Problem 3.11

(a) Multiplying the signal by the local oscillator gives:

$$s_1(t) = A_c m(t)\cos(2\pi f_c t)\cos[2\pi(f_c+\Delta f)t]$$

$$= \frac{A_c}{2}m(t)\{\cos(2\pi\Delta f t) + \cos[\cancel{2\pi 2(f_c+\Delta f)t}]\}$$

$\cos[2\pi(2f_c+\Delta f)t]$

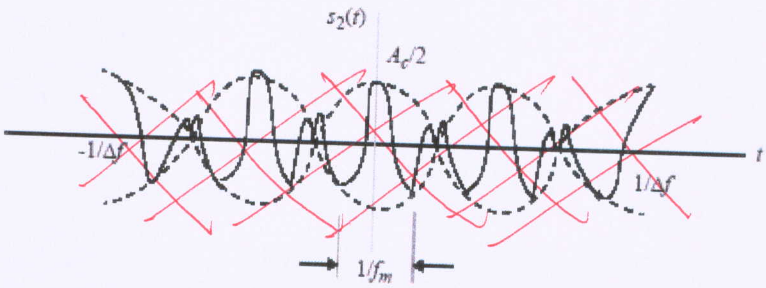
Low pass filtering leaves:

$$s_2(t) = \frac{A_c}{2}m(t)\cos(2\pi\Delta f t)$$

Thus the output signal is the message signal modulated by a sinusoid of frequency Δf.

(b) If $m(t) = \cos(2\pi f_m t)$,

then $s_2(t) = \frac{A_c}{2}\cos(2\pi f_m t)\cos(2\pi\Delta f t)$



See Figure on ~~next~~ page (4).

Problem 3.12

(a) $y(t) = s^2(t)$

$$= A_c^2\cos^2(2\pi f_c t)m^2(t)$$

$$= \frac{A_c^2}{2}[1+\cos(4\pi f_c t)]m^2(t)$$

Therefore, the spectrum of the multiplier output is

$$Y(f) = \frac{A_c^2}{2}\int_{-\infty}^{\infty} M(\lambda)M(f-\lambda)d\lambda + \frac{A_c^2}{4}\left[\int_{-\infty}^{\infty} M(\lambda)M(f-2f_c-\lambda)d\lambda + \int_{-\infty}^{\infty} M(\lambda)M(f+2f_c-\lambda)d\lambda\right]$$

where $M(f) = F[m(t)]$.

Fourier Transform

(Note: this page is taken from the Solutions Manual for "Communication Systems" 5th Ed. by Simon Haykin et. al)

(b) At $f = 2f_c$, we have

$$Y(2f_c) = \frac{A_c^2}{2}\int_{-\infty}^{\infty} M(\lambda)M(2f_c-\lambda)d\lambda$$

$$+\frac{A_c^2}{4}\Big[\int_{-\infty}^{\infty} M(\lambda)M(-\lambda)d\lambda + \int_{-\infty}^{\infty} M(\lambda)M(4f_c-\lambda)d\lambda\Big]$$

Since $M(-\lambda) = M^*(\lambda)$, we may write

$$Y(2f_c) = \frac{A_c^2}{2}\int_{-\infty}^{\infty} M(\lambda)M(2f_c-\lambda)d\lambda$$

$$+\frac{A_c^2}{4}\Big[\int_{-\infty}^{\infty} |M(\lambda)|^2 d\lambda + \int_{-\infty}^{\infty} M(\lambda)M(4f_c-\lambda)d\lambda\Big] \tag{1}$$

With $m(t)$ limited to $-W \le f \le W$ and $f_c > W$, we find that the first and third integrals reduce to zero, and so we may simplify Eq. (1) as follows

$$Y(2f_c) = \frac{A_c^2}{4}\int_{-\infty}^{\infty} |M(\lambda)|^2 d\lambda$$

$$= \frac{A_c^2 E}{4}$$

where E is the signal energy (by Rayleigh's energy theorem). Similarly, we find that

$$Y(-2f_c) = \frac{A_c^2}{4}E$$

The band-pass filter output, in the frequency domain, is therefore defined by

(assuming that the Δf is so small that the spectrum of $y(t)$ is nearly constant (i.e., $\frac{A_c^2}{4}E$) in the passband of the filter [($2f_c - \frac{\Delta f}{2}$, $2f_c + \frac{\Delta f}{2}$) and ($-2f_c - \frac{\Delta f}{2}$, $-2f_c + \frac{\Delta f}{2}$)])

$$\cancel{V(f) \approx \frac{A_c^2}{4}E\Delta f[\delta(f-2f_c)+\delta(f+2f_c)]}$$

~~Hence,~~

$$\cancel{v(t) \approx \frac{A_c^2}{4}E\Delta f\cos(4\pi f_c t)}$$

With the assumption above we get:

$$V(f) = \begin{cases} \frac{A_c^2}{4}E, & |f-2f_c| < \frac{\Delta f}{2} \\ \frac{A_c^2 E}{4}, & |f+2f_c| < \frac{\Delta f}{2} \\ 0, & \text{otherwise.} \end{cases}$$

