Noise in AM Receivers.

LECTURES. (~~32~~, 33, 34, 35, ~~36~~)

Dr. SAIF KHAN MOHAMMED.
(I.I.T. DELHI).

Consider a DSB-SC signal $s(t) = m(t) \cos 2\pi f_c t$. The received signal model that we have been treating so far was that $r(t) = G\, s(t-\tau_d)$ i.e., the received signal $r(t)$ is just a scaled down and delayed version of $s(t)$ (τ_d is the delay and G is the channel gain)

In practice, the receiver circuits generate noise which is modeled as an additive white Gaussian random process (stationary) with power spectral density $\frac{N_0}{2}$. That is the received signal is

$$r(t) = G\, s(t-\tau_d) + n(t). \quad \text{(1)}$$

$$= G\, m(t-\tau_d) \cos 2\pi f_c (t-\tau_d) + n(t)$$

where $n(t)$ is a white Gaussian (stationary) random process with power spectral density.

$$S_N(f) = \frac{N_0}{2}.$$

For sake of analysis we ~~model~~ assume that $m(t)$ is a W.S.S. random process with power spectral density $S_M(f)$. Since $m(t)$ is band-limited and baseband, let $S_M(f) = 0$ for $|f| > W$.

The received signal power can be found by computing the PSD of the received signal component, i.e., $c_1 m(t-\tau_d) \cos 2\pi f_c (t-\tau_d)$. From previous lecture we know that ~~$c_1 m(t-\tau_d)$~~ since $m(t)$ is W.S.S., $c_1 m(t-\tau_d) \cos 2\pi f_c (t-\tau_d)$ is also a W.S.S. random process. Note that here we assume τ_d to be a constant (i.e., does not change with time). Subsequently, since τ_d does not affect our analysis and computation of the effect of noise, we assume that $\tau_d = 0$.

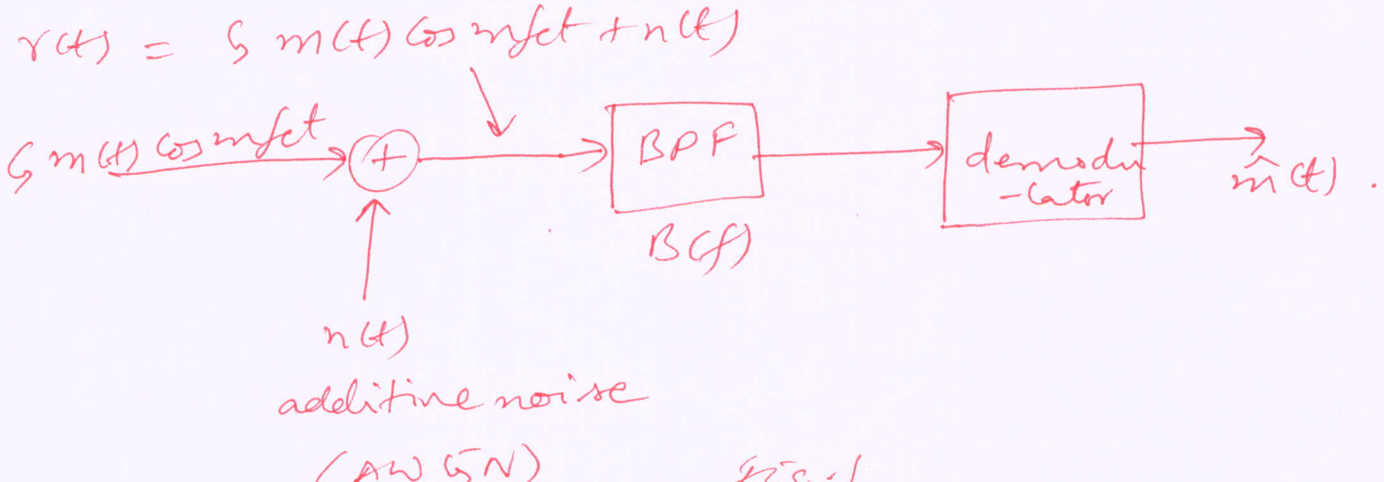


Fig. 1

In the receiver diagram above the band pass filter (BPF) has the frequency response

$$B(f) = \begin{cases} 1, & |f - f_c| < W \\ 1, & |f + f_c| < W \\ 0, & \text{otherwise.} \end{cases}$$

where W is the bandwidth of $m(t)$. The BPF is required to filter out noise and other sources of interference which are outside the useful band $[f_c - W, f_c + W]$ (this is the band where the signal $S\, m(t) \cos 2\pi f_c t$ lies).

The demodulator then takes the output of the BPF and estimates the message signal.

