $H(f)$ for $f > f_c + W$.

From the figure of $H(f)$ it follows that we do allow a small portion ("VESTIGE") of the lower side band (f_v, f_c) to be transmitted, but by imposing the constraint $H(f - f_c) + H(f + f_c) = 1$, $|f| < W$ we are able to guarantee perfect recovery by a coherent receiver.