(Note, this page is taken from the solutions Manual for "Communication systems" 5th ed, Simon Haykin et al)

$$\therefore \quad v(t) = \int_{-\infty}^{\infty} V(f)\, e^{j2\pi f t}\, df$$

$$= \frac{A_c^2 E}{4}\left[\int_{2f_c-\frac{\Delta f}{2}}^{2f_c+\frac{\Delta f}{2}} e^{j2\pi f t}\, df + \int_{-2f_c-\frac{\Delta f}{2}}^{-2f_c+\frac{\Delta f}{2}} e^{j2\pi f t}\, df\right]$$

$$= \frac{A_c^2 E}{2}\left[\int_{2f_c-\frac{\Delta f}{2}}^{2f_c+\frac{\Delta f}{2}} \cos 2\pi f t\, df\right]$$

$$= \frac{A_c^2 E}{2}\left[\frac{\sin 2\pi f t}{2\pi t}\right]_{2f_c-\frac{\Delta f}{2}}^{2f_c+\frac{\Delta f}{2}}$$

$$= \frac{A_c^2 E}{2\pi t} \sin \pi \Delta f t \cos 4\pi f_c t$$

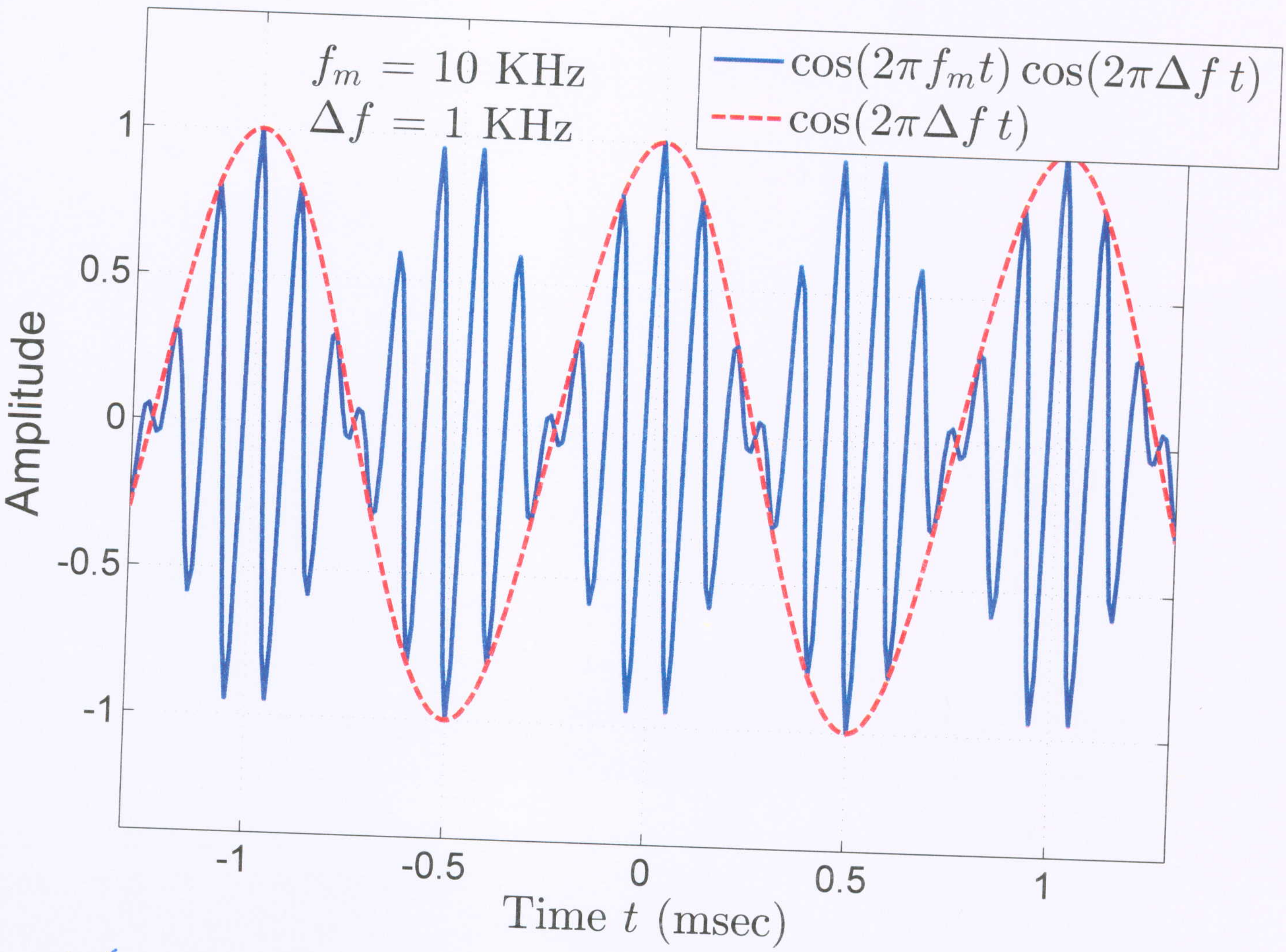
$$= \frac{A_c^2 E \Delta f}{2} \cos 4\pi f_c t \left(\frac{\sin \pi \Delta f t}{\pi \Delta f t}\right)$$

$$\approx \frac{A_c^2 E \Delta f}{2} \cos 4\pi f_c t \quad , \text{ for small } t$$

i.e., $|t| \ll \frac{1}{\Delta f}$.

