1

3.2

a) $i = I_0 [e^{-v/V_T} - 1]$

$V_T = 0.026$ v.

Taylor series expansion of e^{-x} around $(x=0)$ is

$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} +$ higher order terms.

$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} +$ higher order terms.

$\therefore$ for small $(x = v/V_T)$

$$e^{-v/V_T} \simeq 1 - \left(\frac{v}{V_T}\right) + \frac{(v/V_T)^2}{2} - \frac{(v/V_T)^3}{6}$$

$$\therefore i \simeq I_0 \left[\left(\frac{-v}{V_T}\right) + \frac{(v/V_T)^2}{2} - \frac{(v/V_T)^3}{6}\right] \quad \text{— (1)}$$

b) $v = A(\cos 2\pi f_m t + \cos 2\pi f_c t)$ volts.

$A = 0.01$ volts. — (2)

$$\therefore \left(\frac{v}{V_T}\right)^2 = \left(\frac{A}{V_T}\right)^2 \{\cos^2 2\pi f_m t + \cos^2 2\pi f_c t + 2\cos 2\pi f_m t \cos 2\pi f_c t\}$$

$$= \left(\frac{A}{V_T}\right)^2 \{1 + \cos 4\pi f_m t + \cos 4\pi f_c t + \cos 2\pi (f_m + f_c)t + \cos 2\pi (f_m - f_c)t\} \quad \text{— (3)}$$

$$\left(\frac{V}{V_T}\right)^3 = \left(\frac{A}{V_T}\right)^3 \left\{ \cos 2\pi f_m t + \cos 2\pi f_c t \right\}^3 \quad (2)$$

$$= \left(\frac{A}{V_T}\right)^3 \Big\{ \cos^3 2\pi f_m t + \cos^3 2\pi f_c t$$

$$+ 3 \cos^2 2\pi f_m t \cos 2\pi f_c t$$

$$+ 3 \cos 2\pi f_m t \cos^2 2\pi f_c t \Big\}$$

Using the identities.

$$\cos^3\theta = \frac{3\cos\theta + \cos 3\theta}{4},$$

we get.

$$\left(\frac{V}{V_T}\right)^3 = \left(\frac{A}{V_T}\right)^3 \Big\{ \frac{3}{4}\cos 2\pi f_m t + \frac{3}{4}\cos 2\pi f_c t$$

$$+ \frac{\cos 6\pi f_m t}{4} + \frac{\cos 6\pi f_c t}{4}$$

$$+ \frac{3}{2}\cos 2\pi f_c t + \frac{3}{4}\cos(2\pi f_c + 4\pi f_m)t$$

$$+ \frac{3}{4}\cos 2\pi(f_c - 2f_m)t$$

$$+ \frac{3}{2}\cos 2\pi f_m t + \frac{3}{4}\cos(2\pi f_m + 4\pi f_c)t$$

$$+ \frac{3}{4}\cos 2\pi(2f_c - f_m)t \Big\}$$

$$\text{--- } (4)$$

③

$$\therefore \; i \approx I_0 \left[\left(-V/V_T\right) + \frac{(V/V_T)^2}{2} - \frac{(V/V_T)^3}{6} \right] \quad \text{—— (1)}$$

using (2), (3) and (3) in (1) we get

$$\frac{i(t)}{I_0} \approx (\cos 2\pi f_m t)\left(\left(\frac{-A}{V_T}\right) - \left(\frac{3}{8}\right)\left(\frac{A}{V_T}\right)^3\right)$$

$$+ (\cos 2\pi f_c t)\left(\left(\frac{-A}{V_T}\right) - \left(\frac{3}{8}\right)\left(\frac{A}{V_T}\right)^3\right)$$

$$+ \frac{1}{2}\left(\cos 2\pi \cdot 2 f_m t + \cos 2\pi \cdot 2 f_c t\right)\left(\frac{A}{V_T}\right)^2$$

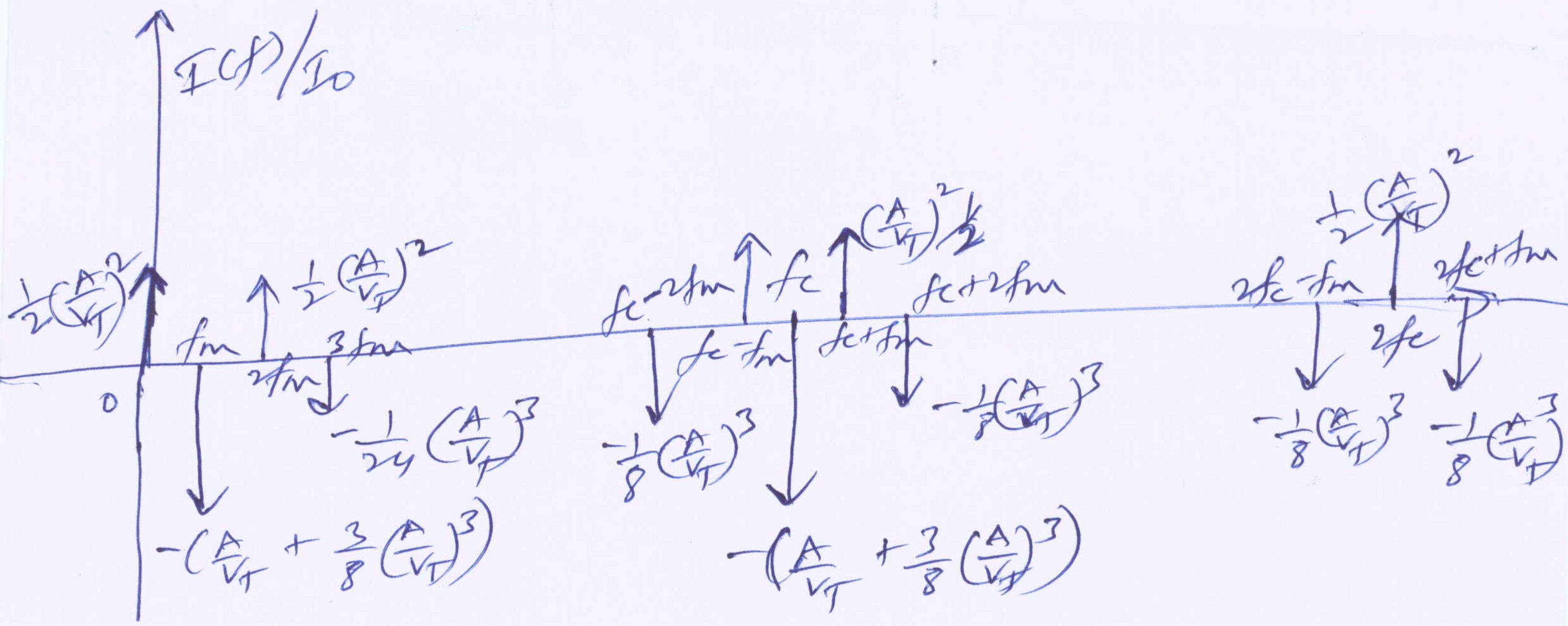
$$+ \frac{1}{2}\left(\frac{A}{V_T}\right)^2 + \frac{1}{2}\left(\frac{A}{V_T}\right)^2 \left(\cos 2\pi (f_m + f_c) t + \cos 2\pi (f_m - f_c) t\right)$$