The output of the demodulator is usually described as

$$\hat{m}(t) = m(t) + e(t)$$, and the

Signal to noise ratio (SNR) at the output of the demodulator is given by

$$SNR_{(o)} = \frac{\mathbb{E}[m^2(t)]}{\mathbb{E}[(\hat{m}(t) - m(t))^2]}.$$

Eventually, in this lecture we would like to derive a simple expression for the ratio between the SNR at the output of the demodulator and the SNR at the input to the demodulator.

~~At the input to the~~ For the coherent. 4.1

demodulator the SNR at its input is ~~given by~~ derived in the following. (see Fig. 1 on page 3)

the BPF o/p

$$z(t) = G\, m(t) \cos 2\pi f_c t + n_b(t)$$

where $n_b(t) = n(t) \circledast b(t)$

impulse response of BPF.

Since $n(t)$ is a stationary gaussian ~~random~~ process, so is $n_b(t)$.

Further their PSD's are related by

$$S_{N_b}(f) = \frac{N_0}{2} |B(f)|^2$$

$$= \begin{cases} \frac{N_0}{2}, & |f+f_c| < W \text{ or } |f-f_c| < W \\ 0, & \text{otherwise.} \end{cases}$$

$\therefore$ SNR at the input to the demodulator

is $SNR_{(i)}^{coh} = \dfrac{G^2 E\left[m^2(t) \cos^2 2\pi f_c t\right]}{E\left[n_b^2(t)\right]}$

$=$

5.2

$$SNR_{(i)}^{coh} = \frac{G^2 E\left[\frac{m^2(t)}{2} + \frac{m^2(t)}{2}\cos 4\pi f_c t\right]}{\int_{-\infty}^{+\infty} S_{N_b}(f)}$$

$$= \frac{G^2 E[m^2(t)]}{2 \times 2 N_0 W}$$

$$= \frac{G^2 \int_{-\infty}^{\infty} S_M(f)\, df}{4 N_0 W} = \frac{G^2 \int_{-W}^{W} S_M(f)\, df}{4 N_0 W}$$

where $S_M(f)$ is the P.S.D of $m(t)$.

Coherent demodulator for DSB-SC using PLL.

Message phase.

(assuming perfect synchronization)

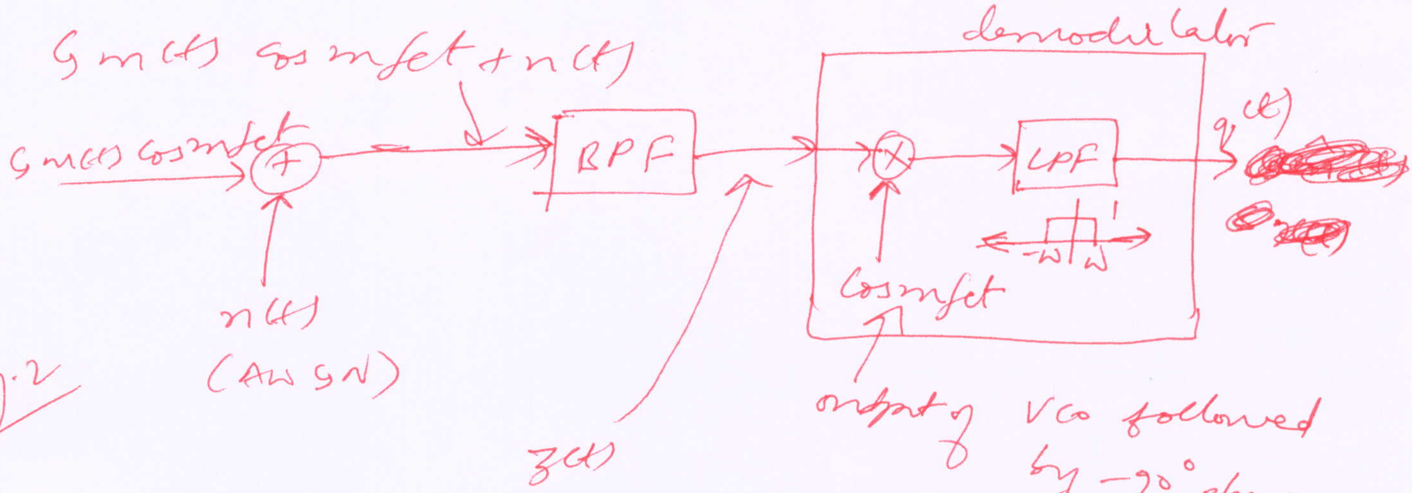


Fig. 2

$$z(t) = s\,m(t)\cos 2\pi f_c t + \underbrace{n(t) \circledast b(t)}_{n_b(t)}.$$

where $b(t)$ is the impulse response of the BPF.

clearly $n_b(t) = \int n(\tau)\, b(t-\tau)\, d\tau$ is a Gaussian random process since $n(t)$ is a Gaussian random process and $b(t)$ is a stable linear filter. (Refer to property 1, chapter 5.7 of text book). Since $n(t)$ is zero mean and stationary, $n_b(t)$ is also zero mean and stationary.

