*Hint*: Use Schwarz's inequality:

$$\left\{\int_{-\infty}^{\infty}[g_1^*(t)g_2(t)+g_1(t)g_2^*(t)]dt\right\}^2 \le 4\int_{-\infty}^{\infty}|g_1(t)|^2\,dt \times \int_{-\infty}^{\infty}|g_2(t)|^2\,dt$$

in which we set

$$g_1(t) = tg(t)$$

and

$$g_2(t) = \frac{dg(t)}{dt}$$

**2.15** Let $x(t)$ and $y(t)$ be the input and output signals of a linear time-invariant filter. Using Rayleigh's energy theorem, show that if the filter is stable and the input signal $x(t)$ has finite energy, then the output signal $y(t)$ also has finite energy. That is, given that

$$\int_{-\infty}^{\infty}|x(t)|^2\,dt < \infty$$

then show that

$$\int_{-\infty}^{\infty}|y(t)|^2\,dt < \infty$$

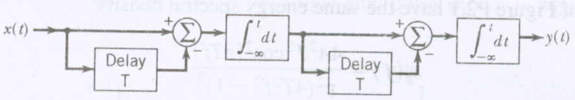


**Figure P2.16**

**2.16** Evaluate the transfer function of a linear system represented by the block diagram shown in Figure P2.16.

**2.17**

**(a)** Determine the overall amplitude response of the cascade connection shown in Figure P2.17 consisting of $N$ identical stages, each with a time constant $RC$ equal to $\tau_0$.

**(b)** Show that as $N$ approaches infinity, the amplitude response of the cascade connection approaches the Gaussian function $\exp\left(-\frac{1}{2}f^2T^2\right)$, where for each value of $N$, the time constant $\tau_0$ is selected so that

$$\tau_0^2 = \frac{T^2}{4\pi^2 N}$$

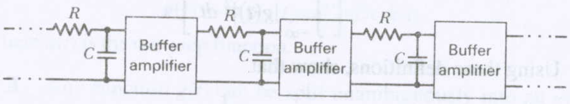


**Figure P2.17**

**2.18** Suppose that, for a given signal $x(t)$, the integrated value of the signal over an interval $T$ is required, as shown by

$$y(t) = \int_{t-T}^{t} x(\tau)\,d\tau$$

**(a)** Show that $y(t)$ can be obtained by processing the signal $x(t)$ with a filter having the transfer function

$$H(f) = T\,\mathrm{sinc}(fT)\exp(-j\pi fT)$$

**(b)** An adequate approximation to this transfer function is obtained by using a low-pass filter with a bandwidth equal to $1/T$, passband amplitude response $T$, and delay $T/2$. Assuming this low-pass filter is created by the combination of an $RC$ circuit followed by a gain of $T$, determine the filter output at time $t = T$ due to a unit step function applied to the filter at $t = 0$, and compare the result with the corresponding output of the ideal integrator.

**2.19** A tapped-delay-line filter consists of $N$ weights, where $N$ is odd. It is symmetric with respective to the center tap, that is, the weights satisfy the condition

$$w_n = w_{N-1-n} \qquad 0 \le n \le N-1$$

**(a)** Find the amplitude response of the filter.

**(b)** Show that this filter has a linear phase response.

**2.20** Consider an ideal band-pass filter with frequency midband $f_c$ and bandwidth $2B$, as defined in Figure P2.20. The carrier wave $A\cos(2\pi f_0 t)$ is suddenly applied to this filter at time $t = 0$. Assuming that $|f_c - f_0|$ is large compared to the bandwidth $2B$, determine the response of the filter.

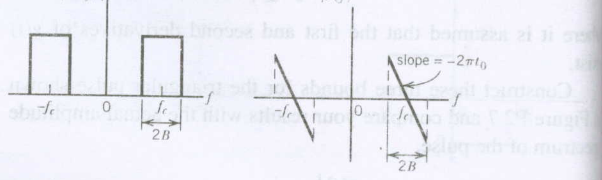


**Figure P2.20**

**2.21** The rectangular RF pulse

$$x(t) = \begin{cases} A\cos(2\pi f_c t), & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$$

is applied to a linear filter with impulse response

$$h(t) = x(T-t)$$

Assume that the frequency $f_c$ equals a large integer multiple of $1/T$. Determine the response of the filter and sketch it.

**2.22** Show that the inverse DFT of a sequence of constant impulses in the frequency domain is a corresponding sequence of impulses in the time domain when the DFT length is even.

## Computer Problems

**2.23** A rectangular pulse $x(t)$ of unit amplitude and duration $T$ is applied to an ideal low-pass filter of bandwidth $B$.

**(a)** What is the impulse response of the ideal low-pass filter?

**(b)** Determine and plot the response $y(t)$ of the filter for $BT = 5$, 10, 20. Using the following Matlab script,

```
% - - - Simulation parameters - - - -
BT       = 5; %BT product
T        = 1;
B        = BT/T;
Delta_t = T/100;
t        = [-6*T: Delta_t: 6*T];
```