$$- \frac{1}{24}\left(\frac{A}{V_T}\right)^3 \left(\cos(2\pi \cdot 3 f_c t) + \cos(2\pi \cdot 3 f_m t)\right)$$

$$- \frac{1}{8}\left(\frac{A}{V_T}\right)^3 \Big(\cos 2\pi (f_c + 2 f_m) t + \cos 2\pi (f_c - 2 f_m) t + \cos 2\pi (2 f_c - f_m) t + \cos 2\pi (2 f_c + f_m) t\Big)$$

Spectrum $i(f)/I_0$

$I(f)/I_0$ (only positive frequencies shown)

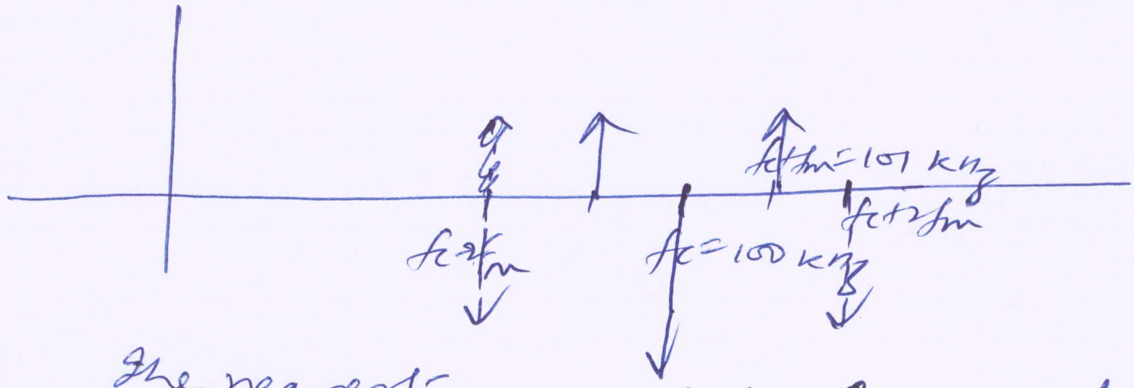


There are also signal components at $3f_c$ (not shown in the above figure)

(c)

The AM signal in $i(t)/I_0$ is essentially the signal components at $f = f_c$, f_c+f_m and f_c-f_m.

With $f_c = 100$ kHz, $f_m = 1$ kHz, the following band pass filter would be sufficient to filter out the AM signal from $i(t)$.



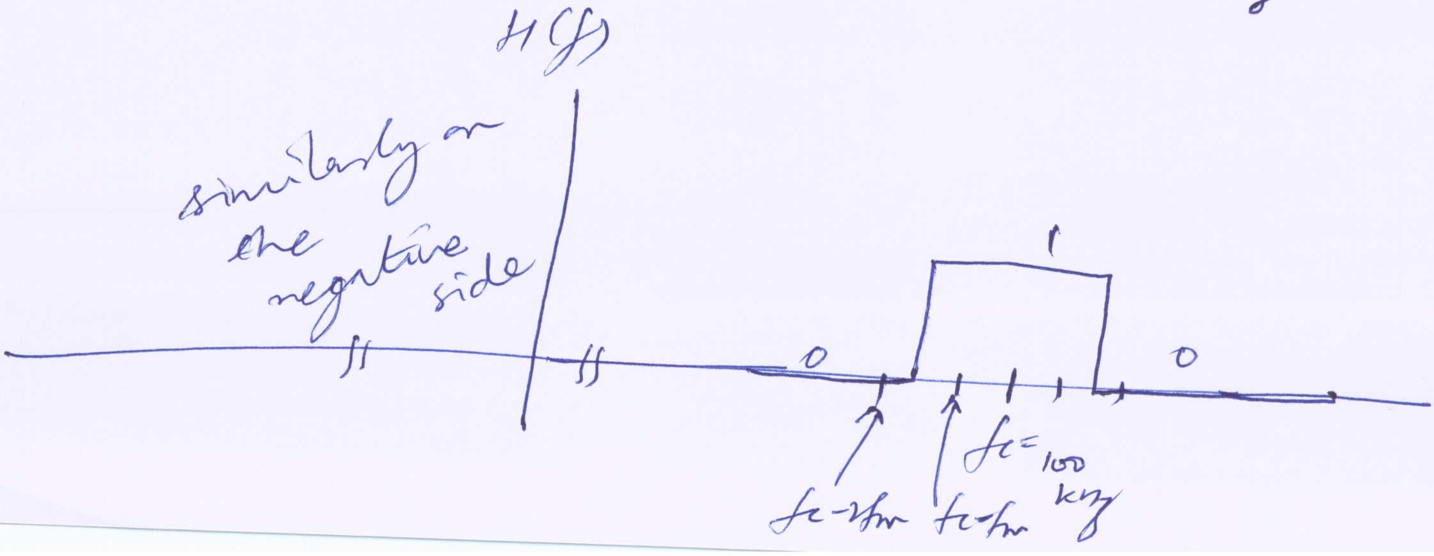
The nearest **non-AM** component is at $f = (f_c - 2f_m)$ and $(f_c + 2f_m)$.