Prob 3.11 (supplement on beats)

BEATS



When two signals (single tone sine waves) of similar frequencies, say $f_m + \Delta f$ and $f_m - \Delta f$ $(\Delta f \ll f_m)$ are added together, i.e.;

$$s(t) = \cos 2\pi(f_m + \Delta f)t + \cos 2\pi(f_m - \Delta f)t$$

$$= 2\cos 2\pi f_m t \cos 2\pi \Delta f\, t,$$

the envelope (i.e., $\cos 2\pi \Delta f t$) varies with time t and fades to zero every $\frac{1}{2\Delta f}$ seconds.

Prob 3.19. Weaver's method for generating a SSB signal. (5)

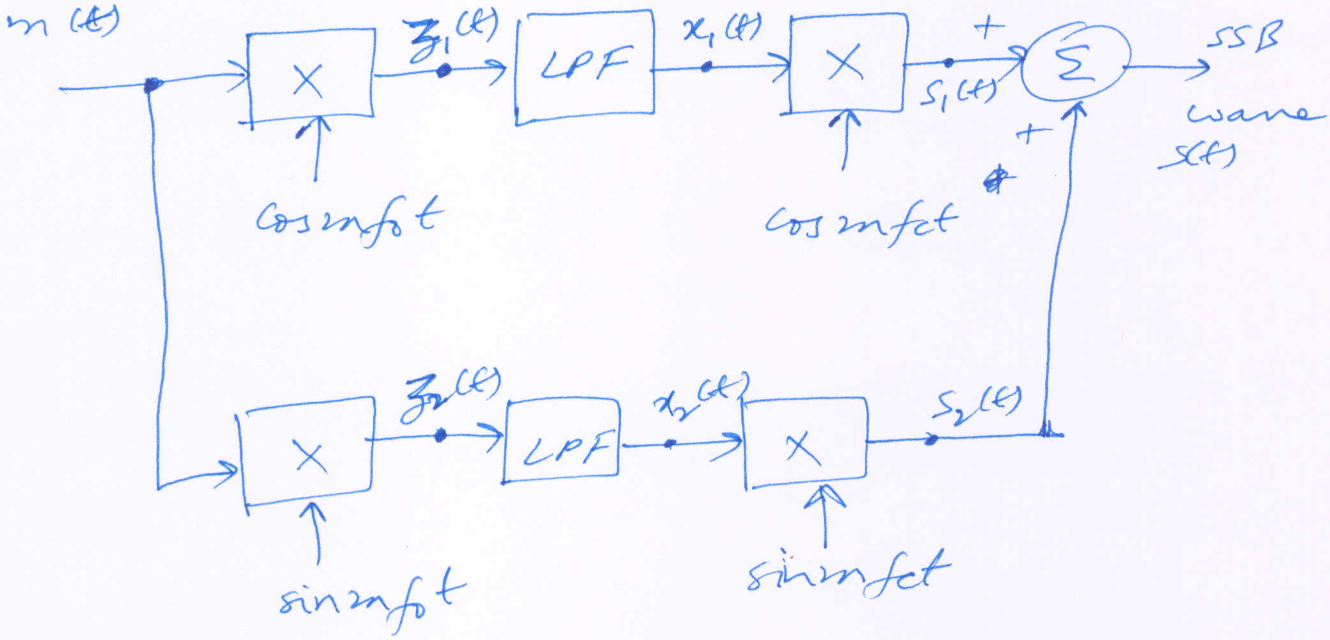
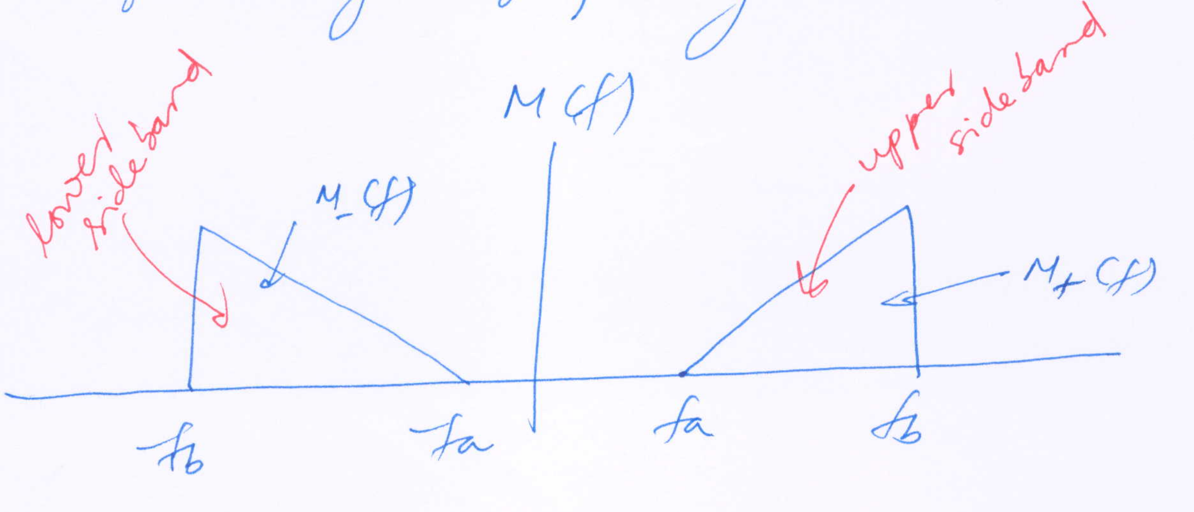