APPENDIX.

In the ~~following~~, from pages 7 to 21 we will prove that.

$n_b(t)$ can be represented as

$$n_b(t) = n^I(t)\cos 2\pi f_c t + n^Q(t)\sin 2\pi f_c t$$

where $n^I(t)$ and $n^Q(t)$ are zero mean jointly Gaussian random processes whose power spectral densities (PSD) are limited to baseband $[-W, W]$. We will show that

$$S_{N^I}(f) = S_{N^Q}(f) = \begin{cases} N_0, & |f| < W \\ 0, & |f| \geq W \end{cases}$$

and also that $n^I(t_1)$ and $n^Q(t_2)$ are independent for any t_1, t_2.

Using the representation of $n_b(t)$ discussed on page 6 we will move ahead and derive an expression for the SNR at the output of the coherent demodulator.

Looking at Figure 2 on page 5,

$$g(t) = LPF\left\{ z(t)\cos 2\pi f_c t \right\}$$

$$= LPF\left\{ \big(\overbrace{S\, m(t)\cos 2\pi f_c t + n_b(t)}^{z(t)}\big) \cos 2\pi f_c t \right\}$$

$$= LPF\left\{ S\, m(t)\cos^2 2\pi f_c t \right\} + LPF\left\{ n_b(t)\cos 2\pi f_c t \right\}$$

$$= LPF\left\{ \frac{S\, m(t)}{2}\left(1+\cos 4\pi f_c t\right) \right\} + LPF\left\{ n_b(t)\cos 2\pi f_c t \right\}$$

$$= LPF\left\{ \frac{S\, m(t)}{2} \right\} + LPF\left\{ \frac{S\, m(t)\cos 4\pi f_c t}{2} \right\} + LPF\left\{ n_b(t)\cdot\cos 2\pi f_c t \right\}.$$

6.2

Since $m(t)$ is band-limited to $[-W, W]$ and LPF only allow frequencies in $[-W, W]$ to pass through, it follows that

$$LPF\left\{\frac{S\,m(t)}{2}\right\} = \frac{S\,m(t)}{2}.$$

the next term $\frac{S\,m(t)\cos 4\pi f_c t}{2}$ is bandlimited to $[2f_c - W, 2f_c + W]$ and gets rejected by the LPF since $f_c > W$ and therefore the bands $[-W, W]$ and $[2f_c - W, 2f_c + W]$ are disjoint.

$\therefore$ the output of the LPF is

$$y(t) = \frac{S\,m(t)}{2} + LPF\{n_b(t)\cos 2\pi f_c t\}.$$

using the representation of $n_b(t)$ on page ⑥ we have

$$n_b(t)\cos 2\pi f_c t = \left(n^I(t)\cos 2\pi f_c t + n^Q \sin 2\pi f_c t\right)\cos 2\pi f_c t$$

$$= \frac{n^I(t)}{2} + \frac{\left(n^I(t)\cos 4\pi f_c t + n^Q(t)\sin 4\pi f_c t\right)}{2}$$

It is easy to see that

$n^I(t)\cos 4\pi f_c t + n^Q(t)\sin 4\pi f_c t$ is a passband Gaussian random process whose P.S.D is band-limited to $[2f_c - W, 2f_c + W]$. Infact using the representation on page ⑥ the PSD of $w(t) \triangleq n^I(t)\cos 4\pi f_c t + n^Q(t)\sin 4\pi f_c t$ is

$$S_W(f) = \begin{cases} \frac{N_0}{2}, & |f + 2f_c| < W, \\ & |f - 2f_c| < W \\ 0, & \text{otherwise.} \end{cases}$$

(6.4)

$\therefore$ LPF $\{n_b(t)\cos 2\pi f_c t\}$

$= \text{LPF}\left\{\frac{n^I(t)}{2}\right\}$

$+ \frac{1}{2}\,\text{LPF}\{n^I(t)\cos 4\pi f_c t$

$- n^Q(t)\sin 4\pi f_c t\}$.

Since $n^I(t)\cos 4\pi f_c t + n^Q(t)\sin 4\pi f_c t$ has a PSD limited to $[2f_c - W,\ 2f_c + W]$ and this band is disjoint with $[-W, W]$ (LPF band), it follows that the random process

LPF $\{n^I(t)\cos 4\pi f_c t - n^Q(t)\sin 4\pi f_c t\}$

has a all-zero PSD. Therefore

LPF $\{n_b \cos 2\pi f_c t\} = \text{LPF}\left\{\frac{n^I(t)}{2}\right\}$,

and ~~hence~~ since $n^I(t)$ is zero mean Gaussian random process LPF $\left\{\frac{n^I(t)}{2}\right\}$ is also zero mean Gaussian.