The designed bandpass filter should therefore pass all signals in the frequency band $[f_c - f_m, f_c + f_m]$

$= [99 \text{ kHz}, 101 \text{ kHz}]$

and reject ~~other~~ signals at other frequencies.

$$\therefore H(f) = \begin{cases} 0, & f > 101.5 \text{ kHz} \\ 1, & 98.5 \text{ kHz} \le f \le 101.5 \text{ kHz} \\ 0, & -98.5 < f < 98.5 \text{ kHz} \\ 1, & -101.5 \le f \le -98.5 \text{ kHz} \\ 0, & f < -101.5 \text{ kHz} \end{cases}$$



$$i(t) \longrightarrow \boxed{h(t)} \longrightarrow s(t).$$

$$\therefore \; s(t) = \cancel{I_0 \left(\frac{1}{2}\left(\frac{A}{V_T}\right)^2 \cos 2\pi f_c t + \frac{1}{2}\left(\frac{A}{V_T}\right)^2 \right.}$$

$$s(t) = I_0 \left(\frac{1}{2}\left(\frac{A}{V_T}\right)^2 \cos 2\pi (f_m + f_c) t + \frac{1}{2}\left(\frac{A}{V_T}\right)^2 \cos 2\pi (f_c - f_m) t - \left[\left(\frac{A}{V_T}\right) + \left(\frac{3}{8}\right)\left(\frac{A}{V_T}\right)^3\right] \cos 2\pi f_c t \right)$$

$$= -I_0 \left(-\cos 2\pi f_c t \cos 2\pi f_m t \cdot \left(\frac{A}{V_T}\right)^2 \cdot \frac{1}{4} + \left[\left(\frac{A}{V_T}\right) + \left(\frac{3}{8}\right)\left(\frac{A}{V_T}\right)^3\right] \cos 2\pi f_c t \right)$$

$$= -I_0 \left[\left(\frac{A}{V_T}\right) + \left(\frac{3}{8}\right)\left(\frac{A}{V_T}\right)^3\right] \cos 2\pi f_c t \left\{ 1 - \frac{(A/V_T)^2 \cos 2\pi f_m t}{4\left(\left(\frac{A}{V_T}\right) + \left(\frac{3}{8}\right)\left(\frac{A}{V_T}\right)^3\right)} \right\}$$

carrier signal

message signal

$$= \cancel{-I_0 \left[\left(\frac{A}{V_T}\right) + \left(\frac{3}{8}\right)\left(\frac{A}{V_T}\right)^2\right] \cos 2\pi f_c t} \quad \cancel{\left\{ 1 - \frac{m(t)}{4V_T(} \right.}$$

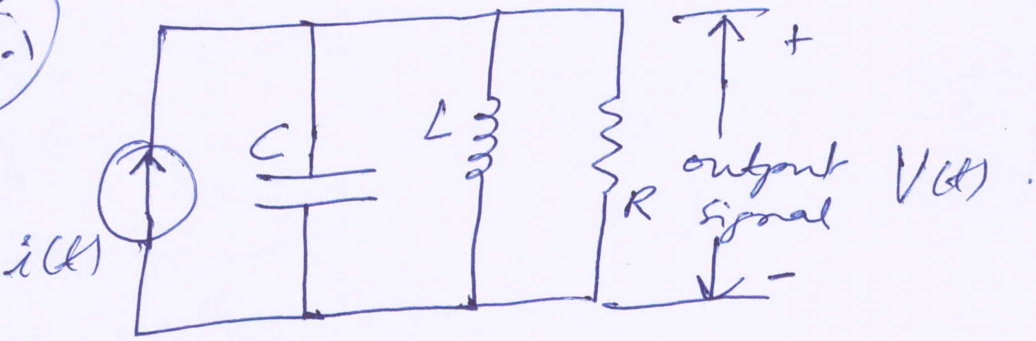
percentage modulation

$$= 100 \times \frac{(A/V_T)^2}{4\left[(A/V_T) + \left(\frac{3}{8}\right)\left(\frac{A}{V_T}\right)^3\right]}$$

$$= 9.11\%$$

B

3.1



Combined impedance (Z)

$$\frac{1}{Z} = j\omega C + \frac{1}{j\omega L} + \frac{1}{R}$$

$$= j2\pi f C + \frac{1}{j2\pi f L} + \frac{1}{R}$$

$$= \frac{R + j2\pi f L - 4\pi^2 f^2 RLC}{j2\pi f RL}$$

$$\therefore Z = \frac{j2\pi f RL}{R(1-(2\pi f)^2 LC) + j2\pi f L} \quad -(1)$$

~~$i(t) = I[$~~

$\therefore$ ~~$V(t)$~~

$$\therefore \frac{V(f)}{I(f)} = Z(f) = \frac{j2\pi f RL}{R(1-(2\pi f)^2 LC) + j2\pi f L}$$

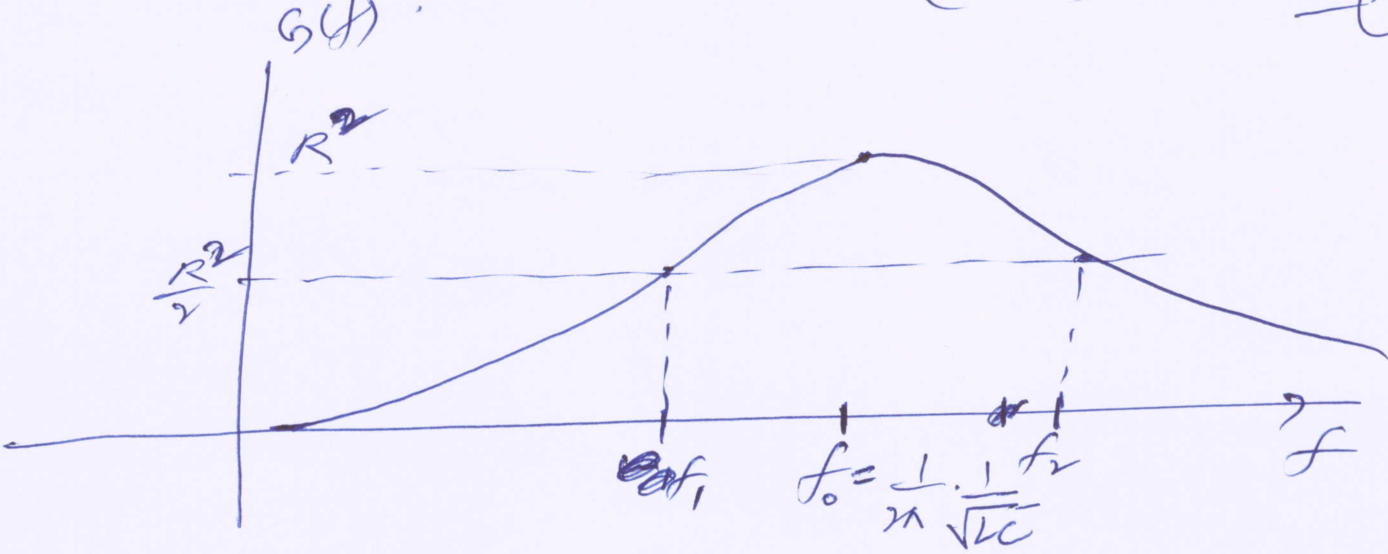
$$= \frac{R \cdot j2\pi f L}{R(1-(2\pi f)^2 LC) + j2\pi f L}$$

$$\frac{V(f)}{I(f)} = \frac{R}{\left(1 - j\frac{R}{2\pi f L}(1-(2\pi f)^2 LC)\right)}$$