Fig. 2

The message signal m(t) has the following frequency transform.



i.e., it has energy in the band $[f_a, f_b]$.

Let $f_0 \triangleq \dfrac{f_a + f_b}{2}$.

Since m(t) is real

$$M_+(f) = M_-^*(-f) \qquad (1)$$

let

$$M_+(f) \triangleq \begin{cases} M(f), & f \geq 0 \\ 0, & f < 0 \end{cases}, \text{ and}$$

$$M_-(f) \triangleq \begin{cases} M(f), & f \leq 0 \\ 0, & f \geq 0. \end{cases}$$

Looking at Fig. 2 above, we see that

$z_1(t) = m(t) \cos 2\pi f_0 t$ and

$z_2(t) = m(t) \sin 2\pi f_0 t$.

$$\therefore z_1(t) = \frac{m(t)}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)$$

$$= \frac{m(t)e^{j2\pi f_0 t}}{2} + \frac{m(t)e^{-j2\pi f_0 t}}{2}$$

and hence taking the Fourier transform on both sides we get

$$Z_1(f) = \frac{M(f-f_0)}{2} + \frac{M(f+f_0)}{2}. \quad \text{— (2)}$$

similarly $z_2(t) = \dfrac{m(t)}{2j}\left(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}\right)$

$$\therefore Z_2(f) = \frac{M(f-f_0)}{2j} - \frac{M(f+f_0)}{2j} \quad \text{— (3)}$$

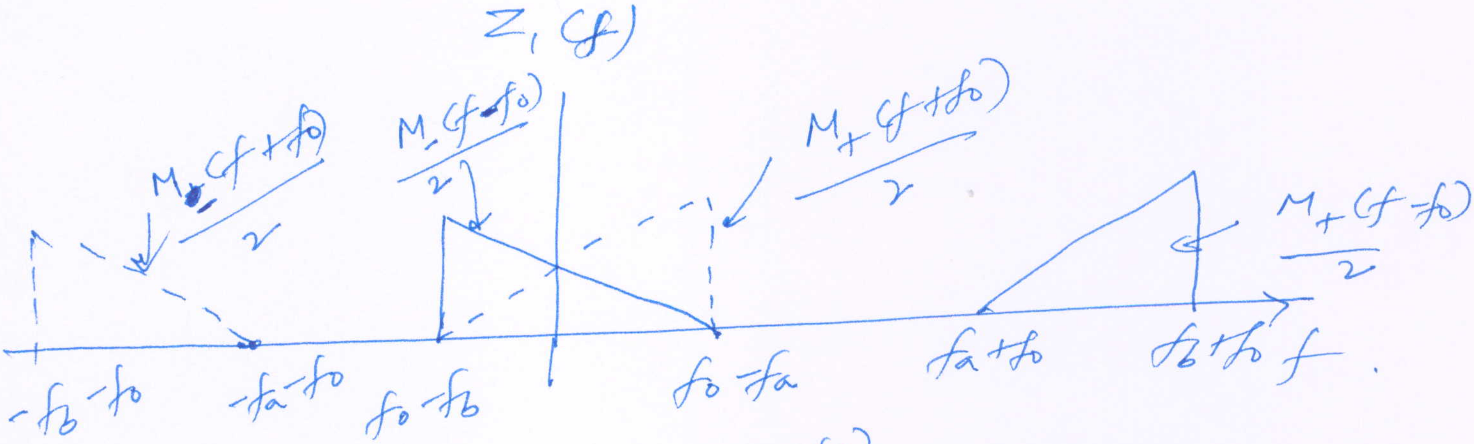


fig 3(a)