$\therefore$ Further the PSD of

$$v(t) \equiv LPF\{ n_s(t) \cos 2\pi f_c t \}$$

is

$$S_v(f) = \frac{S_{N_I}(f)}{4} |H(f)|^2 \text{ where}$$

$H(f)$ is the frequency response of the LPF.

$$H(f) = \begin{cases} 1, & |f| < W \\ 0, & \text{otherwise.} \end{cases}$$

$$\therefore S_v(f) = \begin{cases} \frac{S_{N_I}(f)}{4}, & |f| < W \\ 0, & \text{otherwise.} \end{cases}$$

Since $S_{N_I}(f) = \begin{cases} N_0, & |f| < W \\ 0, & \text{otherwise} \end{cases}$ $\Leftarrow$ See page 6

we therefore have

$$S_v(f) = \begin{cases} \frac{N_0}{4}, & |f| < W \\ 0, & \text{otherwise} \end{cases}$$

$\therefore$ the LPF output is

$$q(t) = \frac{G\, m(t)}{2} + LPF\{n_s(t)\cos 2\pi f_c t\}$$

$$= \frac{G\, m(t)}{2} + v(t)$$

where the zero mean Gaussian noise $v(t)$ has a PSD

$$S_v(f) = \begin{cases} \frac{N_0}{4}, & |f| < W \\ 0, & \text{otherwise.} \end{cases}$$

$\therefore$ the total noise power at the LPF output $= \int_{-\infty}^{\infty} S_v(f)\, df$

$$= \frac{N_0}{4}\int_{-W}^{W} df = \frac{N_0 W}{2}.$$

Total signal power at the LPF output is

$$= \frac{G^2}{4}\int_{-W}^{W} S_M(f)\, df$$

$\therefore SNR_{(o)}^{COH}$ = SNR at the output of the coherent demodulator

$$= \frac{\frac{G^2}{4}\int_{-W}^{W} S_M(f)\, df}{\left(\frac{N_0 W}{2}\right)} = \frac{G^2 \int_{-W}^{W} S_M(f)\, df}{2 N_0 W}.$$

on page 4.2 we had derived the SNR at the input of the coherent demodulator to be

$$SNR_{(i)}^{coh} = \frac{\sigma^2 \int_{-W}^{W} S_M(f)\,df}{4 N_0 W}$$

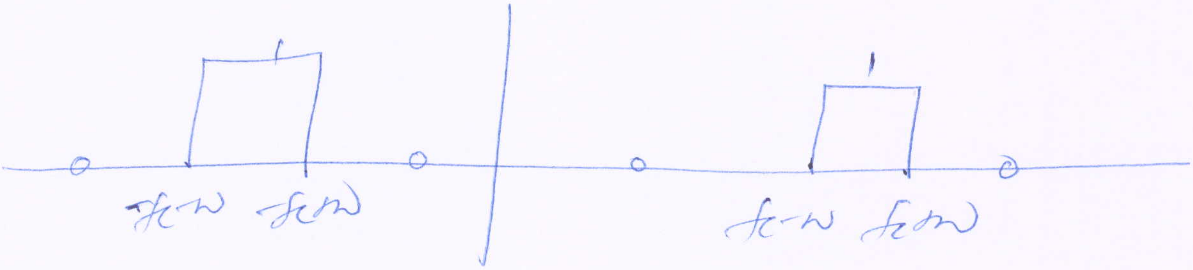
∴ comparing this with the SNR at the output of the coherent demodulator we observe that

$$\frac{SNR_{(o)}^{coh}}{SNR_{(i)}^{coh}} = 2$$, i.e., the coherent demodulator improves the SNR at its input by two times.

APPENDIX.

(i) Since

$B(f)$.



and.

$$\cancel{B(f) = \tilde{B}(f-f_c) + \tilde{B}^*(f+f_c)}$$

we have:

$$b(t) = \mathrm{Re}\left(\tilde{b}(t)\, e^{j2\pi f_c t}\right)$$

$$= \frac{\tilde{b}(t)\, e^{j2\pi f_c t} + \tilde{b}^*(t)\, e^{-j2\pi f_c t}}{2}.$$