8

Power gain $= \dfrac{|V(f)|^2}{|I(f)|^2} = |Z(f)|^2$

$$= \frac{R^2}{1 + \dfrac{R^2\left[1-(2\pi f)^2 LC\right]^2}{(2\pi f L)^2}} = G(f). \quad \text{— (2)}$$



~~Next to find~~ We observe that .

$$f_0 = \arg\max_f G(f) = \frac{1}{2\pi}\cdot\frac{1}{\sqrt{LC}} \quad \text{and}$$

$$G(f_0) = R^2.$$

Note that $\lim_{f\to\infty} G(f) = \lim_{f\to 0} G(f) = 0$, so it is a bandpass filter. — (3)

Next we are interested in finding the bandwidth (3-dB) of this bandpass filter.

The Q-factor of this bandpass filter is defined as (9)

$$Q = \frac{\omega_0}{\Delta\omega},$$

$$= \frac{2\pi f_0}{2\pi \Delta f} = \frac{f_0}{\Delta f}, \text{ where } f_0$$ (4)

is the center bandwidth and Δf is the 3-dB bandwidth of the filter.

The 3-dB bandwidth is defined as the difference $\Delta f = f_2 - f_1$,

where $f_2 > f_0 > f_1$ are such that

$$10 \log_{10} \frac{G(f_2)}{G(f_0)} = 10 \log_{10} \frac{G(f_1)}{G(f_0)} = -3 \text{ dB},$$

i.e., $G(f_2) = G(f_1) = \frac{G(f_0)}{2}$.

i.e., we need to solve for f such that

$$\frac{R^2 \left(1 - (2\pi f)^2 LC\right)^2}{(2\pi f L)^2} = 1$$

or $$1 - (2\pi f)^2 LC = \pm \frac{2\pi f L}{R}.$$ (5)

∴ we have the two solutions.

Since $f_0 = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$ (10)

$$(2\pi f_0)^2 = \frac{1}{LC}$$

$\therefore$ we examine the + equation first.

$$1 - (f/f_0)^2 = +\frac{2\pi f_0 L}{R} (f/f_0).$$

$$\therefore \left(\frac{f}{f_0}\right)^2 + \left(\frac{2\pi f_0 L}{R}\right)(f/f_0) - 1 = 0.$$

$$\frac{f}{f_0} = \frac{-\frac{2\pi f_0 L}{R} \pm \sqrt{\left(\frac{2\pi f_0 L}{R}\right)^2 + 4}}{2}$$

This gives us

$$\frac{f_1}{f_0} = \frac{\sqrt{\left(\frac{2\pi f_0 L}{R}\right)^2 + 4} - \left(\frac{2\pi f_0 L}{R}\right)}{2} \quad \text{— (6)}$$

Taking the negative equation

$$1 - (2\pi f)^2 LC = -\frac{2\pi f L}{R}$$

we get

$$1 - (f/f_0)^2 = -\frac{2\pi f_0 L}{R} (f/f_0)$$

i.e., $$(f/f_0)^2 - \frac{2\pi f_0 L}{R} (f/f_0) - 1 = 0.$$

which gives us

$$\frac{f_2}{f_0} = \frac{\frac{2\pi f_0 L}{R} + \sqrt{4 + \left(\frac{2\pi f_0 L}{R}\right)^2}}{2}. \quad \text{— (7)}$$

using (6) and (7) in (5) we have

$$\frac{\Delta f}{f_0} = \frac{f_2 - f_1}{f_0} = (f_2/f_0) - (f_1/f_0)$$

$$= \left(\frac{2\pi f_0 L}{R}\right)$$

$$\therefore Q = \frac{f_0}{\Delta f} = \frac{R}{(2\pi f_0 L)} = 175. \quad \text{---} (8)$$

It is also given that the filter is tuned to the carrier frequency $f_0 = f_c = 1\ MHz$.

carrier frequency $f_c = ~~8~~\ 1\ MHz$.

To examine the filter output i.e.,

$$m(t) = A \cos 2\pi f_c t\,(1 + 0.5 \cos 2\pi f_m t)$$

where $f_c = 1\ MHz$ and

$f_m = 5\ kHz$.

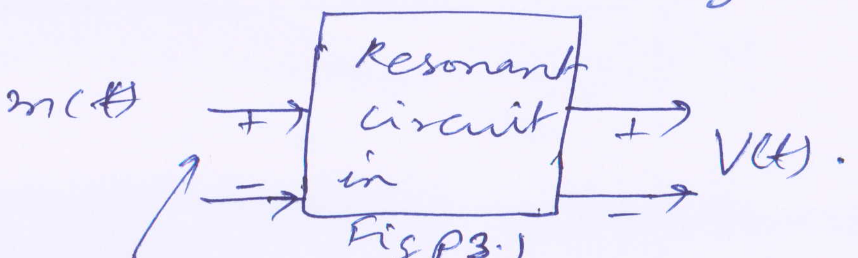


Fig P3.1

input is applied as a current source.

The phase response of the resonant circuit is

$$\frac{V(f)}{I(f)} = \left|\frac{V(f)}{I(f)}\right| e^{j\phi(f)}$$

$$\cos\phi(f) = \frac{1}{\sqrt{1 + \left(\frac{R}{2\pi f L}\right)^2 \left(1-(2\pi f)^2 LC\right)^2}}$$

$$= \frac{1}{\sqrt{1+ \left(\frac{R}{2\pi f_0 L}\right)^2 \cdot \frac{1}{(f/f_0)^2} \left(1-\left(\frac{f}{f_0}\right)^2\right)^2}} \quad \text{--- (9)}$$

$$\sin\phi(f) = \frac{\left(\frac{R}{2\pi f_0 L}\right) \cdot \frac{1}{(f/f_0)} \left(1-(f/f_0)^2\right)}{\sqrt{1+ \left(\frac{R}{2\pi f_0 L}\right)^2 \cdot \frac{1}{(f/f_0)^2}\left(1-(f/f_0)^2\right)^2}}$$

$\therefore$ using the fact that

$\left(\frac{R}{2\pi f_0 L}\right) = 175$ and $f_0 = 10^6$ Hz, we have

$\therefore$ for $f_2 = f_0 + f_m = (10^6 + 5\times10^3)$ Hz.

$= 1.005\times10^6$ Hz.