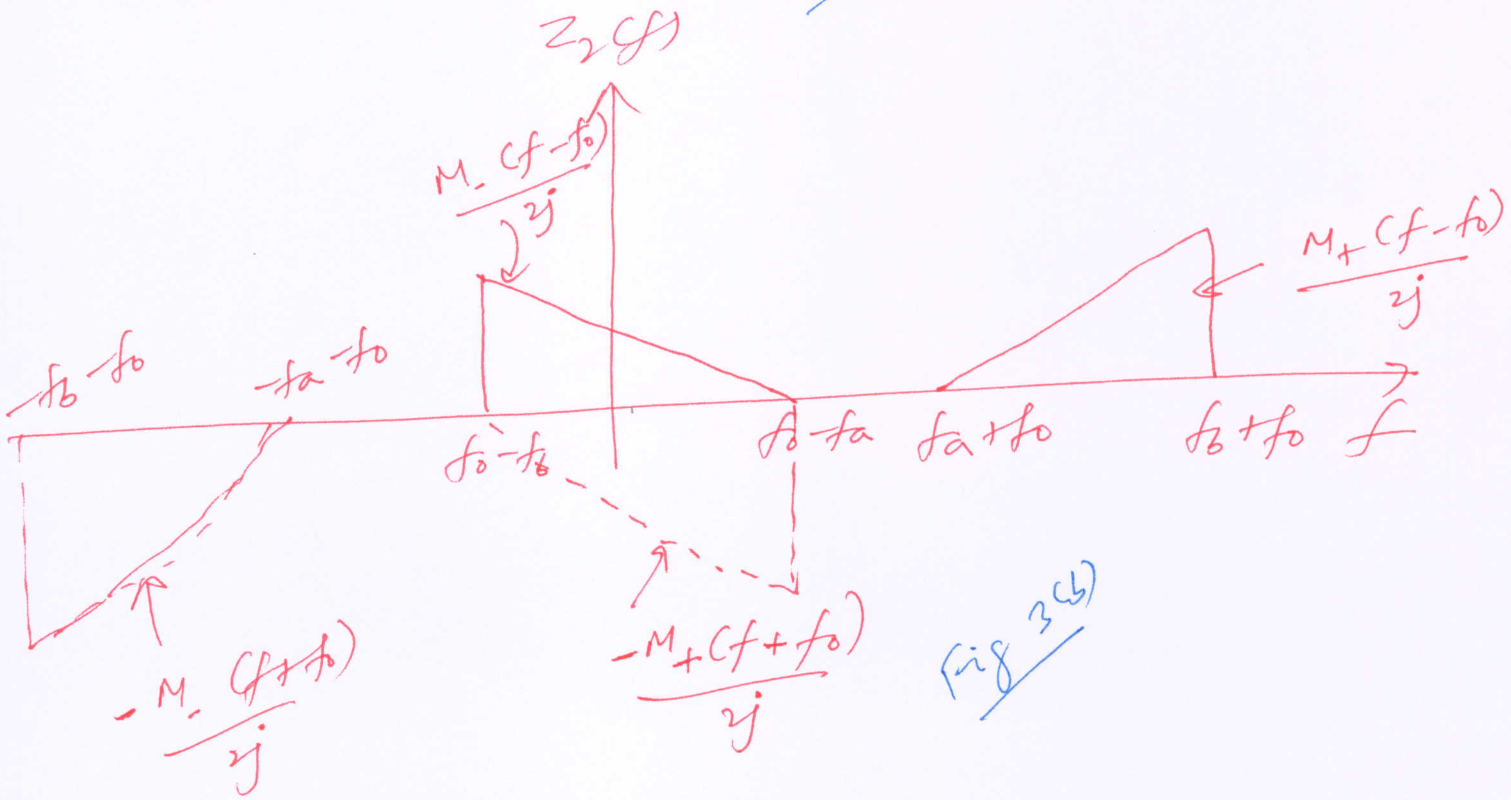


fig 3(b)

The low pass filters (LPF) have a cut-off frequency of $\frac{f_b-f_a}{2} = f_0-f_a = f_b-f_0$.

That is the frequency response of the LPF is

$$H(f) = \begin{cases} 1, & |f| < \frac{f_b-f_a}{2} \\ 0, & \text{otherwise} \end{cases}$$