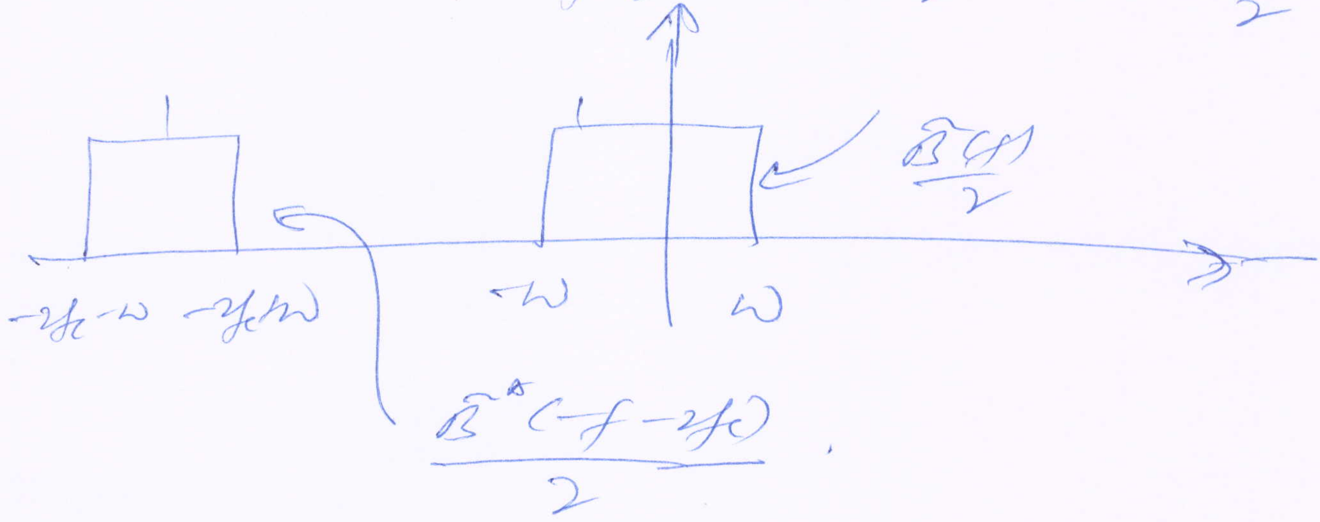
Taking Fourier transforms on both sides we get

$$B(f) = \frac{\tilde{B}(f-f_c) + \tilde{B}^*(-f-f_c)}{2}.$$

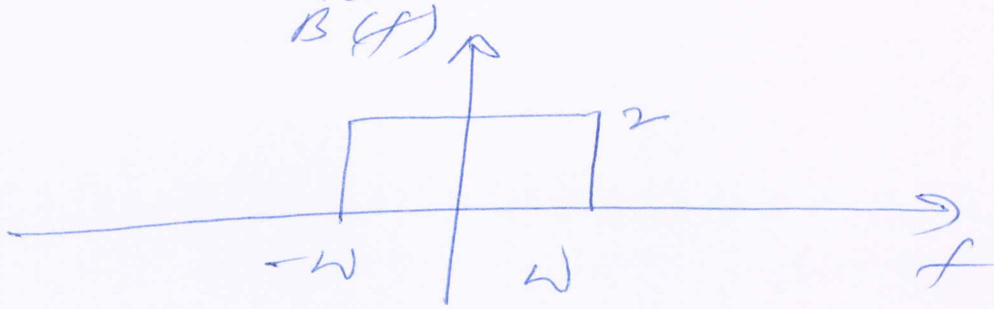
$$\therefore B(f+f_c) = \frac{\tilde{B}(f) + \tilde{B}^*(-f-2f_c)}{2}$$

Since $\tilde{B}(f)$ is band limited to $[-W, W]$, and

$$B(f+f_c) = \frac{\tilde{B}(f)}{2} + \frac{\tilde{B}^*(-f-2f_c)}{2}$$



∴ we have



∴ $\tilde{b}(t)$ = inverse Fourier transform of $\tilde{B}(f)$

$= 4W \operatorname{sinc}(2Wt)$.

∴ $n_b(t) = \int n(\tau)\, b(t-\tau)\, d\tau$.

and $s(t) = \mathrm{Re}\left[\tilde{b}(t) e^{j2\pi f_c t}\right]$

$= \mathrm{Re}\left[4W \operatorname{sinc} 2Wt \; e^{j2\pi f_c t}\right]$

$= 4W \operatorname{sinc} 2Wt \cos 2\pi f_c t$.

hence:

$$n_0(t) = \int n(\tau)\, 4W \operatorname{sinc}(2W(t-\tau)) \cos 2\pi f_c(t-\tau)\, d\tau .$$

$$= \cos 2\pi f_c t \underbrace{\left(\int n(\tau) \cos 2\pi f_c \tau \; 4W \operatorname{sinc} 2W (t-\tau)\, d\tau \right)}_{n^I(t)}$$

$$+ \sin 2\pi f_c t \underbrace{\left(\int n(\tau)\, 4W \operatorname{sinc} 2W(t-\tau) \sin 2\pi f_c \tau \, d\tau \right)}_{n^Q(t)}$$

