In general, if

$Re\{a\} = Re\{b\}$, then

this does not imply that $a = b$.

However in ~~this~~ our case it does, as

we show below.

~~From~~ subtracting (1) from (2) we

get:

$$Re\left\{\left[\tilde{y}(t) - \frac{1}{2}\int \tilde{h}(\tau)\tilde{x}(t-\tau)d\tau\right]e^{j2\pi f_c t}\right\} = 0.$$

Let $v(t) \triangleq \tilde{y}(t) - \frac{1}{2}\int \tilde{h}(\tau)\tilde{x}(t-\tau)d\tau$.

Therefore we have

$$Re\{v(t)\, e^{j2\pi f_c t}\} = 0. \quad -(4)$$

We will show that (4) implies

that $v(t) = 0$ which then proves

that $\tilde{y}(t) = \frac{1}{2}\int \tilde{h}(\tau)\,\tilde{x}(t-\tau)d\tau$.