then from Fig. 3 it is clear that ⑧

$X_1(f)$ and $X_2(f)$ are given by

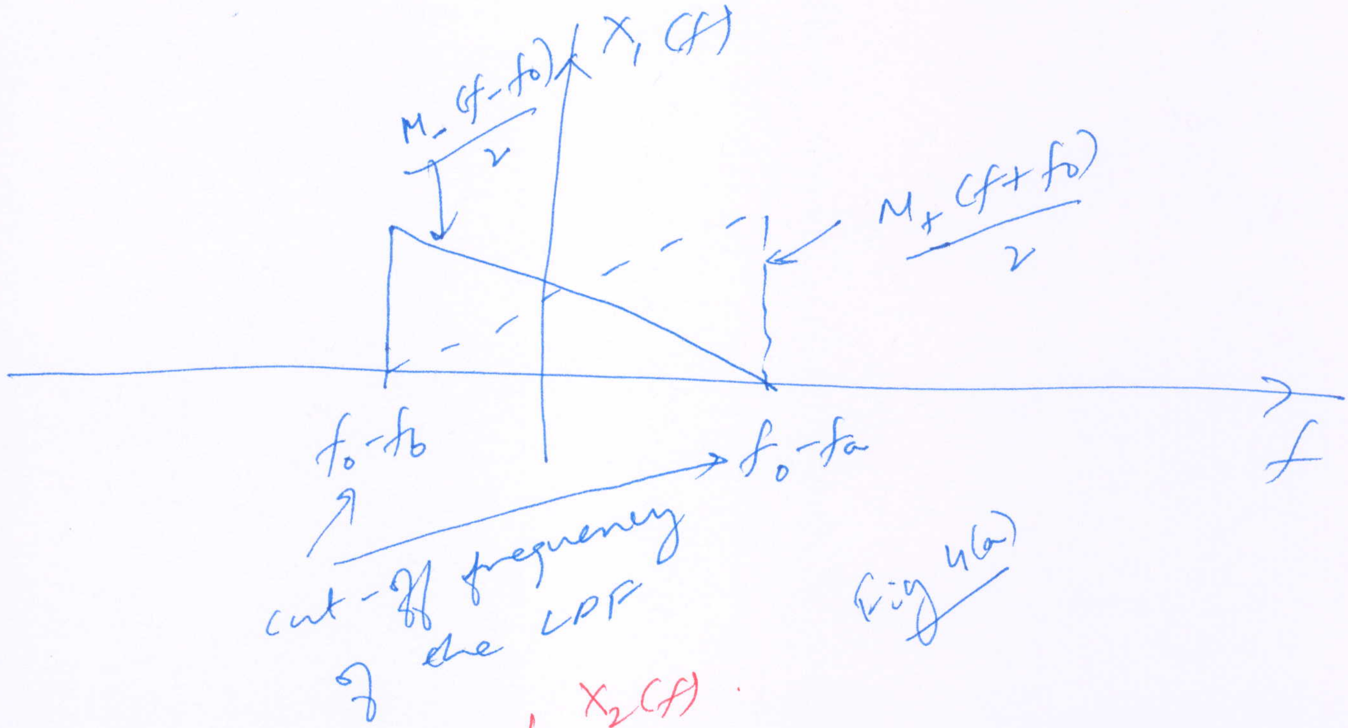


Fig 4(a)

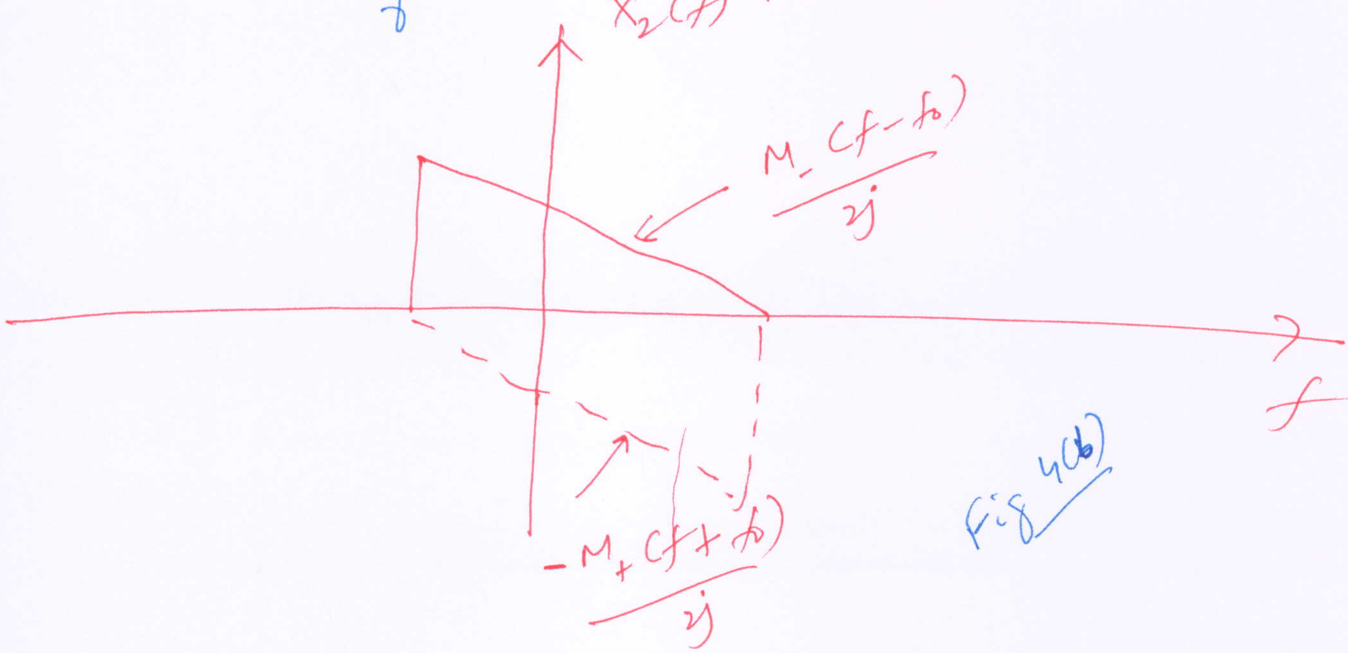


Fig 4(b)