the autocorrelation of $n^I(t)$ is given by

$E[n^I(t_1)\, n^I(t_2)]$

$$= \iint E(n(\tau_1) n(\tau_2)) \cos 2\pi f_c \tau_1 \cos 2\pi f_c \tau_2 \; 4W \operatorname{sinc}(2W(t_1-\tau_1))\; 4W \operatorname{sinc}(2W(t_2-\tau_2))\, d\tau_1\, d\tau_2$$

$$= \iint \frac{N_0}{2} \delta(\tau_1-\tau_2) \cos 2\pi f_c \tau_1 \cos 2\pi f_c \tau_2 \; 4W \operatorname{sinc}(2W(t_1-\tau_1))\; 4W \operatorname{sinc}(2W(t_2-\tau_2))\, d\tau_1\, d\tau_2 .$$

$$= \frac{N_0}{2} \int \cos^2 2\pi f_c \tau \; 4W \operatorname{sinc}(2W(t_1-\tau)) \; 4W \operatorname{sinc}(2W(t_2-\tau)) \, d\tau$$

$$= \underbrace{\frac{N_0}{4} \int_{-\infty}^{\infty} 4W \operatorname{sinc}(2W(t_1-\tau)) \; 4W \operatorname{sinc}(2W(t_2-\tau)) \, d\tau}_{\text{first term}}.$$

$$+ \underbrace{\frac{N_0}{4} \int \cos 4\pi f_c \tau \; 4W \operatorname{sinc}(2W(t_1-\tau)) \; 4W \operatorname{sinc}(2W(t_2-\tau))}_{\text{second term}} \, d\tau.$$

The first term

$$\int_{-\infty}^{\infty} 4W \operatorname{sinc}(2W(t_1-\tau)) \; 4W \operatorname{sinc}(2W(t_2-\tau)) \, d\tau$$

substitute $u = t_1 - \tau$.

$$= 4 \cdot \int_{-\infty}^{\infty} 2W \operatorname{sinc}(2Wu) \; 2W \operatorname{sinc}(2W(t_2-t_1+u)) \, du$$

$$= 4 \int_{-\infty}^{\infty} 2W \operatorname{sinc}(2Wu) \; 2W \operatorname{sinc}(2W((t_1-t_2)-u)) \, du$$

since $\operatorname{sinc}(\cdot)$ is an even function i.e.,

$$\operatorname{sinc}(x) = \operatorname{sinc}(-x).$$

If we denote

$$g(u) \triangleq 2W \operatorname{sinc} 2Wu$$

then:

the first term is

$$= 4 \int_{-\infty}^{\infty} g(u)\, g\big((t_1 - t_2) - u\big)\, du$$

i.e., $g(\cdot)$ is convolved with itself.

Let

$$g(u) \circledast g(u) = h(u).$$

$\circledast$ convolution

then

$$4 \int_{-\infty}^{\infty} g(u)\, g(t_1 - t_2 - u)\, du = h(t_1 - t_2).$$

So, we need to essentially find $h(u)$.

Since $h(u) = g(u) \circledast g(u)$

then

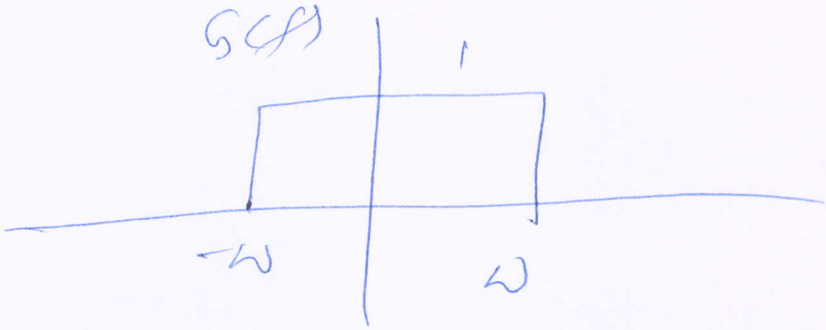
$$H(f) = G(f)\, G(f) = (G(f))^2.$$

Sol:

$$g(u) = 2W\,\mathrm{sinc}(2Wu)$$

$$\therefore\ G(f) = \int_{-\infty}^{\infty} g(u)\, e^{+j2\pi f u}\, du$$

$$= \begin{cases} 1, & |f| < W \\ 0, & \text{otherwise} \end{cases}$$



$$\therefore\ H(f) = (G(f))^2 = G(f)$$

hence $h(u) = g(u) = 2W\,\mathrm{sinc}(2Wu)$.