$\cos\phi(f_2) = 0.4971$

$\sin\phi(f_2) = -0.8677$

$\therefore \phi(f_2) \simeq -\pi/3$

Similarly.

$$f_1 = f_0 - f_m = 0.995 \times 10^{6}\ Hz.$$

using equation 9 we get

$$\cos\phi(f_1) = 0.4952.$$

$$\sin\phi(f_1) = 0.8688$$

$$\therefore \quad \phi(f_1) \approx \pi/3.$$

$\therefore$ Magnitude response

$$\left|\frac{V(f)}{I(f)}\right| = \frac{R}{\sqrt{1 + \left(\frac{R}{2\pi f_0 L}\right)^2 \cdot \frac{1}{(f/f_0)^2}\left(1 - (f/f_0)^2\right)^2}}$$

$$\therefore \left|\frac{V(f_2)}{I(f_2)}\right| \approx \frac{R}{2} \quad (0.4971\,R \text{ to be precise})$$

and

$$\left|\frac{V(f_1)}{I(f_1)}\right| \approx \frac{R}{2} \quad (0.4952\,R \text{ to be precise})$$

$\therefore$ ~~the output of the resonant circuit is~~

$$\cancel{r(t) = \frac{R}{2}\cos\left(2\pi(f_c+f_m)t - \pi/3\right) + \frac{R}{2}\cos\left(2\pi(f_c-f_m)t + \pi/3\right)}$$

The input to the resonant circuit is a AM signal with 50% modulation, i.e.,

$$m(t) = A\cos 2\pi f_c t\,(1 + 0.5\cos 2\pi f_m t)$$

$$f_c = 1\text{ MHz}, \quad f_m = 5\text{ kHz}.$$

$$\therefore m(t) = A\cos 2\pi f_c t + \frac{A}{4}\cos 2\pi(f_c + f_m)t + \frac{A}{4}\cos 2\pi(f_c - f_m)t \qquad (10)$$

The output to $m(t)$ is the ~~linear~~ sum of the outputs to all the three signal terms in (10).

$$A\cos 2\pi f_c t \longrightarrow \boxed{\text{Resonant circuit}} \longrightarrow AR\cos 2\pi f_c t$$

(gain at $f_0 = f_c$ is R and phase shift is 0)

$$\frac{A}{4}\cos 2\pi(f_c + f_m)t \longrightarrow \boxed{\text{Resonant circuit}} \longrightarrow \frac{AR}{8}\cos(2\pi(f_c + f_m)t - \pi/3)$$

since gain at $f_c + f_m = f_2$ is $\frac{R}{2}$ and phase shift is $-\pi/3$.

$$\frac{A}{4}\cos 2\pi(f_c - f_m)t \longrightarrow \boxed{\text{Resonant circuit}} \longrightarrow \frac{AR}{8}\cos[2\pi(f_c - f_m)t + \pi/3]$$

since gain at $f_1 = f_c - f_m$ is $R/2$.

$\therefore$ The output of the resonant circuit is

$$s(t) = AR\cos 2\pi f_c t + \frac{AR}{8}\cos\left[2\pi(f_c+f_m)t - \pi/3\right] + \frac{AR}{8}\cos\left[2\pi(f_c-f_m)t + \pi/3\right]$$

$$= AR\cos 2\pi f_c t + \frac{AR}{8}\left[2\cos 2\pi f_c t \cos(2\pi f_m t - \pi/3)\right]$$

$$= AR\cos 2\pi f_c t\left[1 + \frac{1}{4}\cos(2\pi f_m t - \pi/3)\right]$$

This is also an AM signal, but whose percentage modulation is 1/4.

Note that the percentage modulation has dropped due to the higher filtering loss/gain of the signal components at (f_c+f_m) and (f_c-f_m) when compared to the filter loss/gain at $f=f_c$.

$$s(t) = A_c\,[1+\mu \cos(2\pi f_m t)] \cos 2\pi f_c t.$$

$$\mu = 2, \; f_c \gg f_m.$$

a) The output of the ideal envelope detector is

$$v(t) = A_c\,|1+\mu\cos(2\pi f_m t)|.$$

Note that $v(t)$ is periodic with period $T = \frac{1}{f_m}$.

$$v(t) = v(t+T).$$

$\therefore$ the Fourier series representation of $v(t)$ is given by

$$v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_m t) + \sum_{n=1}^{\infty} b_n \sin(2\pi n f_m t).$$

$$b_n = (2f_m)\int_{-\frac{1}{2f_m}}^{1/2f_m} v(t)\sin(2\pi n f_m t)\,dt.$$

$$a_n = 2f_m \int_{-1/2f_m}^{1/2f_m} v(t)\cos(2\pi n f_m t)\,dt.$$

Since $V(t) = V(-t)$, $b_n = 0$.

$$a_n = 2 f_m A_c \int_0^{1/f_m} |1 + \mu \cos 2\pi f_m t| \cos(2\pi n f_m t)\, dt$$

Note that

$$1 + \mu \cos 2\pi f_m t > 0$$

for

$$\cos 2\pi f_m t > -1/\mu$$

~~i.e.~~ $\cancel{2\pi f_m t}$

i.e.,

$$\frac{\pi}{2} + \theta < 2\pi f_m t < \frac{3\pi}{2} - \theta$$

where $\sin\theta = 1/\mu$. (assuming $\mu > 1$)

Let $w = 2\pi f_m t$, then $dt = \dfrac{dw}{2\pi f_m}$

$$\therefore a_n = \frac{A_c}{\pi}\left[\int_0^{2\pi} |1 + \mu \cos w| \cos(nw)\, dw\right]$$

$$= \frac{A_c}{\pi}\left[\int_0^{\frac{\pi}{2}+\theta} (1 + \mu\cos w)\cos nw\, dw + \int_{\frac{3\pi}{2}-\theta}^{2\pi} (1 + \mu\cos w)\cos nw\, dw - \int_{\frac{\pi}{2}+\theta}^{\frac{3\pi}{2}-\theta} (1 + \mu\cos w)\cos nw\, dw\right]$$

∴ for $n=1$. (using $\mu=2$, $\theta=\pi/6$)

$$a_1 = 2A_c\left(\frac{1}{3} + \frac{\sqrt{3}}{2\pi}\right).$$

$$a_0 = \frac{A_c}{3} + \frac{4A_c}{\pi}\sin\left(\frac{2\pi}{3}\right)$$

The second harmonic is given by

$$a_2 = \frac{A_c\sqrt{3}}{\pi}.$$

∴ The ratio of the second harmonic amplitude to the fundamental harmonic amplitude in $v(t)$ is

$$\frac{a_2}{a_1} = \frac{\sqrt{3}/\pi}{\frac{2}{3} + \frac{\sqrt{3}}{\pi}} = \frac{\sqrt{3}}{\sqrt{3} + \frac{2\pi}{3}} = \frac{3\sqrt{3}}{3\sqrt{3} + 2\pi}$$

$$= 0.4522.$$

Note that since $|\mu|>1$, we get harmonic distortion. Also note that there is no harmonic distortion if $|\mu|<1$.