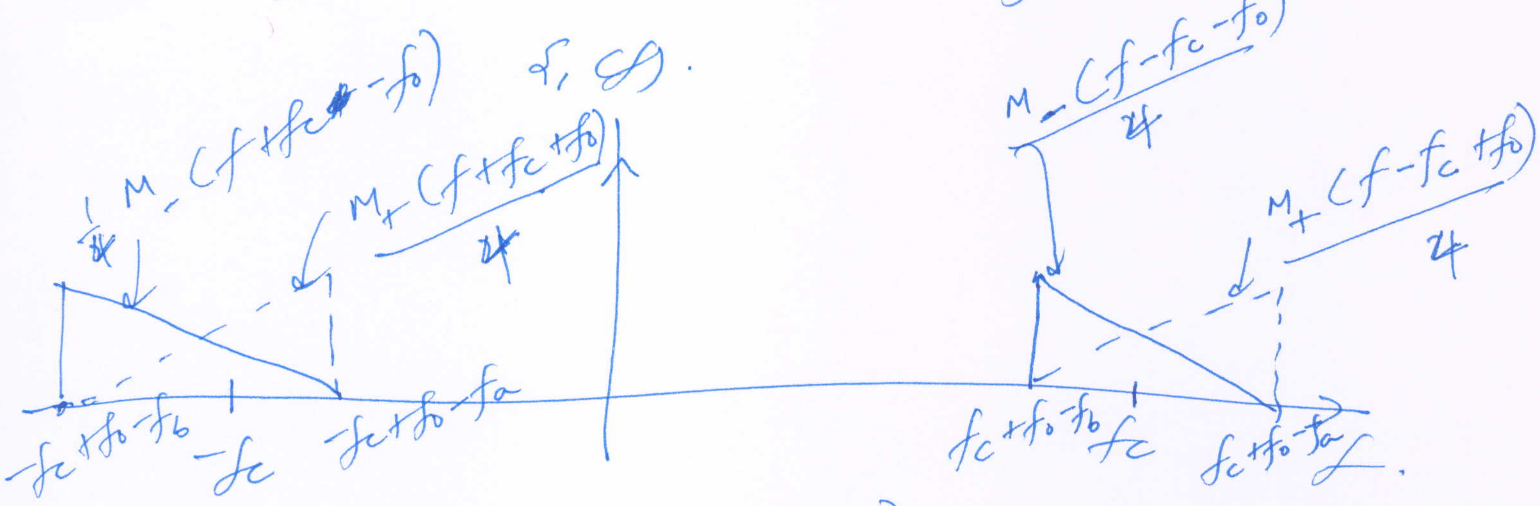
Further $s_1(t) = x_1(t)\cos 2\pi f_c t$.

$$\therefore s_1(t) = \frac{x_1(t)\, e^{j2\pi f_c t}}{2} + \frac{x_1(t)\, e^{-j2\pi f_c t}}{2}$$

$$\therefore S_1(f) = \frac{X_1(f-f_c)}{2} + \frac{X_1(f+f_c)}{2}$$

Similarly we get.

$$S_2(f) = \frac{X_2(f-f_c)}{2j} - \frac{X_2(f+f_c)}{2j}$$



$S_2(f)$. fig 5(a)

$S(f)$.

