where

$$t_g \triangleq -\frac{1}{2\pi} \left. \frac{d\tilde{\theta}(f)}{df} \right|_{f=0}$$

$$= -\frac{1}{2\pi} \left. \frac{d\,\theta(f)}{df} \right|_{f=f_c}$$

Taking the inverse Fourier tranform of 6 we get

$$\tilde{z}(t) = |H(f_c)|\, e^{j\theta(f_c)}\, \tilde{y}(t-t_g) \quad \text{— (7)}$$

Hence

$$z(t) = \operatorname{Re}\left(\tilde{z}(t)\, e^{j2\pi f_c t}\right)$$

$$= |H(f_c)| \operatorname{Re}\left[\tilde{y}(t-t_g)\, e^{j(2\pi f_c t + \theta(f_c))}\right]$$

$$= |H(f_c)| \operatorname{Re}\left[\tilde{y}(t-t_g)\, e^{j2\pi f_c (t-t_p)}\right]$$

where $t_p \triangleq -\frac{1}{2\pi} \frac{\theta(f_c)}{f_c}$.