FREQUENCY MODULATION (FM)

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Angle modulated wave

$$s(t) = A_c \cos[\theta_i(t)]$$

Angle of $s(t)$ carries information

Earlier, in case of AM (Amplitude Modulation), $s(t) = \underbrace{(1 + k_a m(t))}_{\text{Amplitude carries information}} \cos 2\pi f_c t$

In the presence of additive noise i.e., when the received signal is

$$\cancel{r(t) = s(t) + n(t)}$$

$$r(t) = g\, s(t-\tau) + n(t),$$

the Angle modulated signal is more robust towards additive noise when compared to Amplitude Modulation. This however comes at a cost — angle modulated signal would require a larger transmission bandwidth.

Definitions:

The change in phase of the angle modulated signal between time t and $t+\Delta t$ is

$$\theta_{\Delta t}(t) = \theta_i(t+\Delta t) - \theta_i(t)$$

$\therefore$ The average frequency in the time interval $[t, t+\Delta t]$ is

$$f_{\Delta t}(t) = \frac{\theta_{\Delta t}(t)}{2\pi \Delta t} = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t}$$

The instantaneous frequency at time t is defined as

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \frac{1}{2\pi} \lim_{\Delta t \to 0} \frac{\theta_i(t+\Delta t) - \theta_i(t)}{\Delta t}$$

$$= \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}.$$

$$\therefore \theta_i(t) - \theta_i(0) = 2\pi \int_0^t f_i(t)\,dt.$$

i.e., $\phi_i(t) = \phi_i(0) + 2\pi \int_0^t f_i(t)\,dt.$

Un modulated signal (carrier)

No information is carried,

i.e., $f_i(t) = f_c$ (constant w.r.t time)

$\therefore \quad \theta_i(t) = \theta_i(0) + 2\pi f_c t$

i.e.,

$s(t) = A_c \cos(2\pi f_c t + \theta_i(0))$

Modulated signal

The message signal $m(t)$ can be can be communicated by varying $\theta_i(t)$ using $m(t)$. This can be done in several ways.

Two commonly used transmission schemes are

a) **phase Modulation:**

$\theta_i(t)$ is varied linearly with $m(t)$

$\theta_i(t) = 2\pi f_c t + k_p m(t)$

$k_p > 0$ is a constant called the phase sensitivity of the modulator.

5) Frequency Modulation (FM)

The instantaneous frequency $f_i(t)$ of $s(t)$ is varied linearly with $m(t)$, i.e.,

$$f_i(t) = f_c + k_f\, m(t)$$

where $k_f \geq 0$ is a constant called the frequency sensitivity of the FM modulator.

$$\therefore \theta_i(t) = \theta_i(0) + 2\pi \int_0^t f_i(t)\, dt$$

$$= \theta_i(0) + 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt.$$

and hence the transmitted FM signal is

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt + \theta_i(0) \right].$$

FM is a non-linear modulation scheme.

Amplitude Modulation was a linear modulation scheme, i.e.,

$$m_1(t) \longrightarrow \boxed{\text{AM modulator}} \longrightarrow s_1(t)$$

$$m_2(t) \longrightarrow \boxed{\text{AM modulator}} \longrightarrow s_2(t)$$

then for any constants $\alpha, \beta \in \mathbb{R}$

$$\alpha m_1(t) + \beta m_2(t) \longrightarrow \boxed{\text{AM modulator}} \longrightarrow \alpha s_1(t) + \beta s_2(t).$$

However for the FM modulator this is not true, i.e., the FM modulator is a non-linear modulation scheme.