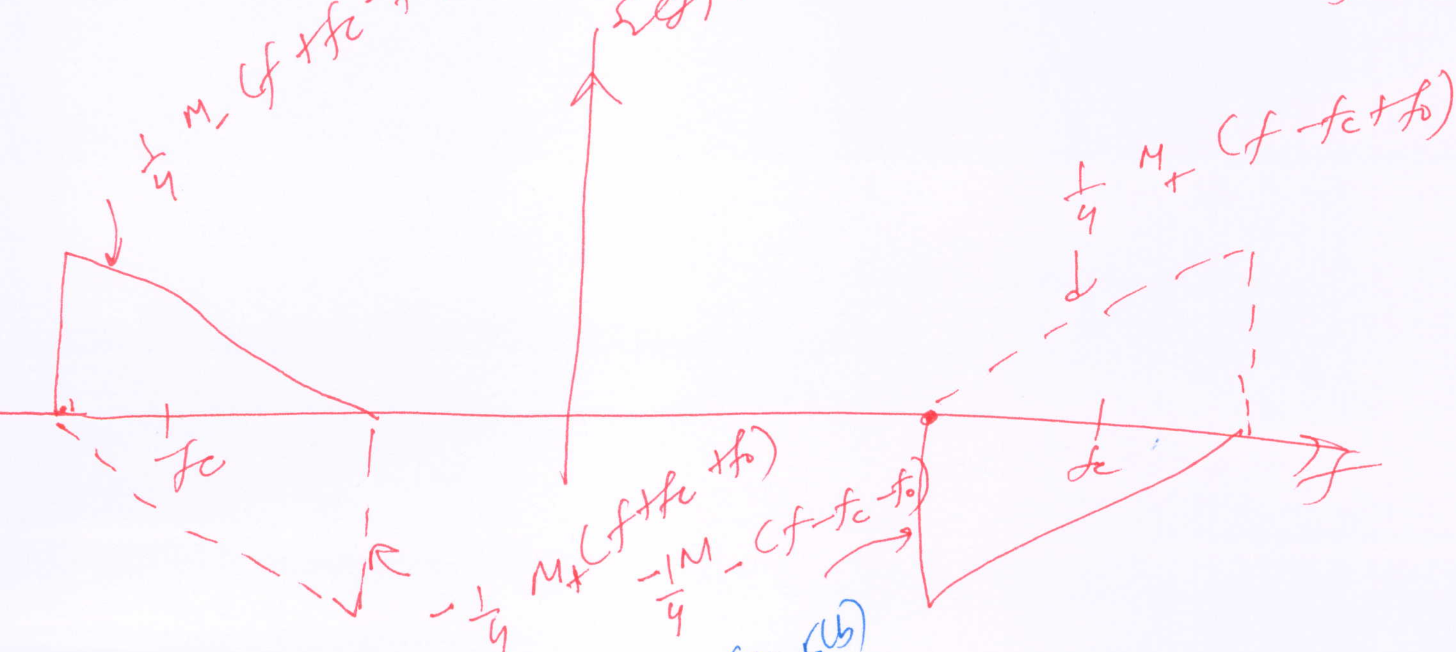


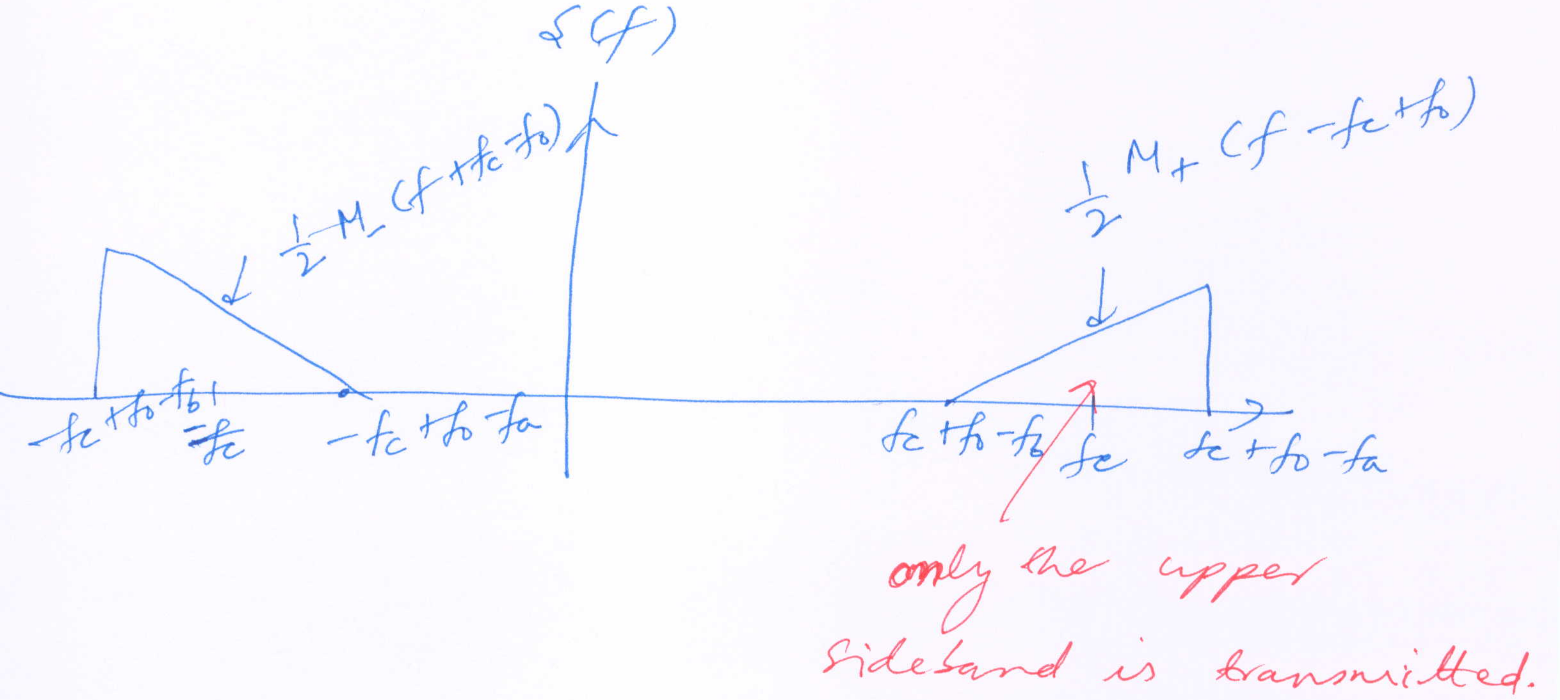
fig 5(b)

Since $s(t) = s_1(t) + s_2(t)$.

we have

$$S(f) = S_1(f) + S_2(f).$$

Therefore ~~adding~~ from Fig 5(a) and Fig 5(b) we finally get.



Note that in the final Σ module, instead of adding $s_1(t)$ and $s_2(t)$, if we subtract $s_2(t)$ from $s_1(t)$, i.e.,

if $s(t) = s_1(t) - s_2(t)$, then

$S(f) = S_1(f) - S_2(f)$ and from Fig 5(a&b) it is clear that only the lower sideband will be transmitted in this case.