Since $H(f)$ is narrow band we also assume that

$|H(f)|$ is almost constant in the pass band, i.e.,

$$|H(f)| \approx |H(f_c)| \quad \text{for all } f_c - W \le f \le f_c + W \qquad (5)$$

With this approximation and the approximation in (4) we have

$$\tilde{H}(f) = 2|H_+(f+f_c)| e^{j\theta_+(f+f_c)} \quad (\text{from } 2)$$

$$\approx 2|H_+(f_c)| e^{j\tilde{\theta}(f)} \quad \leftarrow \text{using } 5$$

$$\approx 2|H_+(f_c)| e^{j(\tilde{\theta}(f_c) + f \left.\frac{d\tilde{\theta}(f)}{df}\right|_{f=f_c})} \quad \leftarrow \text{using } 5$$

$$= 2|H(f_c)| e^{j\theta(f_c)} e^{-j\pi f \left(-\frac{1}{\pi} \left.\frac{d\tilde{\theta}(f)}{df}\right|_{f=f_c}\right)}$$

Hence using (1) now we set

$$\tilde{Z}(f) = \frac{1}{2}\tilde{H}(f)\tilde{Y}(f)$$

$$\approx |H(f_c)| e^{j\theta(f_c)} \tilde{Y}(f) e^{-j\pi f t_g} \qquad (6)$$