FM has constant amplitude

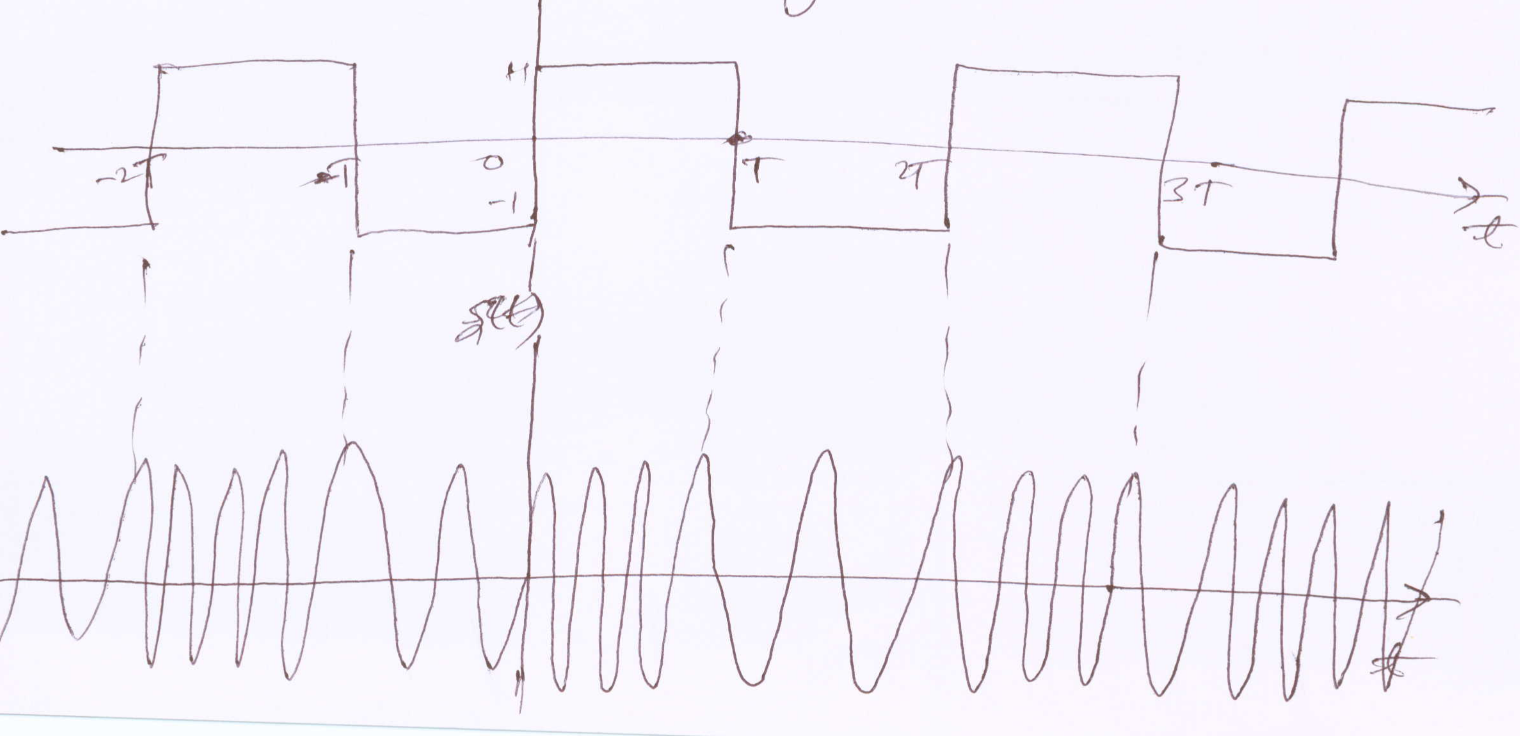
the amplitude of the FM signal is however constant A_c as compared to the AM signal.

when $f_c \gg$ highest frequency ~~content~~ content of $m(t)$,

then it can be argued that the information content of $m(t)$ is ~~carried by~~ resides in the zero crossings of $s(t)$.

This is because the instantaneous frequency can be inferred from the rate of zero crossings of $s(t)$ (i.e., how many times ~~does~~ ~~$s(t)$ cross zero~~ is $s(t) = 0$ in a given interval $\left(t - \frac{\Delta t}{2}, t + \frac{\Delta t}{2}\right)$ where $\frac{1}{f_c} \ll \Delta t \ll \frac{1}{W}$.

$m(t)$ Fig. 1



In Fig. 1, the instantaneous frequency is $f_c + k_f$ when

$t \in [2nT, (2n+1)T]$, $n \in \mathbb{Z}$, and
(even intervals)

is $f_c - k_f$ when

$t \in [(2n+1)T, 2(n+1)T]$, $n \in \mathbb{Z}$.
(odd intervals)

Since the instantaneous frequency is higher in the even intervals, the rate of zero crossings is higher than in the odd intervals.

FM.

Since the FM modulator

$m(t) \longrightarrow s(t)$ is

a non-linear system ~~it is difficult~~ as compared to AM, it is difficult to write a simple equation relating $S(f)$ and $M(f)$.

So, spectral analysis of $S(f)$ is not as easy as that for the AM signal.

Spectral analysis of $S(f)$ is required to estimate the transmission bandwidth of $s(t)$.

It turns out that even spectral analysis using a single tone message signal

$m(t) = A_m \cos(2\pi f_m t)$

provides enough insights to accurately estimate the bandwidth of the transmitted FM signal.

FM with a single tone

message signal $m(t) = A_m \cos 2\pi f_m t$.

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau + \theta_i(0)\right).$$

Instantaneous frequency is

$$f_i(t) = f_c + k_f A_m \cos 2\pi f_m t$$

$$= f_c + \Delta f \cos 2\pi f_m t$$

$\Delta f = k_f A_m$ is called the frequency deviation representing the maximum departure of $f_i(t)$ from f_c.

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) + \theta_i(0).$$

$$\therefore\ s(t) = A_c \cos\left(2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) + \theta_i(0)\right).$$

$\beta = \frac{\Delta f}{f_m}$ is called the modulation index represents the ~~phase~~ maximum deviation of the instantaneous phase from $2\pi f_c t + \theta_i(0)$.