(9)

Since $H(f)$ is narrow band

(the signal is narrow band and therefore we can equivalently say that the channel is narrow band),

we have

$$\cancel{\tilde{\theta}(f) \approx \tilde{\theta}(f=0) + f}$$

$$\theta_+(f) \approx \theta_+(f_c) + (f-f_c) \left.\frac{d\,\theta_+(f)}{df}\right|_{f=f_c} \quad \text{--- (3)}$$

(first order Taylor series Expansion of $\theta_+(f)$ around $f=f_c$)

~~Since~~ using $\tilde{\theta}(f) = \theta_+(f+f_c)$, (From eqn. (2)) ~~we have~~ in equation (3)

we set.

$$\tilde{\theta}(f) = \theta_+(f+f_c) = \theta_+(f_c) + f \left.\frac{d\,\theta_+(f+f_c)}{df}\right|_{f+f_c=f_c}$$

i.e.,

$$\tilde{\theta}(f) = \theta_+(f_c) + f \left.\frac{d\,\theta_+(f+f_c)}{df}\right|_{f=0} \quad \text{--- (4)}$$

$$= \tilde{\theta}(0) + f \left.\frac{d\,\tilde{\theta}(f)}{df}\right|_{f=0}$$ since $\tilde{\theta}(f) = \theta_+(f+f_c)$