~~∴ the first term integral in the on page (10)~~

$$\therefore\ \int 2W\,\mathrm{sinc}\,2Wu\ \ 2W\,\mathrm{sinc}(2W(t_1 - t_2 - u))\, du$$

$$= 2W\,\mathrm{sinc}\left[2W(t_1 - t_2)\right]$$

∴ on page 10 the first term

is therefore given by:

$$\frac{N_0}{4}\int_{-\infty}^{\infty} 4W\ \mathrm{sinc}\,(2W(t_1-\tau))$$
$$4W\ \mathrm{sinc}\ (2W(t_2-\tau))\,d\tau$$

$$= N_0\int_{-\infty}^{\infty} 2W\ \mathrm{sinc}\,(2Wu)\ 2W\ \mathrm{sinc}\,(2W(t_1-t_2)-u)\,du$$

$$= N_0(2W)\ \mathrm{sinc}\,(2W(t_1-t_2))$$

using the result on page 12

On page 10 the second term

is given by

$$N_0\int \cos 4\pi f_c \tau\ 2W\ \mathrm{sinc}(2W(t_1-\tau))$$
$$2W\ \mathrm{sinc}\ (2W(t_2-\tau))\,d\tau$$

let $e(\tau) \triangleq 2W\ \mathrm{sinc}\ (2W(t_1-\tau))$
$$2W\ \mathrm{sinc}\ (2W(t_2-\tau))$$

then the second term is

$$N_0\int \cos 4\pi f_c \tau\ e(\tau)\,d\tau$$

$$= N_0 \, \mathrm{Re}\Big[\underbrace{\int e(\tau)\, e^{-j4\pi f_c \tau}\, d\tau}_{\text{Fourier transform of } e(t) \text{ at } f = 2f_c.} \Big]$$

$$= N_0 \, \mathrm{Re}\left[E(f = 2f_c) \right]$$

where
$$E(f) \triangleq \int e(\tau)\, e^{-j2\pi f \tau}\, d\tau$$
is the Fourier transform of $e(t)$.

Now,

$$e(\tau) = \underset{\uparrow \atop 2W \,\mathrm{sinc}(2W(t_1-\tau))}{x_1(\tau)} \times \underset{\uparrow \atop 2W \,\mathrm{sinc}(2W(t_2-\tau))}{x_2(\tau)}$$

$$\therefore E(f) = X_1(f) \xleftarrow{} \otimes X_2(f).$$ convolution in frequency domain

both $X_1(f)$ and $X_2(f)$ are limited to $[-W, W]$ and are zero outside this interval.

$\therefore$ $E(f)$ is limited to the frequency band $[-2W, 2W]$

i.e., $E(f)=0$ for $|f| > 2W$.

~~So~~ Since $f_c > W$, $2f_c > 2W$

$\therefore$ $E(f=2f_c)=0$ (using)

this leads us to conclude that the second term on page 10 is 0.

$\therefore$ the autocorrelation of $n^I(t)$ (on page 9)

is given by

$E[n^I(t_1) n^I(t_2)]$

$= N_0 \, 2W \, \text{sinc}(2W(t_1-t_2))$ (see page 13)

Since the autocorrelation only depends on the time difference $t_1 - t_2$, we can conclude that

(wide sense stationary)

$n^I(t)$ is a W.S.S. Gaussian random process (zero mean).

~~Ref~~ Its autocorrelation function is

$$R_{N^I}(\tau) \triangleq E\left[n^I(t)\, n^I(t-\tau)\right]$$

$$= N_0\, 2W \operatorname{Sinc}(2W\tau).$$

$\therefore$ the p.s.d of $n_I(t)$ will be

$$S_{N^I}(f) = \int R_{N^I}(\tau)\, e^{-j2\pi f\tau}\, d\tau$$

$$= \begin{cases} N_0, & |f| < W \\ 0, & |f| \geq W. \end{cases}$$

Similarly for $n^Q(t)$ (defined on page 9), also it can be shown that it is also wide sense stationary (W.S.S) and zero mean Gaussian.

with autocorrelation and P.S.D. given by

$$R_{N^Q}(\tau) \triangleq E\left[n^Q(t)\, n^Q(t-\tau)\right]$$

$$= N_0\, 2W \operatorname{sinc}(2W\tau)$$

and

$$S_{N^Q}(f) = \begin{cases} N_0, & |f| < W \\ 0, & |f| \geq W. \end{cases}$$

It can also be shown that actually $n^I(t)$ and $n^Q(t)$ are <u>jointly gaussian</u> random process (i.e., for any real number α and β, and any t_1, t_2, $\alpha n^I(t_1) + \beta n^Q(t_2)$ is also a , zero mean gaussian random process).

. We will further prove that $n^I(t)$ and $n^Q(t)$ are independent.