3.6.

$$V_2(t) = a_1 V_1(t) + a_2 V_1^2(t).$$

$$V_1(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t.$$

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 A_c k_a m(t) \cos 2\pi f_c t + a_2 A_c^2 (1 + k_a m(t))^2 \cos^2 2\pi f_c t$$

$$= a_1 \overbrace{A_c \cos 2\pi f_c t}^{T_1} + \overbrace{a_1 A_c k_a m(t) \cos 2\pi f_c t}^{T_2} + \frac{a_2 A_c^2}{2} (1 + \cos 4\pi f_c t)(1 + k_a m(t))^2 \quad (1)$$

If $m(t)$ is bandlimited to $[-W, W]$ and $f_c \gg W$ such that.

$f_c - W > 2W$, then we can use a low pass filter with frequency response $H(f) = \begin{cases} 1, & |f| < 2W \\ 0, & \text{otherwise} \end{cases}$

to retrieve $(1 + k_a m(t))^2$, i.e.,

$$V_2(t) \longrightarrow \boxed{h(t)} \longrightarrow \frac{a_2 A_c^2}{2} (1 + k_a m(t))^2$$ (20)

To see this note that (see (1) for T_1, T_2)

$T_1(t) = a_1 A_c \cos 2\pi f_c t$

has $T_1(f) = \frac{a_1 A_c}{2} (\delta(f - f_c) + \delta(f + f_c))$

(i.e., has power at $f = \pm f_c$).

Similarly $T_2(t)$ has power only

in the band $(f_c - W, f_c + W)$

the term $\frac{a_2 A_c^2}{2} \cos(4\pi f_c t) [1 + k_a m(t)]^2$

has power in the band

$(2f_c - 2W, 2f_c + 2W)$.

The useful term

$\frac{a_2 A_c^2}{2} (1 + k_a m(t))^2$ has

power in the band

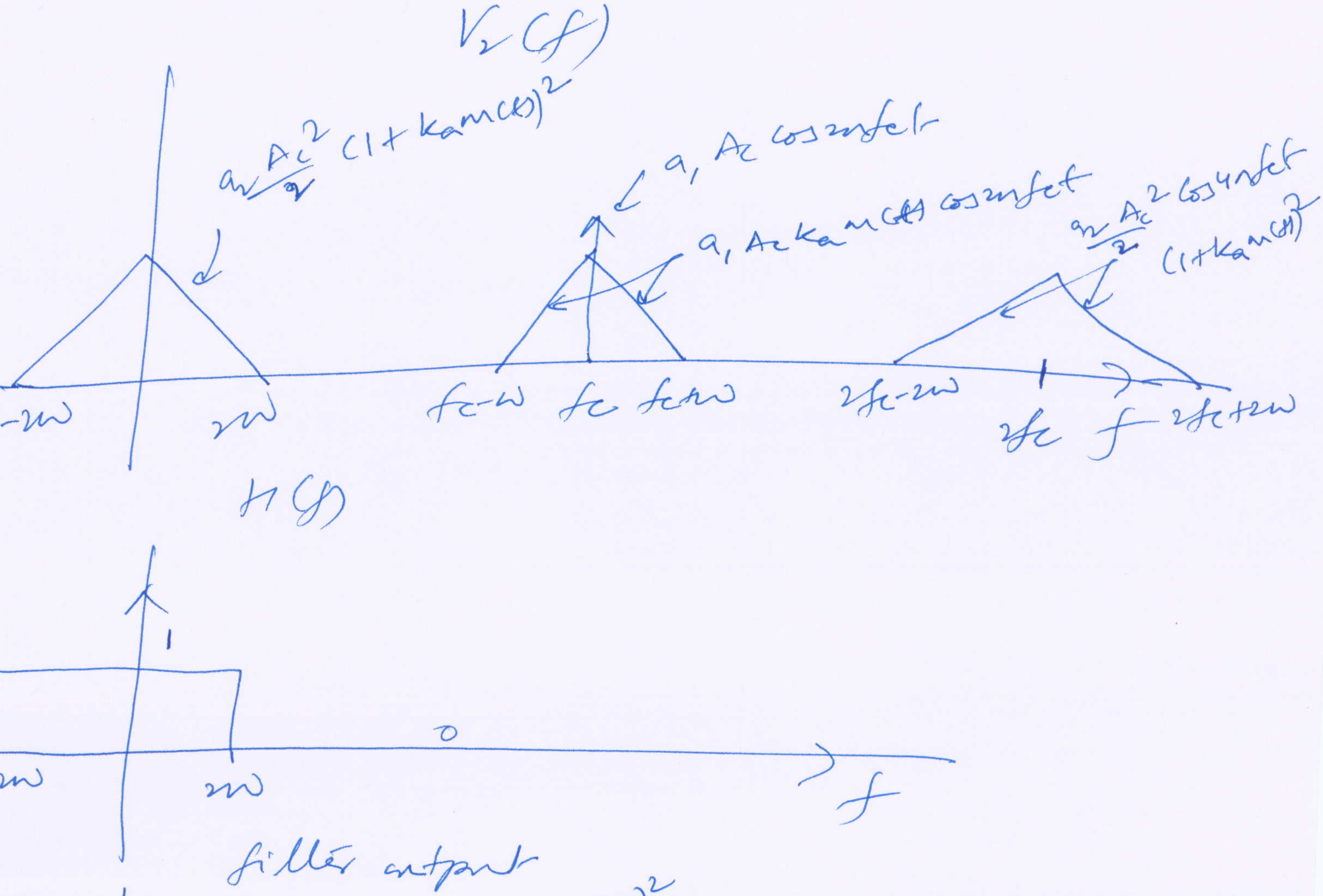
$(-2W, 2W)$.

