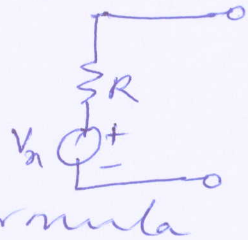
Lecture 7. (Jan 24, 2014)

EEL 306.

On Johnson - Nyquist noise in electrical circuits.

Electrical noise due to Brownian motion of electrons in circuits.

A resistor R (noisy) is modeled (Thevenin equivalent) as R in series with a random voltage source V_n.



Nyquist came up with a formula for the total power in the noise signal V_n in the frequency band $\left[f - \frac{\Delta f}{2}, f + \frac{\Delta f}{2}\right]$, to be

$$E\left[V_n^2(f-\Delta f/2, f+\Delta f/2)\right] = \frac{4Rhf\,\Delta f}{e^{hf/k_BT} - 1}$$

Note: The reason why we are interested in band $\left[f-\frac{\Delta f}{2}, f+\frac{\Delta f}{2}\right]$ is because typical receivers usually have a band pass filter [to filter out interference] where h is the Planck's constant

$h = 6.626 \times 10^{-34}$ Js, and

k_B is the Boltzmann's constant

$k_B = 1.38 \times 10^{-23}$ J/K

T is the ambient temperature in kelvin.

For $f \ll \frac{k_B T}{h}$, we have

$$E\left[V_n^2\left(f-\frac{\Delta f}{2}, f+\frac{\Delta f}{2}\right)\right] \simeq 4 k_B T R \Delta f$$

At room temperature (~ 300 K),

$$\frac{k_B T}{h} = 6.248 \times 10^{12} \text{ Hz}$$

$$\approx 6248 \text{ GHz}.$$

For ~~communi~~ communication systems that we use f is usually a few GHz, and therefore the approximation above $\left(E\left[V_n^2\left(f-\frac{\Delta f}{2}, f+\frac{\Delta f}{2}\right)\right] = 4 k_B T R \Delta f\right)$ is valid.

Note the power is independent of f and only depends upon Δf.

The maximum noise power that can be transferred to a load is under matching impedance conditions, and is given by

$$E\left(\frac{V_n^2\left(f-\frac{\Delta f}{2}, f+\frac{\Delta f}{2}\right)}{4R}\right)$$

$$= k_B T \Delta f.$$

What is the typical value of the noise power in communication systems we use?

Consider a GSM system where $\Delta f = 200$ kHz, then the noise power in the pass band of the GSM signal $= 1.38 \times 10^{-23} \times 300 \times 200 \times 10^3$

$= 8.28 \times 10^{-16}$ W.

This power level is too low, but then why do we study it then.

The noise power levels might appear low to us, but it is significant for the communication circuits since the power of the GSM signal received from the base station is also very weak (about 10^{-14} W). This happens due to the fact that the base station only radiates about 10 W and then there is propagation loss especially when the mobile terminal is several kms from the base station).