(c) $\displaystyle\int_{-\infty}^{\infty} t^n g(t)\,dt = \left(\frac{j}{2\pi}\right)^n G^{(n)}(0)$

(d) $\displaystyle g_1(t)g_2^*(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2^*(\lambda - f)\,d\lambda$

(e) $\displaystyle\int_{-\infty}^{\infty} g_1(t)g_2^*(t)\,dt = \int_{-\infty}^{\infty} G_1(f)G_2^*(f)\,df$

2.7 The Fourier transform $G(f)$ of a signal $g(t)$ is bounded by the following three inequalities:

$$|G(f)| \le \int_{-\infty}^{\infty} |g(t)|dt$$

$$|j2\pi f G(f)| \le \int_{-\infty}^{\infty} \left|\frac{dg(t)}{dt}\right| dt$$

and

$$|(j2\pi f)^2 G(f)| \le \int_{-\infty}^{\infty} \left|\frac{d^2g(t)}{dt^2}\right| dt$$

where it is assumed that the first and second derivatives of $g(t)$ exist.

Construct these three bounds for the triangular pulse shown in Figure P2.7 and compare your results with the actual amplitude spectrum of the pulse.

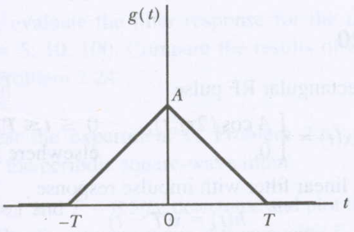

Figure P2.7

2.8 Prove the following properties of the convolution process:

(a) The commutative property:

$$g_1(t) \star g_2(t) = g_2(t) \star g_1(t)$$

(b) The associative property:

$$g_1(t) \star [g_2(t) \star g_3(t)] = [g_1(t) \star g_2(t)] \star g_3(t)$$

(c) The distributive property:

$$g_1(t) \star [g_2(t) + g_3(t)] = g_1(t) \star g_2(t) + g_1(t) \star g_3(t)$$

2.9 Consider the convolution of two signals $g_1(t)$ and $g_2(t)$. Show that

(a) $\displaystyle\frac{d}{dt}[g_1(t) \star g_2(t)] = \left[\frac{d}{dt}g_1(t)\right] \star g_2(t)$

(b) $\displaystyle\int_{-\infty}^{t} [g_1(\tau) \star g_2(\tau)]d\tau = \left[\int_{-\infty}^{t} g_1(\tau)d\tau\right] \star g_2(t)$

2.10 A signal $x(t)$ of finite energy is applied to a square-law device whose output $y(t)$ is defined by

$$y(t) = x^2(t)$$

The spectrum of $x(t)$ is limited to the frequency interval $-W \le f \le W$. Hence, show that the spectrum of $y(t)$ is limited to $-2W \le f \le 2W$. *Hint*: Express $y(t)$ as $x(t)$ multiplied by itself.

2.11 Evaluate the Fourier transform of the delta function by considering it as the limiting form of (1) a rectangular pulse of unit area, and (2) a sinc pulse of unit area.

2.12 The Fourier transform $G(f)$ of a signal $g(t)$ is defined by

$$G(f) = \begin{cases} 1, & f > 0 \\ \dfrac{1}{2}, & f = 0 \\ 0, & f < 0 \end{cases}$$

Determine the signal $g(t)$.

2.13 Show that the two different pulses defined in parts (*a*) and (*b*) of Figure P2.1 have the same energy spectral density:

$$\mathcal{E}_g(f) = \frac{4A^2T^2\cos^2(\pi T f)}{\pi^2(4T^2f^2 - 1)^2}$$

2.14

(a) The *root mean-square (rms) bandwidth* of a low-pass signal $g(t)$ of finite energy is defined by

$$W_{\text{rms}} = \left[\frac{\int_{-\infty}^{\infty} f^2|G(f)|^2\,df}{\int_{-\infty}^{\infty} |G(f)|^2\,df}\right]^{1/2}$$

where $|G(f)|^2$ is the energy spectral density of the signal. Correspondingly, the *root mean-square (rms) duration* of the signal is defined by

$$T_{\text{rms}} = \left[\frac{\int_{-\infty}^{\infty} t^2|g(t)|^2\,dt}{\int_{-\infty}^{\infty} |g(t)|^2\,dt}\right]^{1/2}$$

Using these definitions, show that

$$T_{\text{rms}}W_{\text{rms}} \ge \frac{1}{4\pi}$$

Assume that $|g(t)| \to 0$ faster than $1/\sqrt{|t|}$ as $|t| \to \infty$.

(b) Consider a Gaussian pulse defined by

$$g(t) = \exp(-\pi t^2)$$

Show that, for this signal, the equality

$$T_{\text{rms}}W_{\text{rms}} = \frac{1}{4\pi}$$

can be reached.