Therefore by putting a filter with

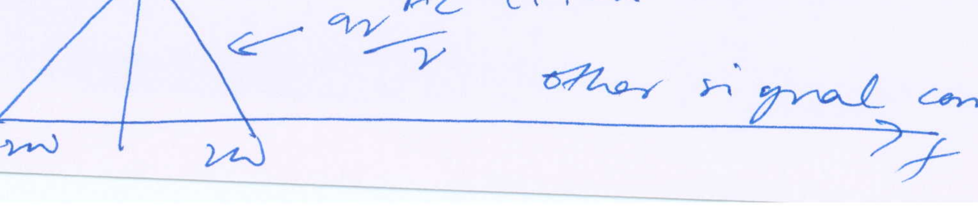
passband $(-2W, 2W)$ and assuming

that the other signal terms

don't overlap with the useful term in the frequency domain (i.e., for this to be satisfied $f_c - W > 2W$, i.e. $f_c > 3W$) we can get $\frac{a_2 A_c^2}{2}(1 + k_a m(t))^2$ at the filter output.



filter output



other signal components in $V_2(f)$ get filtered out.

The next question is how to extract the message signal $m(t)$ from the filter output $\frac{a_2 A_c^2}{2}(1+k_a m(t))^2$.

If $|k_a m(t)| < 1$ for all t, then

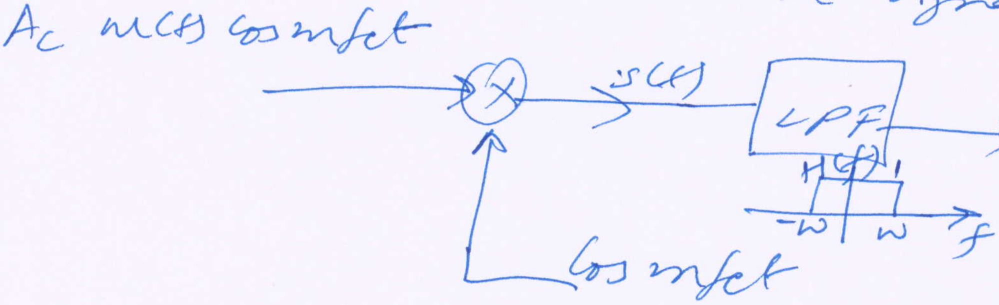
$1 + k_a m(t) > 0$

$\therefore$ a square root circuit will have as its output.

$$\frac{a_2 A_c^2}{2}(1+k_a m(t))^2 \longrightarrow \boxed{\text{SQUARE ROOT}} \longrightarrow \alpha(1+k_a m(t))$$

where α is some constant.

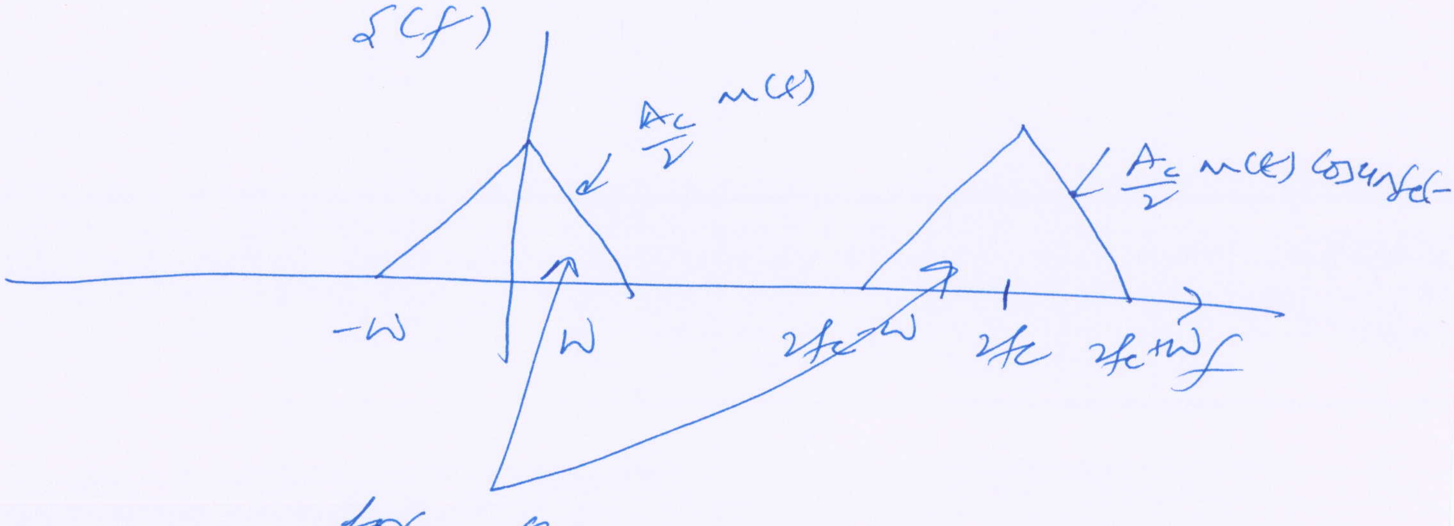
3.8 Assuming perfect synchronization at the receiver, the coherent receiver for the DSB-SC signal is



$$s(t) = A_c\, m(t) \cos^2 2\pi f_c t$$

$$= \frac{A_c\, m(t)}{2} (1 + \cos 4\pi f_c t)$$

$$= \frac{A_c\, m(t)}{2} + \frac{A_c}{2} m(t) \cos 4\pi f_c t$$



for the receiver to detect $m(t)$ ~~some~~ correctly, the bands $[-W, W]$ and $[2f_c - W, 2f_c + W]$ must be disjoint i.e.,

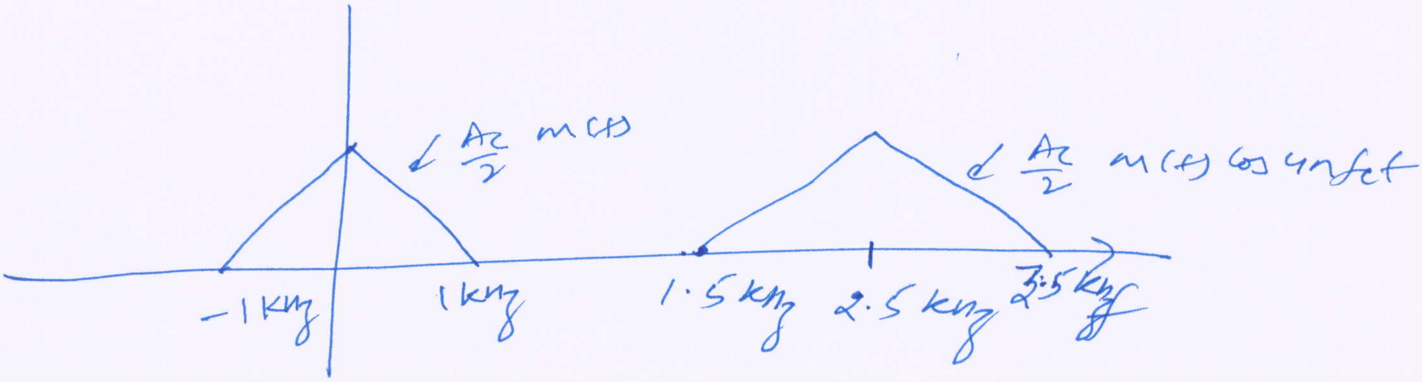
$$2f_c - W > W \quad \text{or} \quad f_c > W.$$

$\therefore$ the lowest frequency f_c which allows correct detection is $f_c = W = 1$ kHz.

