Tutorial - II

EEL306 - Communication Engineering (II-Sem 2013-14)

Instructor: Dr. Saif Khan Mohammed, saifkm@ee.iitd.ac.in, 011-26591067

1) Find the complex baseband envelope of each of the following real-valued band-limited band-pass signals.

a)

$$x(t) \,=\, 0.5\,\cos\Big(20000\pi t \,+\, \frac{\pi}{6}\Big). \tag{1}$$

b)

$$x(t) \,=\, 0.25\,\cos\Big(20000\pi t \,+\, \frac{\pi}{4}\Big) \,+\, \sin\Big(20100\pi t \,-\, \frac{\pi}{3}\Big). \tag{2}$$

c)

$$x(t) \,=\, 2\,\cos\Big(50\pi t\Big)\,\sin\Big(25000\pi t\Big). \tag{3}$$

d)

$$x(t) \,=\, m(t)\,\sin\Big(2\pi f t \,+\, \phi\Big), \tag{4}$$

where $m(t)$ is a real-valued baseband signal bandlimited to $[-W, W]$, and $\phi \in [-\pi\,,\,\pi)$ is a constant.

2) If $\tilde{x}(t)$ is the complex baseband envelope of the real-valued bandpass signal $x(t)$, then find an expression for the complex baseband envelope of $y(t) = x(t-\tau)$ in terms of $\tilde{x}(t)$.

3) Before transmitting the real-valued band-limited bandpass signal $s(t)$, one would like to pass it through a bandpass filter (having impulse response $h(t)$) (This is usually done in order to adhere to radiation constraints specified by standards/regulatory bodies). The filtered output is given by

$$x(t) \;=\; \int_{-\infty}^{\infty} h(\tau)\, s(t-\tau)\, d\tau. \tag{5}$$

In practice, filtering of the transmit signal is not done in pass band, but in baseband due to implementation related issues (low cost, low power consumption). Let $s(t) = sinc(2Wt)\cos(2\pi f_c t)$ and $h(t) = sinc^2(2Wt)\,\sin(2\pi f_c t)$.

Derive an expression for the impulse response of the equivalent complex baseband filter ($h_b(t)$) such that

$$\tilde{x}(t) \;=\; \int_{-\infty}^{\infty} h_b(\tau)\, \tilde{s}(t-\tau)\, d\tau, \tag{6}$$

i.e., we can get the same output $x(t)$ in (5) by simply filtering $\tilde{s}(t)$ in complex baseband followed by an upconversion (no need for any filtering in pass band).

Finally derive an expression for $x(t)$ (first derive an expression for $\tilde{x}(t)$).

4) (Group/Envelope delay and Phase delay)

The signal received at the receiver ($y(t)$) is usually passed through a band pass filter $h(t)$ to get rid of out of band interference (i.e., outside the band $[f_c - W \,,\, f_c + W]$). Assuming that $y(t)$ is narrow band (i.e., $f_c \gg W$, the in-band phase response of $h(t)$ is almost linear, and $|H(f)| \approx |H(f_c)|$ inside the pass band $|f - f_c| < W$), you are required to show that the filtered output $z(t)$ is given by

$$z(t) \;\approx\; |H(f_c)|\, \mathrm{Re}\Big(\tilde{y}(t - t_g) e^{j2\pi f_c (t - t_p)}\Big), \tag{7}$$

$$\begin{aligned} t_g &\;\stackrel{\Delta}{=}\; -\frac{1}{2\pi}\frac{d\theta(f)}{df}\Big|_{f=f_c} \\ t_p &\;\stackrel{\Delta}{=}\; -\frac{1}{2\pi}\frac{\theta(f_c)}{f_c} \end{aligned} \tag{8}$$

where $\theta(f) \stackrel{\Delta}{=} \arg\Big(H(f)/|H(f)|\Big)$ is the phase response of $h(t)$. Since the signal $y(t)$ is narrow band we can assume that inside the pass band

$$\theta(f) \;\approx\; \theta(f_c) \,+\, (f - f_c)\frac{d\theta(f)}{df}\Big|_{f=f_c}. \tag{9}$$

Note that this is the first order Taylor series expansion of $\theta(f)$ around $f = f_c$.

From (7) it can be concluded that, due to band pass filtering the complex envelope of the received signal gets delayed by t_g, whereas the carrier signals $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ are delayed by t_p seconds.