It can be shown that

$$E\left[n^I(t_1)\, n^Q(t_2)\right]$$

(using the integral definition of $n^I(t)$ and $n^Q(t)$ on page 9).

$$= \iint E(n(\tau_1)\, n(\tau_2)) \cos 2\pi f_c \tau_1 \sin 2\pi f_c \tau_2 \; 4W \operatorname{sinc} 2W(t_1-\tau_1) \; 4W \operatorname{sinc} 2W(t_2-\tau_2) \; d\tau_1 d\tau_2$$

delta function

$$= \frac{N_0}{2} \iint \delta(\tau_1-\tau_2) \cos 2\pi f_c \tau_1 \sin 2\pi f_c \tau_2 \; 4W \operatorname{sinc} 2W(t_1-\tau_1) \; 4W \operatorname{sinc} 2W(t_2-\tau_2)\, d\tau_1 d\tau_2$$

$$= \frac{N_0}{2} \int \sin 2\pi f_c \tau \cos 2\pi f_c \tau \; 4W \operatorname{sinc} 2W(t_1-\tau) \; 4W \operatorname{sinc} 2W(t_2-\tau)\, d\tau$$

$$= N_0 \int \sin 4\pi f_c \tau \; 2W \operatorname{sinc} 2W(t_1-\tau) \; 2W \operatorname{sinc} 2W(t_2-\tau)\, d\tau$$

$$= -N_0 \operatorname{Im}\left[\int 2W \operatorname{sinc} 2W(t_1-\tau)\; 2W \operatorname{sinc}(2W(t_2-\tau))\, e^{-j4\pi f_c \tau} d\tau \right]$$

$$= -N_0\, \mathrm{Im}\left[\underbrace{\int_{-\infty}^{\infty} x_1(\tau)\, x_2(\tau)\, e^{-j4\pi f_c \tau}\, d\tau}_{\text{Fourier transform of } x_1(t)\,x_2(t) \text{ at } f = 2f_c} \right]$$

where $x_1(\tau) \triangleq 2W \operatorname{sinc}(2W(t_1-\tau))$

and $x_2(\tau) \triangleq 2W \operatorname{sinc}(2W(t_2-\tau))$

This will be zero since $x_1(t)\,x_2(t)$ will be band-limited to $[-2W, 2W]$ and $2f_c > 2W$.

$\therefore$

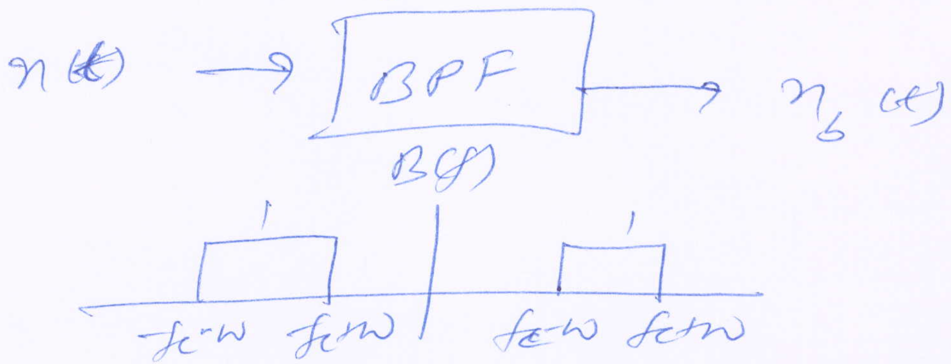
$$E\left[n^I(t_1)\, n^Q(t_2)\right] = 0$$

Since $n^I(t)$ and $n^Q(t)$ are zero mean, it mean that the ~~correlation~~ covariance between $n^I(t_1)$ and $n^Q(t_2)$ is zero for any t_1, t_2. However since $n^I(t_1), n^Q(t_2)$ are jointly Gaussian, zero ~~correlation~~ covariance implies that

$n^I(t_1)$ and $n^Q(t_2)$ are <u>independent</u>.

(Note that correlation and covariance between two random variable is <u>not</u> the same thing).

$\therefore$ we finally have



If $n(t)$ is zero mean w.s.s. Gaussian random process and is white, then

$n_b(t)$ is zero mean, w.s.s. Gaussian random process which can be written as

$$n_b(t) = n^I(t) \cos 2\pi f_c t + n^Q(t) \sin 2\pi f_c t$$

where $n^I(t)$ and $n^Q(t)$ are independent zero mean jointly Gaussian random processes.

having the p.s.d.'s

$$S_{N^I}(f) = S_{N^Q}(f) = \begin{cases} N_0, & |f| < W \\ 0, & |f| \geq W. \end{cases}$$

the representation

$$n_b(t) = n^I(t) \cos 2\pi f_c t + n^Q(t) \sin 2\pi f_c t$$

reminds us of the baseband representation of deterministic pass-band signals. Here ~~also~~ instead of deterministic signals we have random processes, but the result is of the same kind, i.e., a passband random process whose psd is limited to $[f_c - W, f_c + W]$ and $[-f_c - W, -f_c + W]$ can be expressed (as above) with $n^I(t)$ and $n^Q(t)$ being random processes having psd's limited to baseband i.e., $[-W, W]$.