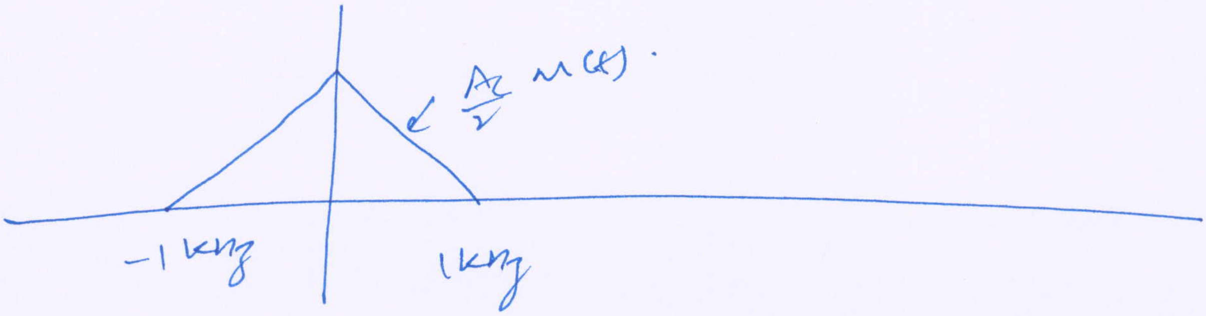
Prob. 3.8 continued.

a) When $f_c = 1.25$ kHz.

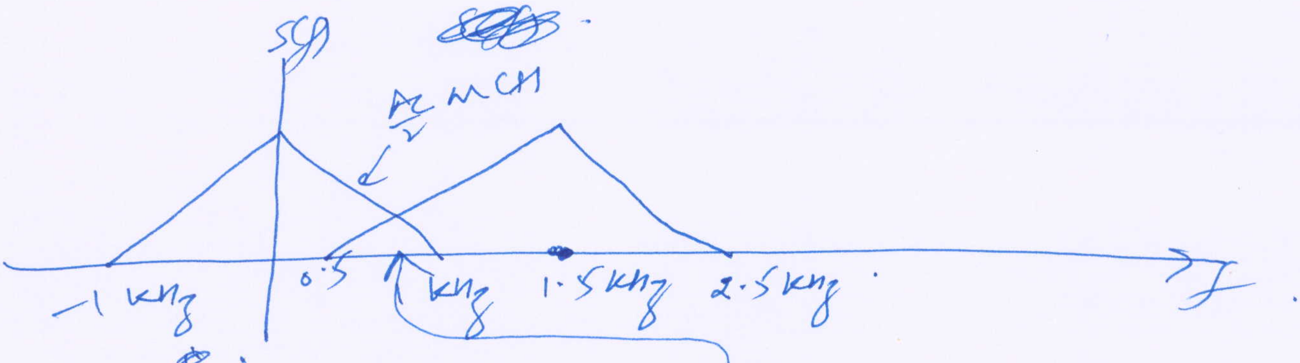
S(f).



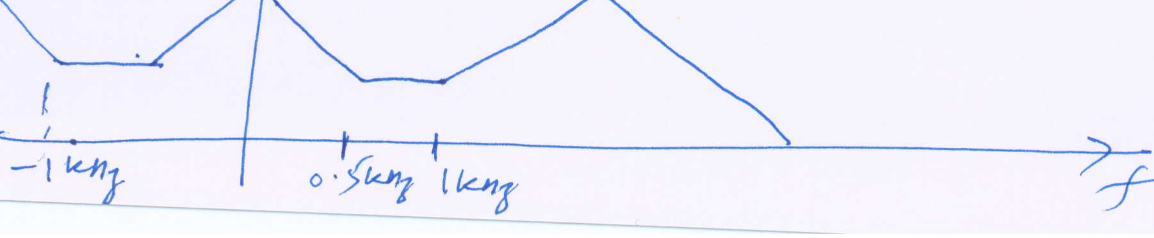
X(f)



When $f_c = 0.75$ kHz we have



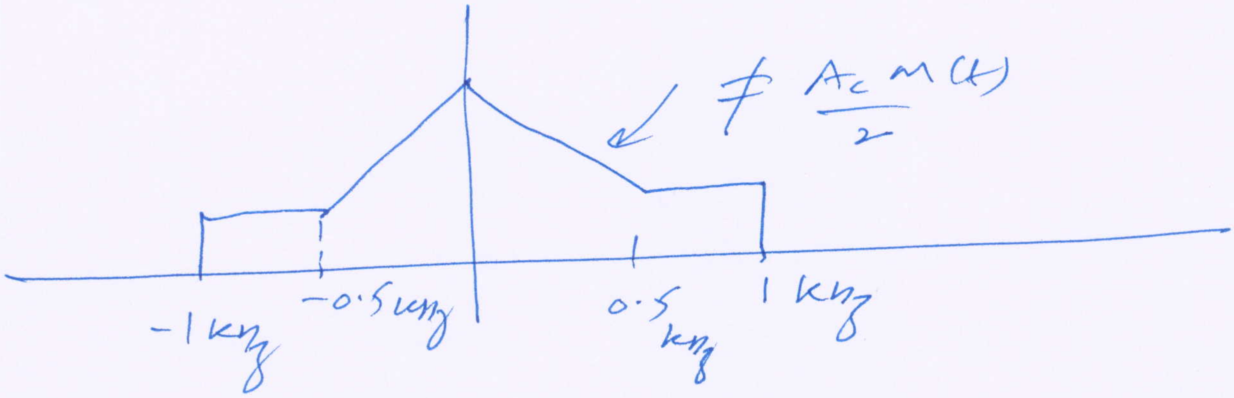
i.e., due to overlap S(f) appears as



and therefore

for $f_c = 0.75$ kHz

$$X(f) = S(f)H(f).$$



which is clearly not the spectrum of $\frac{A_c}{2} m(t)$.

therefore for $f_c = 0.75$ kHz $< W$ the coherent detector is unable to detect the message signal.