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3. We know that if

$$x(t) = \int h(\tau)\, s(t-\tau)\, d\tau \quad , \text{ then} \quad (1)$$

$$\tilde{x}(t) = \frac{1}{2} \int \tilde{h}(\tau)\, \tilde{s}(t-\tau)\, d\tau , \quad (2)$$

and therefore we can choose

$$h_b(t) = \frac{1}{2} \tilde{h}(t).$$

Now, $h(t) = \operatorname{sinc}^2(2wt) \sin 2\pi f_c t$

$$= \operatorname{Re}\left(-j \operatorname{sinc}^2(2wt)\, e^{j 2\pi f_c t}\right)$$

$$= \operatorname{Re}\left(\tilde{h}(t)\, e^{j 2\pi f_c t}\right)$$

$\therefore$ $\tilde{h}(t) = -j \operatorname{sinc}^2(2wt)$

Hence

$$h_b(t) = \frac{-j}{2} \operatorname{sinc}^2(2wt).$$

We also have

$$\tilde{X}(f) = H_b(f)\, \tilde{S}(f) \quad (\text{Fourier transforming } (2))$$